



(RESEARCH ARTICLE)



IBM-2 configuration mixing in Mo nuclei

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World Journal of Advanced Engineering Technology and Sciences, 2022, 05(02), 114–122

Publication history: Received on 26 February 2022; revised on 18 April 2022; accepted on 20 April 2022

Article DOI: <https://doi.org/10.30574/wjaets.2022.5.2.0046>

Abstract

The even-even Mo isotopes are studied using an extension of the neutron-proton interacting boson model that accounts for the breaking of shell or sub-shell closures via two-particle-two-hole excitations. The energy levels and electromagnetic transitions are explored. The results back up the idea that these isotopes are interacting between two configurations.

Keywords: Mo isotopes; Boson model; Energy Levels; BM-2 CM Hamiltonian Parameters

1. Introduction

For many years, the structure of nuclei with proton numbers of 38, 40, and 42, as well as neutron numbers larger than 50, has presented a problem to theoretical interpretations. In this region, nuclei exhibit very low first excited 0^+ states connected by strong E2 transitions to the first 2^+ states, as well as a highly rapid change from a vibrational to a rotational-like structure. Sheline *et al.*, [1] sought to explain the energy-level systematics of $_{40}\text{Zr}$ in terms of the interaction of two configurations a few years ago. The spherical minimum of the nuclear potential surface was associated with one configuration of vibrational-like character, whereas the other, of rotational-like character, was connected with a severely deformed secondary minimum expected on the basis of shell-model considerations. However, the Strutinsky method computations of potential energy surfaces did not support this view. Federman and Pittel [2], using the shell model, investigated the rotational structure of Zr isotopes more recently. Due to a strong monopole-monopole interaction between protons and neutrons, the conclusion reached in this scenario is that this rotating structure eventually evolves as neutrons are put on top of the first excited 0^+ states of the lighter isotopes. This contact causes a deformation of this state as well as a significant reduction in its energy. The same authors came to similar conclusions about Mo isotopes using the Hartree-Fock-Bogoliubov approach.

It is far from straightforward to conduct a quantitative and extensive analysis of the coexistence of various communal formations. However, under the framework of the interacting boson model, Duval and Barrett [3] have recently proposed a viable technique to simply attack this problem. We followed Duval and Barrett's instructions and conducted a thorough analysis of the Mo isotopes, the results of which we'd like to provide here. Our conclusion is that mixing effects do indeed dominate the structure of these isotopes, as suggested in refs.[1,2]. Abood *et al.*, [4] studied the mixed symmetry states in ^{92}Zr and ^{94}Mo isotopes within IBM-2, in this work founded a few mixed-symmetry states in these isotopes, such as 2^+_2 , 2^+_3 , 3^+_1 , and 1^+ . These isotopes have a vibrational structure that corresponds to the SU(5) symmetry, according to the results.

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2. The Interacting Boson Model

The fact that ${}^{90}_{40}\text{Zr}_{50}$ behaves like a closed shell nucleus is well known. If this were the case, an interaction boson model calculation of the ${}_{42}\text{Mo}$ isotopes would use $N_{\pi} = 1$ proton boson (one active proton pair above $Z = 40$) and $N_{\pi} = 1, 2, 3, 4$ neutron bosons for ${}^{94}\text{Mo}$, ${}^{96}\text{Mo}$, ${}^{98}\text{Mo}$ and ${}^{100}\text{Mo}$. However, it is commonly recognized that it isn't actually a closed shell, but rather a sub-shell closure. This suggests that excitations of pairs from below to above proton number 40 are easily possible, according to Duval and Barrett [3]. The interacting boson configuration has $N_{\pi} = 3$ if only one pair is stimulated from below to above proton number 40. Both configurations are processed in the mixing method according to the framework of the neutron-proton interacting boson model. We employ the same Hamiltonian as in most computations [4-6].

In the simplest form of the IBM-1, it is assumed that:

- Low-lying collective states in medium and heavy even-even nuclei away from closed shells are dominated by the excitations of the valence protons and the valence neutrons only, while the closed-shell core stays inert.
- The identical valence nucleons are coupled into pairs of angular momenta $J = 0$ (s-boson) or $J = 2$ (d-boson)
- The number of bosons is always half the number of valence nucleons (or holes) counted from the nearest closed shell.
- No distinction is made between protons and neutrons.
- IBM-2 distinguishes between proton and neutron bosons.

Both configurations are processed in the mixing method according to the framework of the neutron-proton interacting boson model. We employ the same Hamiltonian as in most computations [4-6].

The IBM-2 Hamiltonian is given as:

$$H = H_{\pi} + H_{\nu} + V_{\pi\nu} \dots\dots\dots(1)$$

where

$$H = \varepsilon_{\pi} n_{d\pi} + \varepsilon_{\nu} n_{d\nu} + 2\kappa_{\pi\nu} Q_{\pi}^{\chi_{\pi}} Q_{\nu}^{\chi_{\nu}} + V_{\pi\nu} + M_{\pi\nu}(\xi_1, \xi_2, \xi_3) \dots\dots\dots(2)$$

$$V_{\rho\rho} = \sum_{L=0,2,4} \frac{1}{2} C_{L\rho} (2L+1)^{1/2} [(d_{\rho}^{+} d_{\rho}^{+})^{(L)} (d_{\rho}^{-} d_{\rho}^{-})^{(L)}]^{(0)} \dots\dots\dots(3)$$

Additionally, the Majorana operator is used to describe the interaction between protons and neutrons. For mixed symmetry states, this operator induces an overall energy change.

$$M_{\pi\nu}(\xi_1, \xi_2, \xi_3) = \xi_2 [(d_{\pi}^{+} s_{\nu}^{+} - s_{\pi}^{-} d_{\nu}^{-})^{(2)} (d_{\pi}^{-} s_{\nu}^{-} - s_{\pi}^{+} d_{\nu}^{+})^{(2)}]^{(0)} - 2 \sum_{k=1,3} \xi_k [(d_{\nu}^{+} d_{\pi}^{+})^{(k)} (d_{\nu}^{-} d_{\pi}^{-})^{(k)}]^{(0)} \dots\dots(4)$$

$$Q_{\rho} = [(d^{+} \times s^{-})_{\rho}^{(2)} + (s^{+} \times d^{-})_{\rho}^{(2)}] - \chi_{\rho} [d^{+} \times d^{-}]_{\rho}^{(2)} \dots\dots\dots(5)$$

We introduce the mixing Hamiltonian described in ref. [3] to admix the setups.

$$H_{mix} = \alpha (s_{\pi}^{+} s_{\pi}^{+} + s_{\pi}^{-} s_{\pi}^{-})^{(0)} + \beta (d_{\pi}^{+} d_{\pi}^{+} + d_{\pi}^{-} d_{\pi}^{-})^{(0)} \dots\dots\dots(6)$$

There are two steps to the computations. The Hamiltonian (1) is diagonalized for each configuration in the standard basis in the first [6].

$$\left[(d_{\pi}^+)^{n_{d\pi}} (L_{\pi} \psi_{\pi}) (s_{\pi}^+)^{N_{\pi} - n_{d\pi}} (d_{\nu}^+)^{n_{d\nu}} (L_{\nu} \psi_{\nu}) (s_{\nu}^+)^{N_{\nu} - n_{d\nu}} \right] |0\rangle \dots\dots\dots (7)$$

Where ψ_{π} and ψ_{ν} are additional quantum numbers that describe the states. In the second, $H + H_{mix}$ is diagonalized using the first four eigenstates of the $N_{\pi} = 1$ configuration and the first four eigenstates of the $N_{\pi} = 3$ configuration as a basis. In this basis, H is already diagonal. An energy d is added to the energies of the states of the configuration with $N_{\pi} = 3$ in this stage. The energy separation between the two configurations is denoted by Δ , which is designed to decrease practically linearly with the number of neutrons. According to the analysis in ref. [2], this behavior is due to the effects of a monopole-monopole interaction that primarily affects protons in the $1g_{7/2}$ orbit and neutrons in the $1g_{7/2}$ orbit. With increasing neutron number, this interaction becomes more active and promotes protons into the $1g_{7/2}$ orbit, favoring the $N_{\pi} = 3$ configuration.

3. Results and discussion

3.1. Energy Levels

Table 1a IBM-2 Hamiltonian Parameters for normal Configuration $N_{\pi} = 1$, all parameters in MeV units except $\chi_{\nu}(\chi_{\pi})$ are dimensionless

isotopes	⁹⁶ Mo	⁹⁸ Mo	¹⁰⁰ Mo	¹⁰² Mo
ε	1.010	1.599	1.170	0.900
κ	-0.161	-0.150	-0.140	-0.150
χ_{ν}	-0.900	-1.00	-1.10	-0.190
χ_{π}	0.388	0.388	0.388	0.388
$C_{0\nu}$	0.311	0.300	-0.200	-0.120
$C_{2\nu}$	0.120	0.080	0.00	-0.05
ξ_2	0.13	0.13	0.13	0.13
$\xi_1 = \xi_3$	0.033	0.033	0.033	0.033

$$C_{4\nu} = 0.0 \text{ MeV}, C_{0\pi} = C_{2\pi} = C_{4\pi} = 0.0 \text{ MeV}$$

Table (1) lists all of the parameters that appear in Eqns. (1) and (6). These parameters match those found in prior computations in this mass range [5]. The quantity $\varepsilon = \varepsilon_{\pi} = \varepsilon_{\nu}$, which has a maximum at neutron number 56, is the only slightly anomalous behavior. The strange behavior is due to the occurrence of another subshell closure at 56, which is produced by the $d_{3/2}$ neutron orbital being filled. We also notice that, because only a few states of the measured spectra can usually be attributed to the configuration with $N_{\pi} = 1$, certain of the configuration's properties are not clearly defined. In the following section, we'll go over this topic in further depth finally for all isotopes the mixing parameters, α and β , have been kept equal and constant.

Table 1b IBM-2 CM Hamiltonian Parameters for intruder configuration $N_\pi = 3$, all Parameters in MeV units except χ_ν (χ_π) are dimensionless

isotopes	⁹⁶ Mo	⁹⁸ Mo	¹⁰⁰ Mo	¹⁰² Mo
ε	0.561	0.770	0.660	0610
κ	-0.161	-0.150	-0.140	-0.150
χ_ν	-0.900	-1.00	-1.10	-0.190
χ_π	0.388	0.388	0.388	0.388
$C_{0\nu}$	0.311	0.300	-0.200	-0.120
$C_{2\nu}$	0.120	0.080	0.00	-0.05
ξ_2	0.00	0.00	0.00	0.00
$\xi_1 = \xi_3$	-0.086	-0.086	-0.086	-0.086
α	0.161	0.161	0.161	0.161
β	0.161	0.161	0.161	0.161
Δ	1.330	0.700	0520	-0.110

$$C_{4\nu} = 0.0 \text{ MeV}, C_{0\pi} = C_{2\pi} = C_{4\pi} = 0.0 \text{ MeV}$$

In Figure (1), we compare the calculated and experimental energy levels. It looks that the deal is satisfactory. We discover weak mixing for ⁹⁶Mo, strong mixing for ⁹⁸Mo and to ¹⁰⁰Mo, and weak mixing for ¹⁰²Mo, although the roles of the configurations $N_\pi = 1$ and $N_\pi = 3$ have been swapped. That is, the configuration $N_\pi = 3$ has the lowest energy for the heavier isotopes. The scenarios seen in ⁹⁸Mo and ¹⁰⁰Mo, where the mixing is strongest, are the most interesting. To demonstrate the effects of mixing, we present the spectra obtained without mixing ($\alpha = \beta = 0$) and with mixing ($\alpha = \beta = 161$) in the cases of ⁹⁸Mo and ¹⁰⁰Mo, respectively, in figures. (2) and (3), and compare them to the experiment. The spectra of the two configurations are simply superimposed on each other in the absence of mixing. The mixing changes the order of the states, which has a significant impact on the 0_2^+ state. The most striking feature of these spectra is that, as we predicted in the previous section, due to the larger spacing.

Spacing of levels in the configuration $N_\pi = 1$ and the fact that the "unperturbed" ground states of both configurations are relatively close in energy, only a few of the low-lying levels belong to the configuration $N_\pi = 1$. In particular, the greatest component in the 2_2^+ and 4_1^+ states in ⁹⁶Mo belongs to the configuration $N_\pi = 3$, and this nucleus is the only one where the 2_1^+ is anticipated to belong to the configuration $N_\pi = 1$. Furthermore, the level structure of ⁹⁸Mo and ¹⁰⁰Mo is very similar, with two levels, 0_2^+ and 2_1^+ , which are relatively close in energy, and a higher pair of levels, 2_2^+ and 4_1^+ , which are similarly somewhat close in energy but separated from the other levels. The main difference between the two scenarios is that although the 0_2^+ is largely of configuration $N_\pi = 3$ in the lightest isotope, it is mostly of configuration $N_\pi = 1$ in the heaviest.

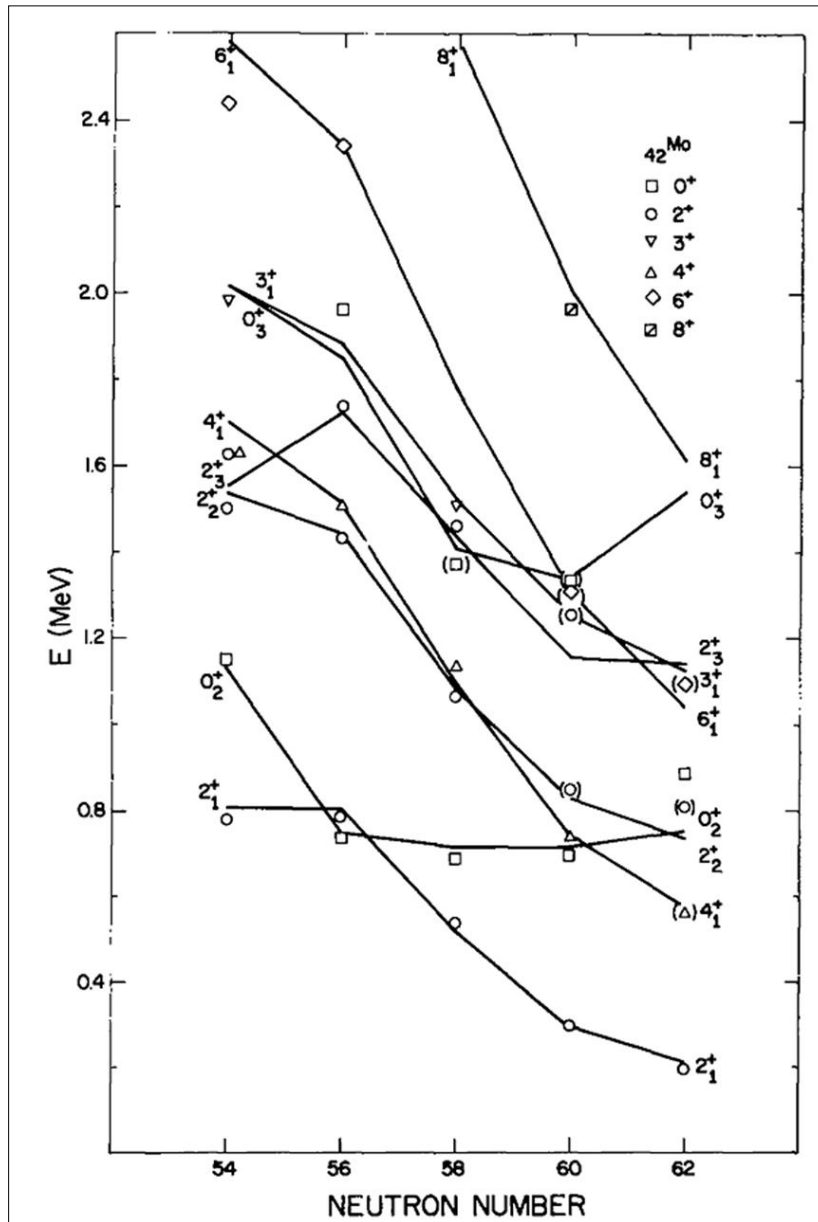


Figure 1 Comparison between experimental data [7,8,9,10,11] and IBM-2 calculated energy levels for Mo isotope

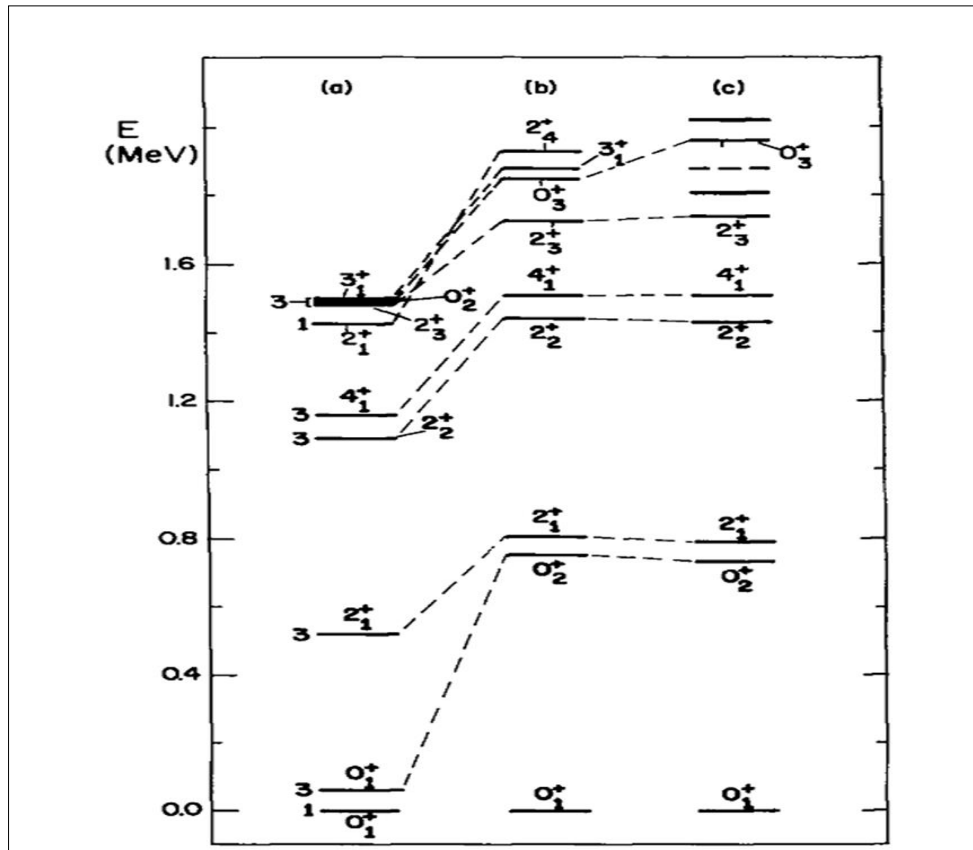


Figure 2 A spectrum of ^{98}Mo with mixing effects. The spectrum produced without mixing is displayed in column (a), mixing is introduced in column (b), and the experimental spectrum is given for comparison in column (c). The configurations ($N_\pi = 1$ and $N_\pi = 3$) to which the levels belong are labeled by the numerals 1 and 3 on the left side of column (a). The experimental levels come from ref [8]

3.2 Electric Transition Probability

In the IBM-2, we also assume the normal form of the electromagnetic transition operators. This is the case for E2 transitions:

$$T^{(E2)} = e_\pi Q_\pi + e_\nu Q_\nu \dots\dots\dots (8)$$

where e_π (e_ν) is the effective charge of the boson E2 and Q_π (Q_ν) is determined by Eq. (5). For the sake of simplicity, we'll choose $e_\pi = e_\nu = e$. The calculation is done by computing the matrix elements in the mixed eigenstates of the E2 operator individually for the two configurations and then combining them together. The value of the effective charges to be used for the configurations $N_\pi = 1$ and $N_\pi = 3$ is an important question. These effective charges are denoted by the letters $e^{(1)}$ and $e^{(3)}$, respectively. The calculated values of various E2 matrix members are quite sensitive to the $e^{(3)} / e^{(1)}$ ratio, especially in the mixing zone.

We discovered that $e^{(3)} / e^{(1)} = 2$ is the optimal ratio for describing the data.

We estimated the $B(E2)$ values for the transitions by keeping this ratio constant for all isotopes $2_1^+ \rightarrow 0_1^+$, $4_1^+ \rightarrow 2_1^+$, $2_2^+ \rightarrow 2_1^+$, $2_3^+ \rightarrow 2_1^+$ and $2_2^+ \rightarrow 0_1^+$. We estimated the effective charge $e^{(1)} = 0.055 e.b$ from the equation [12]:

$$B(E2;2_1^+ \rightarrow 0_1^+) = \frac{5(e_\pi N_\pi + e_\nu N_\nu)^2}{N} \dots\dots\dots(9)$$

Where $B(E2;2_1^+ \rightarrow 0_1^+)$ experimental value, N is the total number of bosons for simplicity $e_\pi = e_\nu = e$, the great value at $N = 56$ is recreated in the former instance, and the sharp climb from $N = 56$ to $N = 62$ (nearly four times) be likewise well reproduced in the later case, but there are discrepancies around neutron number 54. The nature of the states involved can provide a qualitative explanation for the large value of Table (2) at $N = 56$.

Configuration $N_\pi = 3$ and the 0_1^+ is mostly of the configuration $N_\pi = 1$, the transition $B(E2;2_1^+ \rightarrow 0_1^+)$ is rather hindered, while the transition $B(E2;2_2^+ \rightarrow 0_1^+)$ is still large. The quadrupole instances show good agreement as well except for the black square at.

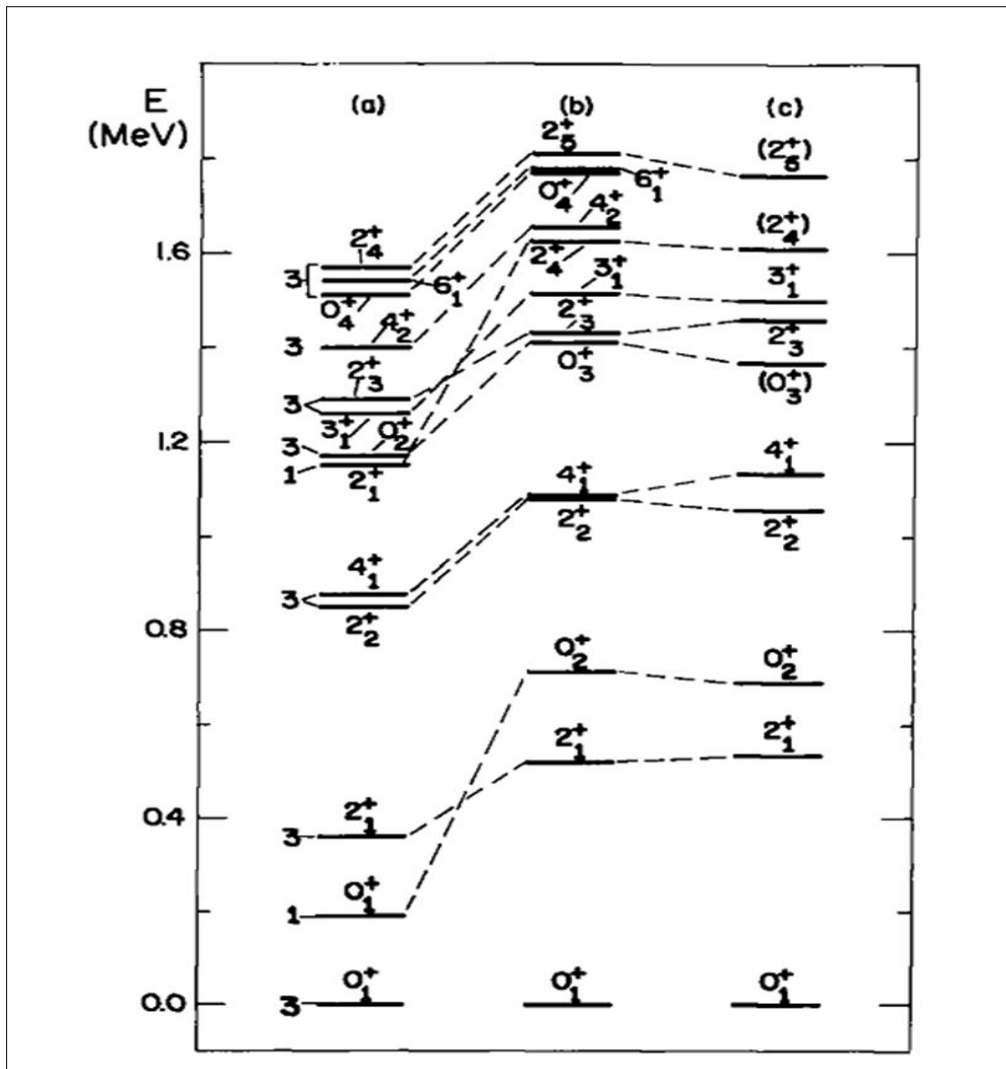


Figure 3 Example of mixing effects in the spectrum of ^{100}Mo . Columns (a), (b), (c) and labels 1 and 3 have the same meaning as in fig.(2) . Experimental levels are taken from ref. [9]

$N = 58$, all of the dots in Table (2) correspond to the same measurement but pertain to separate interference terms. These calculations choose those that correspond to the positive sign of the interference term. These calculations choose those that correspond to the positive sign of the interference term, which is consistent with the experimental data. It is important to note that all of the figures provided are approximate. The results above can be calculated using only two parameters, $e^{(1)}$ and $e^{(3)}$.

Table 2 Comparison between experimental and IBM-2 calculated $B(E2)$ values for Mo isotopes in e^2b^2 units

$J_i^+ \rightarrow J_f^+$	⁹⁶ Mo		⁹⁸ Mo	
	Exp. [7]	IBM-2	Exp. [8]	IBM-2
$2_1^+ \rightarrow 0_1^+$	0.05405(7:8)	0.0552	0.05368(10.7)	0.0552
$4_1^+ \rightarrow 2_1^+$	0.10444(29)	0.1123	0.12346(134)	0.1261
$2_2^+ \rightarrow 2_1^+$	0.0470(78)	0.0501	0.0161(134)	0.0170
$2_3^+ \rightarrow 2_1^+$	-	0.00431	0.11809(107)	0.120
$2_2^+ \rightarrow 0_1^+$	0.00311(2.6)	0.00331	0.011(1)	0.0013
$J_i^+ \rightarrow J_f^+$	¹⁰⁰ Mo		¹⁰² Mo	
	Exp. [9]	IBM-2	Exp. [10]	IBM-2
$2_1^+ \rightarrow 0_1^+$	0.0937(55)	0.0980	0.2094(255)	0.2088
$4_1^+ \rightarrow 2_1^+$	0.1902(110)	0.2110	0.2519(510)	0.2520
$2_2^+ \rightarrow 2_1^+$	0.1406(137.8)	0.1511	-	0.1550
$2_3^+ \rightarrow 2_1^+$	0.00309(22)	0.0030	-	0.0035
$2_2^+ \rightarrow 0_1^+$	0.00171(1.4)	0.0018	-	0.0020

The electric quadrupole moment in IBM-2 is calculated as follows:

$$Q_J = \left[\frac{16\pi}{5} \right]^{1/2} \begin{bmatrix} J & 2 & J \\ -J & 0 & J \end{bmatrix} \langle J_J || T^{(E2)} || J_J \rangle \dots\dots (10)$$

We employed the IBM-2 wave-function and electric transition operator in Eq.(8) to achieve the results in Table (3). The quadrupole moment definition for the nucleus in a state is described by the angular momentum, which is given in Eq.(10). As the number of neutrons increases, these values become more negative (increasing neutron bosons). In the first excited states, the negative sign indicates that these isotopes have an oblate form feature.

Table 3 Quadrupole moments for first excited states $Q(2_1^+)$ in eb units for Mo isotopes

$Q(2_1^+)$	⁹⁴ Mo	⁹⁶ Mo	⁹⁸ Mo	¹⁰⁰ Mo	¹⁰² Mo
Exp.	-0.13(8)	-0.20(8)	-0.26(9)	-0.42(9)	-
IBM-2	-0.131	-0.22	-0.281	-0.452	-0.551

Experimental data are taken from refs. [7,8,9,13]

4. Conclusion

We have provided a configuration-mixing calculation for Mo isotopes and demonstrated that it may be used to obtain a fair description of the known energy spectra and electromagnetic E2 transition rates. These calculations can be expanded to include other features, such as the intensities of two-nucleon and a-transfer processes, as well as isomer and isotope shifts, for which experimental data is currently being collected. The results of this study significantly

support the idea that these isotopes are interacting between two configurations. The proton subshell closures at proton number 40 are assumed to be the source of the majority of configuration mixing in these computations. However, it's probable that the mixed configurations have a more complex structure related to subshell closures in both the proton and neutron levels at 40 and 56 and 64. Indeed, as recently proposed by Iachello and Jackson [6] it's possible that the excited configurations involve α -clustering states. Only microscopic computations can provide a definitive judgment concerning the exact origin of these arrangements. However, we believe that the mixing of configurations, not the specific microscopic structure of the configurations being mixed, is what causes the major effects of spectra and electromagnetic transition rates.

Compliance with ethical standards

Acknowledgments

The author wishes to thank Prof. Dr. Saad A. Hamada for inspiring the subject of this paper and for many interesting discussions.

Disclosure of conflict of interest

No conflict of interest.

References

- [1] R. K. Sheline, I. Regnarsson, S. G Nilsson. Nilsson-Struinsky model of very high spin states Phys. Lett. 1972; 41B: 115.
- [2] P. Federman, S. Pittel. Unified Shell Model Description of Nuclear Deformation Phys. Rev. 1979; C20: 820.
- [3] P. D. Duval, B. R. Barrat Quantitative Description of Configuration Mixing in the Interacting Boson Model Phys. Lett. 1981; 100B: 223.
- [4] G. Puddu, O. Scholten, T. Otsuka Collective Quadrupole States of Xe, Ba and Ce in the Interacting Boson Model Nucl. Phys. 1980; A348: 109.
- [5] P. van Isacker, G. Puddu The Ru and Pd isotopes in the proton-neutron interacting boson model Nucl. Phys. 1980; A348: 125.
- [6] R. Bijker, A. E. L Dieperink, O. Scholten Description of the Pt and Os Isotopes in the Interacting Boson Model Nucl. Phys. 1980; A344: 207.
- [7] D. Abriola, A. A Sonzogni Nuclear Data Sheets 2008; 109: 2558.
- [8] Balraj Singh and Zhiqiang Hu Nuclear Data Sheets 2003; 98: 384.
- [9] Balraj Singh, Zhiqiang Hu Nuclear Data Sheets: 2008; 109: 340.
- [10] D. De Frenne Nuclear Data Sheets 2009; 110: 1774.
- [11] Jean Blachot Nuclear Data Sheets 2007; 108: 2045.
- [12] P. van Isacker, K. Heyde, J. Jolie, A. Sevrin The F-spin symmetric limits of the neutron-proton Interacting Boson Model Ann. Phys. 1986; 171: 253-296.
- [13] D. Abriola, A. A Sonzogni Nuclear Data Sheets 2006; 107: 2493.