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Mechanisms kinematic analysis by geometric and computational approach

Carlos Sergio Pivetta *, Fernando De Azevedo Silva and José Elias Tomazini

Department of Mechanical Engineering, FEG / UNESP - Faculty of Engineering of Guaratinguetá / UNESP - Universidade Estadual Paulista, Brazil.

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Abstract

The kinematics of a four-bar mechanism and others extern elements using one developed method is investigated. The basic four bar mechanism is coupled with a sliding element in a straight line and another sliding element guided by a straight line coupled to a rotating connection. The positions, speeds and accelerations are investigated in the full possible cycle. The PC and the Excel spreadsheet are used for the kinematic solution and simulation of the mechanical system. The kinematic analysis is performed considering specified the initial and end angular positions of the input bar, with an established angular increment. The method used allowed to verify the kinematic of the mechanism with precision and also the working simulation and the visualization, making it a viable alternative and allowing the information of the results.

Keywords: Four Bars Mechanisms; Articulated Mechanisms; Kinematic of Mechanisms; Limits of Mechanisms

1. Introduction

The kinematic analysis of articulated mechanisms developed in this work refers to the determination of speeds and accelerations of points of interest of the elements of an articulated mechanical system and the characteristics of its existing movements are determined. The solution procedure is performed considering motor link movement for the starting and ending position angles chosen with an established increment.

The objectives of this work are to present and apply an alternative computational method for kinematic analysis of articulated mechanisms. The computational methodology adopted is presented and used in this work and allows to illustrate the results and simulations performed, and a comparative analysis with other method is also elaborated.

The procedures, calculations and illustrations are performed using MS EXCEL which has some interesting characteristics, such as: analysis and simulation are accessible, intuitive, enables iterative learning and immediate responses and contributes to the rapid visualization and simulation of movements and the results.

2. Development

The kinematic analysis of mechanisms is generally carried out assuming that the bars are ideal rigid bodies and in several cases ignoring the friction and effects of gaps in pin joints, also without considering the mass of the elements, the dynamic conditions and the small variations due of the necessary dimensional tolerances of the elements.

* Corresponding author: Carlos Sergio Pivetta
Department of Mechanical Engineering, FEG / UNESP - Faculty of Engineering of Guaratinguetá / UNESP - Universidade Estadual Paulista, Brazil.

One iterative method for the analysis of position, the speed and the acceleration of 4-bar and 5-bar mechanisms of the crank-rocker and double-crank types was developed. Using small increments to the "θ_2" angle causing a disturbance of proximity to the previous position, they calculated the position of a generic "P" point of the intermediate bar. In this method the authors used the partial differentiation of the equations of the position and speed, Taylor series expansion and the Newton-Raphson method. For a small variation "$\Delta\theta_2$" to the entry angle "θ_2", the displacement of all points will last a small time "Δ_t", which will tend to zero [1].

Kinematic analysis can be done by graphic, analytical, differential equation systems, vector and matrix methods, numerical, and the relative kinematic. The speed and the acceleration of the interest generic point "P" of the mechanism can be determined using the Eqs. (1)-(2), respectively [2,3].

$$vP = \lim_{\Delta t \to 0} \left(\frac{\Delta RP}{\Delta t}\right) \qquad \text{(equation 1)}$$

$$aP = \lim_{\Delta t \to 0} \left(\frac{\Delta vP}{\Delta t}\right) \qquad \text{(equation 2)}$$

The mechanism of Fig. 1 is used in this work for the analysis aiming to explore additional resources and the results.

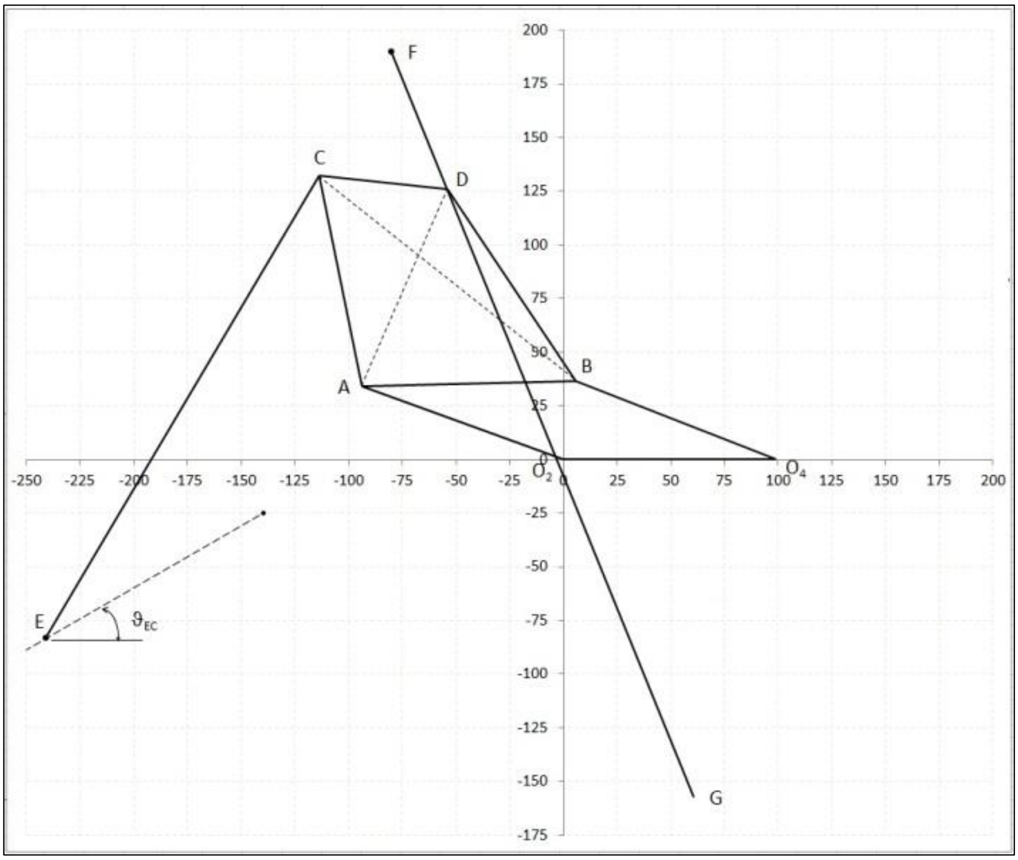

Figure 1 Mechanism (Authors, 2022)

The data of the mechanism of Fig.1 used are: $R_1=O_2O_4= 99$ mm; $R_2=O_2A= 100$ mm; $R_3=BA= 100$ mm; $R_4=O_4B= 100$ mm; $R_5=CA= 100$ mm; $R_6=DA= 100$ mm; EC= 250 mm; xF= -80 mm; yF= 190 mm; GF= 375 mm; $\theta_2= 160^0$; $\theta_{CA} = 100^0$; $\theta_{DA} = 65^0$; $\vartheta_{EC} = 210^0$, B= 56 mm; angular speed $\omega_2 =10$ rad/s cc and angular acceleration $\alpha_2 = 0$ rad/s^2.

The mechanisms in motion are subject at the angular and linear accelerations that generate forces of inertia and torques, according to Newton's second law. In this way, even without external forces or torques applied to the bars, the inertia forces cause reactive forces in the joints [4,5].

The Graphic method and the procedure of kinematic analysis of a 4-bar mechanism based on vector equations were used, considering as a reference the position angle of the entry bar ("$R_2 = \theta_2$") and the nominal lengths of the bars. The graphic vector or differential methods are generally applied when the angular dimensions, velocity and acceleration of the entry bar are known [4,6].

When an output variable of a mechanism reaches a limit position, it is very likely to momentarily have a reversal movement reaching zero speed and the deceleration tends to increase a lot [6]. The general number of critical four bars mechanisms points, in synthesis problems, were analyzed and in which precision and trajectory points combined and result in a finite number of solutions, aiming at reaching desired positions according to points and the orientations established [7].

Problems of possible singularities existing in 4-bar mechanisms were evaluated in order to have solutions for the synthesis and it was found that the number of trajectories changes when there is a critical point or in an extreme position [8].

Several desired points were specified in order to obtain a 4-bar plane mechanism and a coupling point in the intermediate bar with a coupling point on the intermediate bar using trajectory synthesis to perform exact or mixed movements using kinematic mapping [9].

Several studies of the design conditions of crank-to-rocker bar mechanisms with rotational and sliding joints, establishing alternative solutions with adjustable topologies and making the necessary corrections for the optimization of the solution in order to obtain more suitable transmission conditions [10].

The translational conformities ware evaluated using mechanisms in series having one joint of revolution as the reference point, being one link, and three additional joints each, chosen as revolution or prismatic joints, constituting several alternative configurations.

Several alternatives was established for assembling the rigid bodies, the types of links and the location of the relative sliding bar were chosen arbitrarily. Combinations of three joints were established depending of the analysis to be performed and the configuration requirements [11].

Eqs. (1)-(2) define that if the time "Δ_t" tends to zero value, the average speeds and accelerations will tend to the values of the respective snapshots. The lower the value of "Δ_t", the speeds and accelerations in the "x" and "y" directions and the resulting ones will have more accurate values.

For kinematic analysis it is interesting to use the "O_2A" motor bar as an input element for controlling the movements of the system and for the operating conditions of all elements of the mechanism in all desirable and possible positions.

The average velocities and accelerations of a generic point "P" in the "x" and "y" directions of the Cartesian plane and their respective results, in the trajectory interval, can be determined using Eqs. (1)-(2) when replacing the increment of radius "Δ_{RP}" by the relative displacements "Δ_S" in the "x" and "y" directions. The value of "Δ_t" can be determined by Eq. (3), in "s" when using "$\Delta\theta_2$" in degrees and "ω_2" in rad/s.

$$\Delta_t = (2\,\pi\,\Delta\theta_2)/(360\,\omega_2) \qquad \text{(equation 3)}$$

The angular speeds "ω_3" and "ω_4" and the angular accelerations "α_3" and "α_4" can also be obtained in a similar way by replacing "Δ_{RP}" with the angular displacements "$\Delta\theta$" of "θ_3" and "θ_4" or the angular speed variations "$\Delta\omega$" of "ω_3" and "ω_4".

The kinematic conditions established for the mechanism in Fig. 1 and using the angular positions "θ_2" of the input bar "$R_2 = O_2A$" and the angular velocity "ω_2". The "$R_2 = O_2A$" bar is the input element for kinematic control. Thus, in this case it is very interesting to use the "$R_2 = O_2A$" bar for controls and positions measurements. The mechanism coordinates points A, B, C, D, E and G, according of the position angle "θ_2", can be determined by Eq. (4)-(11).

$$xA\,,\,yA = (R_2\ \cos\theta_2\,,\,R_2\ \sin\theta_2) \qquad \text{(equation 4)}$$

$$xB\,,\,yB = (R_1 + R_4\ \cos\theta_4\,,\,R_4\ \sin\theta_4) \qquad \text{(equation 5)}$$

$$xC\,,\,yC = (R_2 \cos\theta_2 + CA \cos(\theta_3 + \theta_{CA}),\ R_2\ \sin\theta_2 + CA \sin(\theta_3 + \theta_{CA})) \qquad \text{(equation 6)}$$

$$xD , yD = (R_2 \cos \theta_2 + DA \cos(\theta_3 + \theta_{DA}), R_2 \sin \theta_2 + DA \sin(\theta_3 + \theta_{DA})) \quad \text{(equation 7)}$$

The E point coordinates can be determined by the Eqs. (8)-(13) and there are two possibilities for E point, and it will be depending on the existing geometric conditions. If the "ϑ_{EC}" <= 90^0 or "ϑ_{EC}" >= 270^0 the Eq. (12) will be used, or if the angle "ϑ_{EC}" >= 90^0 or "ϑ_{EC}" =< 270^0, Eq. (13) must be used.

$$a = 1 + tg^2\, \vartheta_{EC} \quad \text{(equation 8)}$$

$$b = -2\ xC + 2\ tg\, \vartheta_{EC}\ B - 2\ tg\, \vartheta_{EC}\ yC \quad \text{(equation 9)}$$

$$c = B^2 - 2\ yC\ B + xC^2 + yC^2 - EC^2 \quad \text{(equation 10)}$$

$$\Delta = b^2 - 4\ a\ c \quad \text{(equation 11)}$$

$$xE_1 , yE_1 = (-b + \sqrt{\Delta}) / (2\ a)\ ,\ \ tg\, \vartheta_{EC}\ \ xE_1 + B \quad \text{(equation 12)}$$

$$xE_2 , yE_2 = (-b - \sqrt{\Delta}) / (2\ a)\ ,\ \ tg\, \vartheta_{EC}\ \ xE_2 + B \quad \text{(equation 13)}$$

After determining the positions of each point of interest for each established kinematic condition, as a function of time, and also the neighboring positions considering the position angle "θ_2", "$\theta_2 - \Delta\theta_2$" and "$\theta_2 + \Delta\theta_2$", the positions, velocities and accelerations of each point and the accelerations of each bar can be determined, by using the Eqs. (1) to (13), obtaining the kinematic analysis of the mechanism.

3. Results

Theoretical nominal dimensional conditions for the mechanism were established, being rigid, without looseness in the joints and without presents external forces.

According to Eq. (1) to (13) and for each angular position "θ_2" using the kinematic analysis by the proposed method it was possible to obtain the complete results.

Fig. 2 illustrates the possible trajectories of A, B, C, D and E points for the mechanism complete cycle in function to angle "θ_2". Fig. 3 illustrates the results of velocities of the points B, C, D, E, G and sliding point DF.

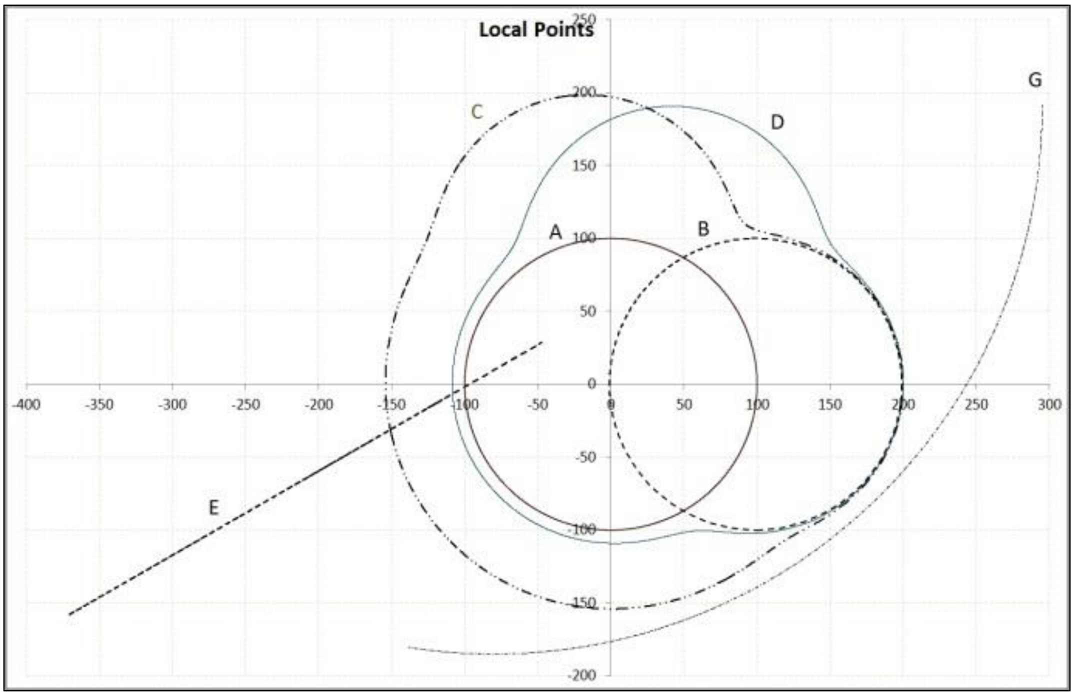

Figure 2 Local positions of the points A, B, C, D, E, F and G as a function of "θ_2" from 0^0 to 360^0 (Authors, 2022)

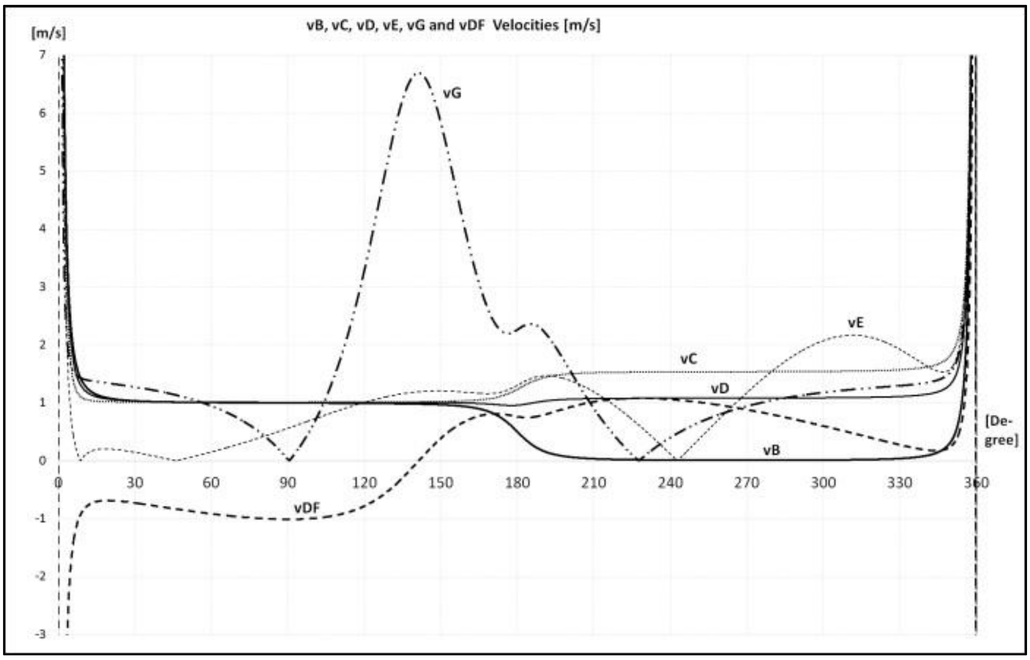

Figure 3 Velocities of the points B, C, D, E, G and sliding DF as a function of "θ_2" from 0^0 to 360^0 for $\Delta\theta_2$ = 0,05 degree (Authors, 2022)

In order to obtain the kinematic analysis of the mechanism, the speeds and the accelerations of the chosen points and the angular speeds and angular accelerations of each bar which were determined for each angular position "θ_2" of the "O_2A" entry bar and for from 0^0 to 360^0 counterclockwise direction, and also all the nominal dimensions and the nominal geometrics conditions were established.

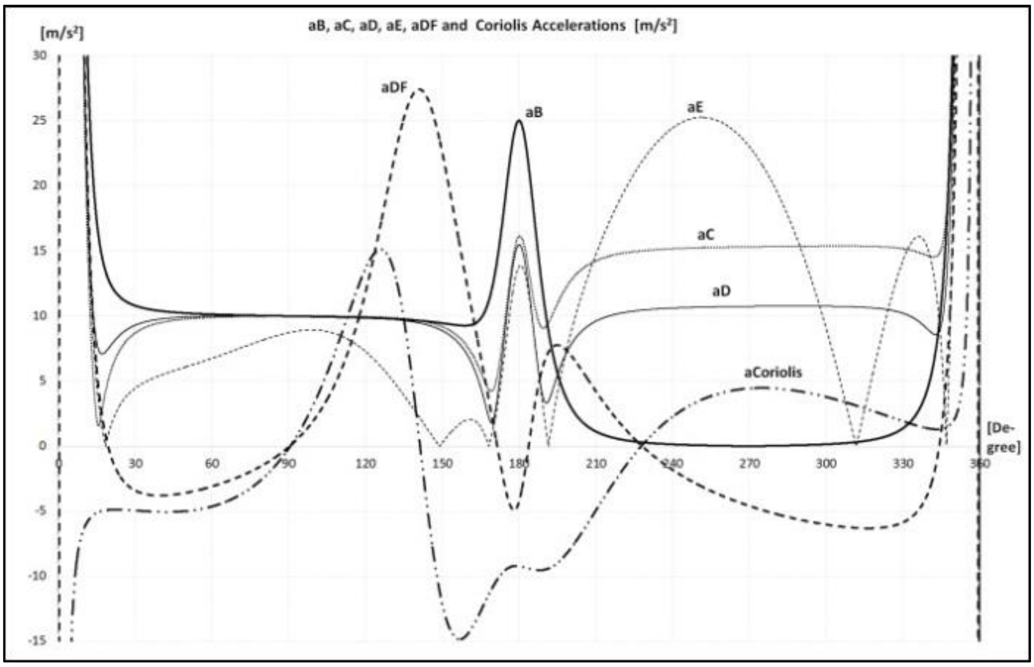

Figure 4 Accelerations of the points B, C, D, E, G sliding DF and Coriolis as a function of "θ_2" from 0^0 to 360^0 (Authors, 2022)

Considering the existing kinematic conditions, the speeds and the accelerations of the chosen points and the respective speeds and angular accelerations of each bar, which were determined for each angular position "θ_2" of the entry bar, for from 0^0 to 360^0 counterclockwise direction. Fig. 4 illustrates the results of the accelerations of the B, C, D, E, G points, D sliding GF point and the Coriolis acceleration as a function the O_2A bar angular position "θ_2" from $\theta_2 = 0^0$ to $\theta_2 = 360^0$ degrees for $\Delta\theta_2$ = 0,05 degree, with the Δt = 0,0000872 seconds (Authors, 2022).

Fig. 5 illustrates the results of the angular velocities "ω_3", "ω_4", "ω_{EC}" e "ω_{DF}" of the "BA", "O_4B", "EC" and "DF" bars as a function the "O_2A" bar angular position "θ_2" from $\theta_2 = 0^0$ to $\theta_2 = 360^0$ for $\Delta\theta_2$ = 0,05 degree (Authors, 2022).

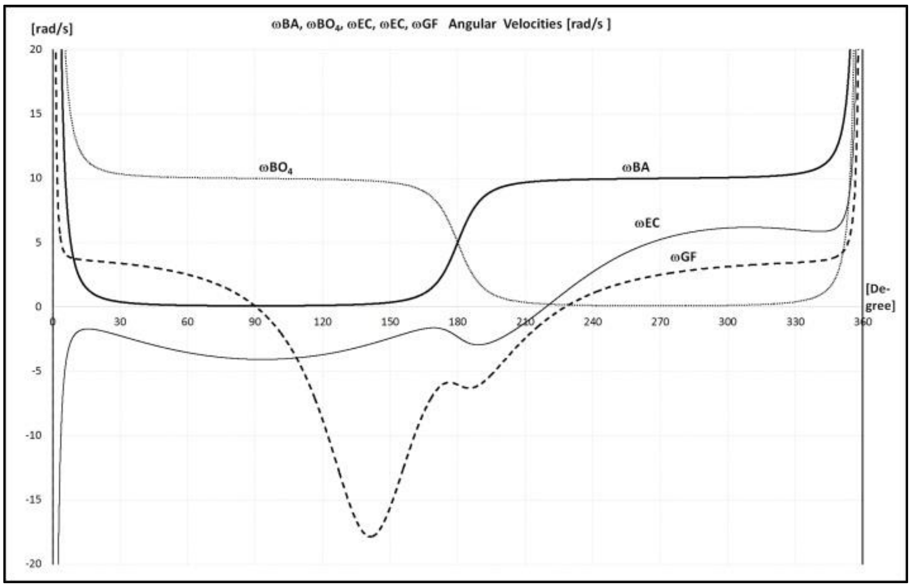

Figure 5 Angular velocities "ω_3", "ω_4", "ωEC" and "ω_{DF}" of the "DF" sliding bar as function of "θ_2" from 0^0 to 360^0 (Authors, 2022)

Fig. 6 illustrates the results of the angular accelerations "α_3", "α_4", "α_{EC}" e "α_{DF}" of the "BA", "O_4B", "EC" and "DF" bars as a function the "O_2A" bar angular position "θ_2" from "θ_2" = 0^0 to "θ_2" = 360^0 (Authors, 2022).

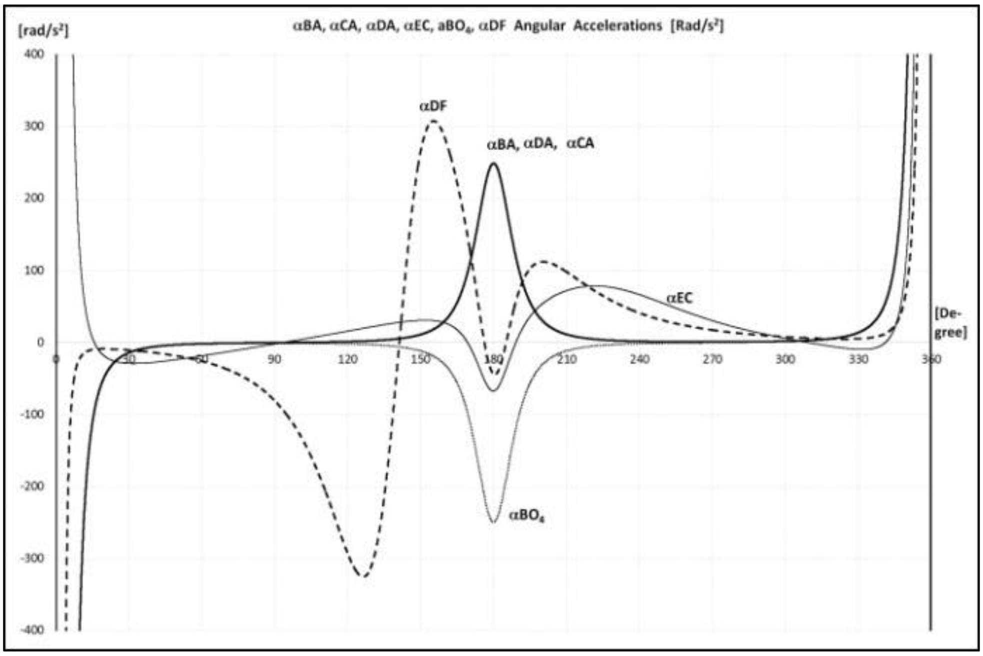

Figure 6 Angular accelerations "α_3", "α_4", "α_{EC}" and "α_{DF}" sliding bar as function of "θ_2" from 0^0 to 360^0 (Authors, 2022)

By observing the results of the velocities of the points, the angular velocities of the bars (Fig. 5), the accelerations of the points and the angular accelerations of the bars of the mechanism (Fig. 6), is possible it will be there critical points for the mechanic system running.it is possible to observe several regions of movements that will could be critical for the mechanical system.

Thus, is probably will be necessary one adequate revision, if desired. The mechanism must to work according to specified and also desired. In order to avoid possible problem not accepted for the design, probably will be necessary actions and revisions of mechanism design will be necessary. Making an evaluation of the possible critical phases of mechanism and observing the Fig. [3-4-5-6] it could to see the regions of "θ_2" that close to zero, it is possible to observe the kinematic behavior of the mechanism. Fig. 7 illustrates the results of the velocities of the B, C, D, E, G points and DF, D point sliding on straight path DF as function the motor bar position angle "θ_2" between -2^0 to 2^0 degrees.

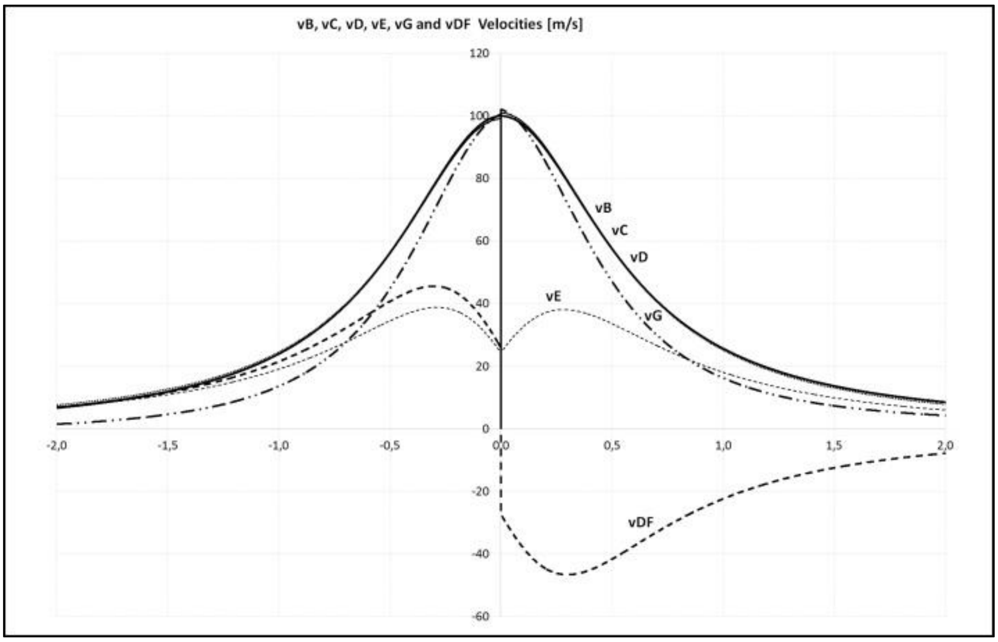

Figure 7 Velocities of the points B, C, D, E, G and sliding DF as a function of "θ_2" = -2^0 to "θ_2" = 2^0 (Authors, 2022)

Fig. 8 illustrates the results of the accelerations of points B, C, D, E, G and DF, point of slippage of point D in relation to F, as a function of the position angle of the motor bar "θ_2" between -2^0 to 2^0 degrees.

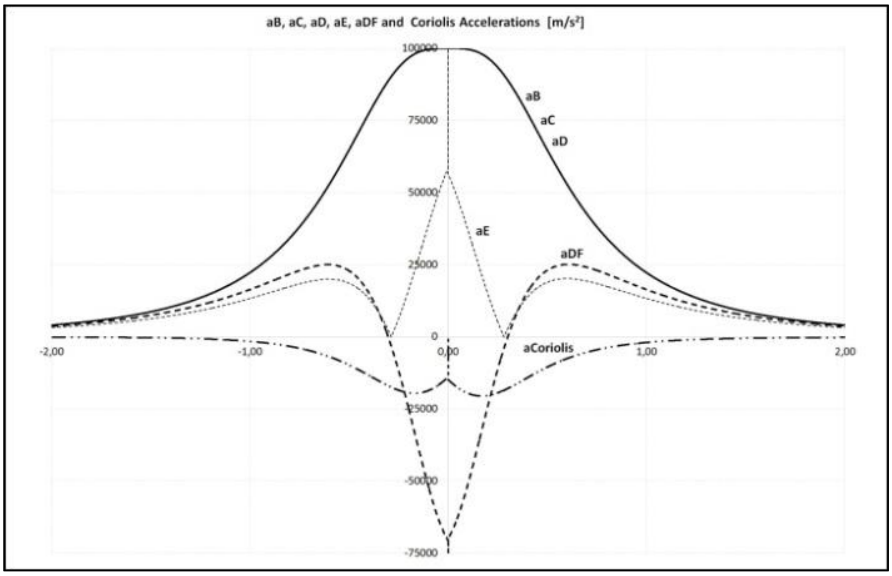

Figure 8 Accelerations of the points B, C, D, E, G sliding DF and Coriolis as a function of "θ_2" bar angle from -2^0 to 2^0 degree (Authors, 2022)

Fig. 9 illustrates the results of the angular velocities of the bars B, C, D, E, G and DF, slip point of the point D in relation to F, as a function of the position angle of the motor bar "θ_2" between -2^0 to 2^0 degree.

Mechanism components can move under predictable conditions, but it is possible that they may present adverse problems during operation due to critical positions of geometric order and restrictions with limits and bar alignments.

These inconvenient situations mentioned could be avoided with changes in the project. The mechanism will theoretically work according to the kinematic analysis presented in Fig. (2-6). Possible problems that arise can be previously detected and avoided if the kinematic analysis is well studied and if possible design changes are made.

An alternative for this study is to first analyze the functioning of the mechanism in the regions of critical points. These critical regions can be identified when there are high increases in velocities and accelerations.

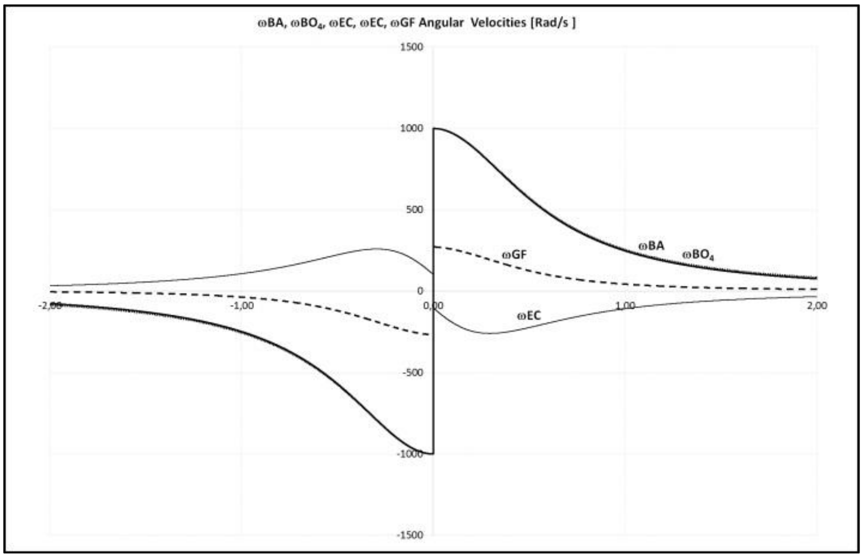

Figure 9 Angular velocities "ωBO4", "ωBA", "ωEC" and "ωDF" sliding bar as function of "θ2" bar angle from -20 to 20 degree (Authors, 2022)

The Fig. 10 illustrate the results of the angular accelerations in order to verify the kinematic of the BA, BO_4, EC bars and the GF/DF bars sliding straight line path. The DF bar position also as function the motor bar position and the angle "θ_2" between -2^0 to 2^0 degree.

It can be seen that a critical running region is "θ_2" close to this phase, and in this way, the kinematics of the "θ_2" mechanism close to zero can be evaluated in more detail by Fig. 8 and Fig. 10.

The possible inconvenient situations mentioned can be previously detected if the kinematic analysis is more detailed and studied and could be avoided with design changes. There are several alternatives to avoid possible malfunctions of the mechanism. Among the possibilities for improvements in the mechanic design would be changes in the dimensions of the mechanism's bars.

An alternative would be a small change in the dimension of the fixed bar O_2A from 99 to 96 mm. When the changing the fixed bar O_2O_4 to 96 mm the kinematic will be changed. The Fig. 11 illustrate the results of the accelerations of the B, C, D, E, G and DF points for the of the position θ_2 angle.

The Fig. 12 illustrate the results of the angular accelerations of the bars O_2A, BA, F the B, C, D, E, G points and DF, D point sliding on straight path DF as function the motor bar position angle "θ_2" between -2^0 to 2^0 degrees.

Fig. 12 illustrates the results of the angular accelerations "α_3", "α_4", "αEC" e "αDF" of the "BA", "O_4B", "EC" and "DF" bars as a function the "O_2A" bar angular position "θ_2" from "θ_2" = -2^0 to "θ_2" = 2^0 for fixed bar O_2A (Authors, 2022).

A small design change of the basic 4-bar mechanism in the dimension of the fixed bar R_1 (O_2O_4) from nominal dimensions 99 mm to 96 mm allowed a great reduction in the accelerations of the mechanism points and the angular accelerations of the bars that can be compared between Figures 11 and 12 against Figures 8 and 10 respectively.

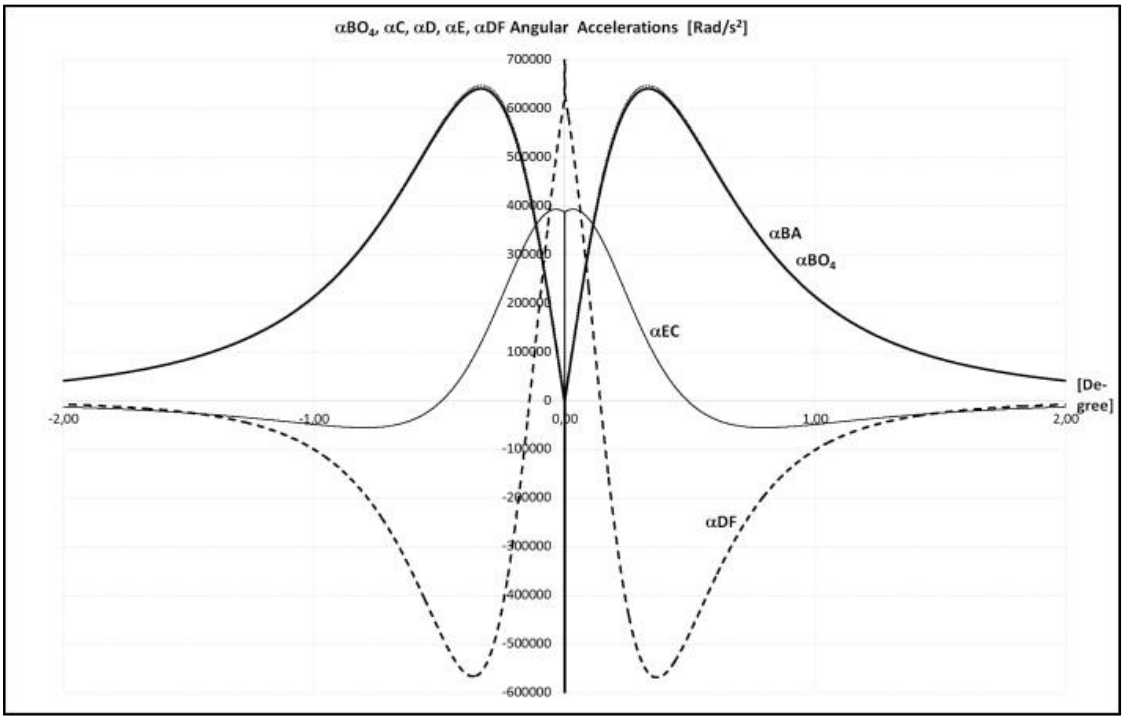

Figure 10 Angular Accelerations "αBO4", "αBA", "αEC" and "αDF" sliding bar for the "θ2" bar angle from -2º to 2º (Authors, 2022)

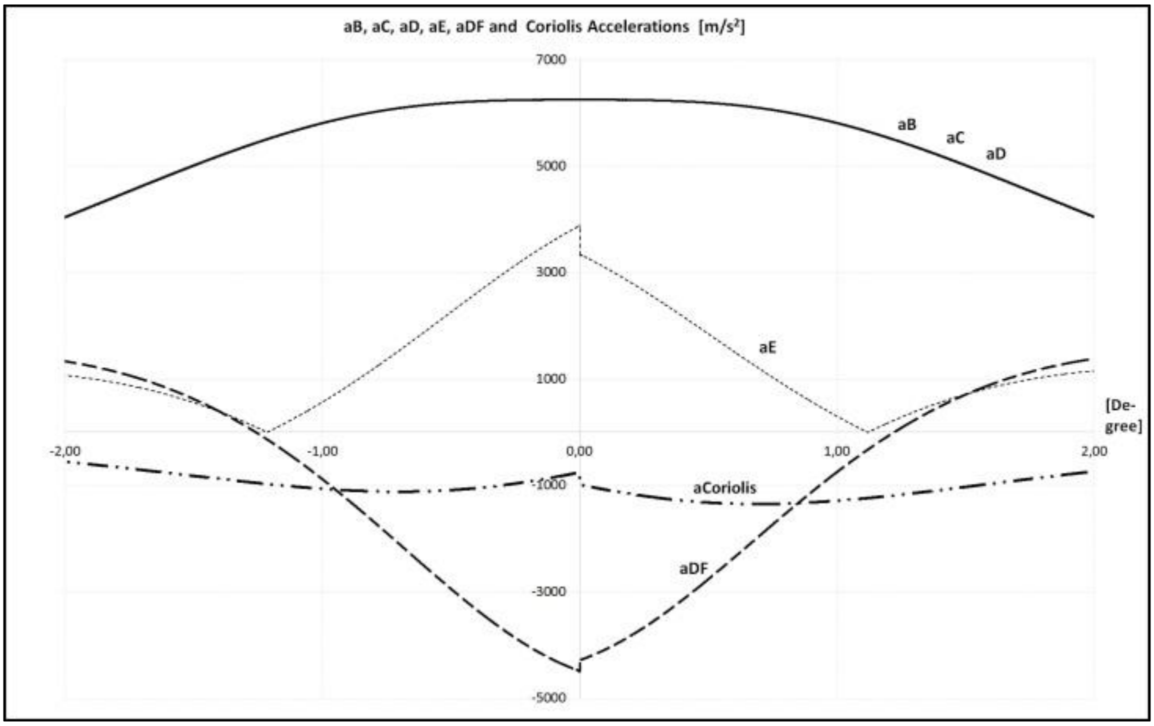

Figure 11 Accelerations of the points B, C, D, E, G sliding DF and Coriolis as a function of "θ2" from -2⁰ to 2⁰ (Authors, 2022)

The running of the mechanism has changed very little from the original design, but the kinematics have become less critical compared to the original design.

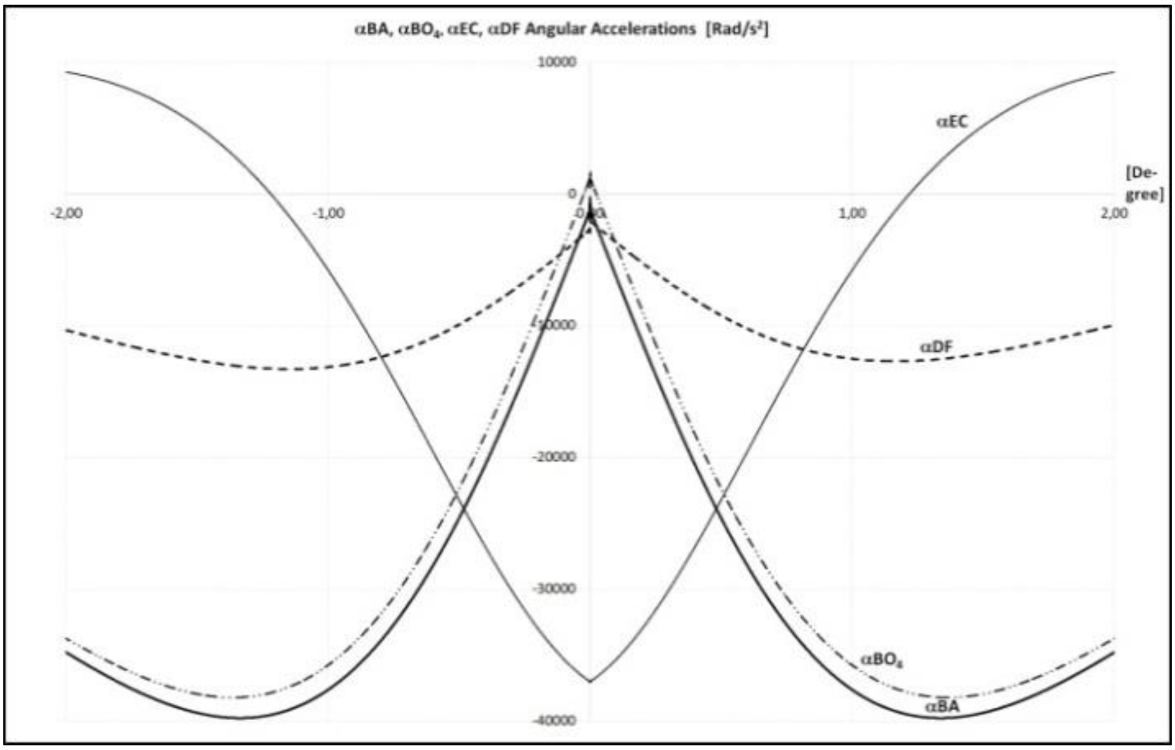

Figure 12 Angular Accelerations "αBO_4", "αBA", "αEC" and "αDF" for R_1 = 96 mm fixed bar and "θ_2" angle from -2^0 to 2^0 Bar (Authors, 2022)

3.1. Nomenclature

α_2: Angular acceleration R_2 bar
αBA: Angular acceleration BA bar
αBO_4: Angular acceleration BO_4 bar
αEC: Angular acceleration EC bar
αDF: Angular acceleration DF
aP : Acceleration of the generic P point
B : y intercept for "x=0" straight line
$\Delta\theta_2$: Crank link θ_2 position angular variation
Δt : Increment time interval
ΔVP : Velocities variation of the generic P point
ΔVxP : Velocities variation of the generic P point on x direction
ΔVyP : Velocities variation of the generic P point on y direction
EC : EC bar length
GF : GF bar length
ϑEC : F point path direction angle
P : Generic P point
θ_2 : R_2 crank link angular position

θ_3 : R_3 coupler link angular position
θ_4 : R_4 rocker link angular position
θCA : CA coupler link angular position
θDA : DA coupler link angular position
R_1 : O_2O_4 crank fixed link length
R_2 : O_2 A crank motor link length
R_3 : BA coupler link length
R_4 : O_4B rocker link length
R_5 : CA coupler link length
R_6 : DA coupler link length
vP : Velocities of the generic P point
ω_2: Angular velocities R_2 bar
ωBA: Angular velocities BA bar
ωBO_4: Angular velocities BO_4 bar
ωEC: Angular velocities EC bar
ωDF: Angular velocities DF bar

4. Conclusion

The results obtained and presented in this work demonstrated that the procedures applied in the kinematic analysis of the articulated mechanism of Fig. 1 and the kinematic specifications was make successful in the complete cycle.

By using the appropriate and objectively applicable computational resources, they allowed us to evaluate the kinematic for the articulated mechanical system in motion conditions with the desired details. More precise and more detailed studies can be carried out according to the specific needs that can determine the criteria of precision and scope of the evaluated parameters.

The theories of mechanisms kinematics were applied through alternative techniques that allowed visibility, precision and reliability according to the needs defined in this work.

Additional evaluations may be carried out considering the operation of the mechanism at angular positions in addition to this work, such as for the return from the maximum position to a desired additional position or condition.

Compliance with ethical standards

Acknowledgments

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Disclosure of conflict of interest

The Authors declares that there is no conflict of interest.

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