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(Research Article)

Unsteady Magnetohydrodynamic (MHD) flow past an accelerated vertical plate in the presence of thermal radiation and chemical reaction

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Abstract

A study on Magnetohydrodynamic (MHD) flow past an accelerated vertical plate with the effects of thermal radiation and chemical reaction has been carried out. The dimensionless governing equations are solved using Laplace transform technique. The solution for the velocity, temperature and concentration field are obtained and analyzed for the different physical parameters like thermal Grashof Number, Modified Grashof Number, permeability parameter, Prandtl Number, Radiation parameter, Schmidt Number, chemical reaction parameter and time. It is observed that the velocity profile increases with increasing parameters like, Sc, Gc, M, R and t and also decrease with the decreases Pr and Gr. the temperature profile shows the decrease with the decreasing in Pr and t. and increase in R. While the concentration profile shows the increase in alpha and decrease in Sc and t.

Keywords: Unsteady; Vertical plate; MHD; Radiation and chemical reaction

1. Introduction

MHD plays an important role in agriculture, petroleum industries, geophysics and in astrophysics. MHD flow has application in metrology, solar physics and in motion of earth's core. Also it has applications in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics. In the field of power generation, MHD is receiving considerable attention due to the possibilities it offers for much higher thermal efficiencies in power plants. Thermal radiation is an important factor in the thermodynamic analysis of many high temperature systems like solar collectors, boilers and furnaces. The simultaneous effect of heat and mass transfer in the presence of thermal radiation plays an important role in manufacturing industries for the design of fins, steel rolling, nuclear power plants, cooling of towers, gas turbines and various propulsion device for aircraft, materials processing, energy utilization, temperature measurements, remote sensing for astronomy and space exploration, food processing and cryogenic engineering, as well as numerous agricultural, health and military applications.

Muthucumaraswamy et al. [6] analyzed theoretical solution of flow past an impulsively started vertical plate with variable temperature and mass diffusion. Jaiswal and Soundalgekar [2] have considered the oscillating plate temperature effects on a flow past an infinite porous plate with constant suction and embedded in a porous medium. Unsteady convection flow of micropolar fluids past a vertical porous plate embedded in a porous medium has been considered [3]. Kumar [4] studied radiative heat transfer with hydro magnetic flow and viscous dissipation over a stretching surface in the presence of variable heat flux. Samad and Rahman [11] studied thermal radiation interaction with unsteady MHD flow past a vertical porous plate immersed in a porous medium. Some effects like radiation and mass transfer on MHD flow were studied [5]. Unsteady MHD flow past a vertical oscillating plate with thermal radiation and variable mass diffusion was investigated [1]. Muthucumaraswamy and Vijayalakshmi [8] studied effects of heat and

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mass transfer on flow past an oscillating vertical plate with variable temperature. Muthucumaraswamy et al. [7] studied unsteady flow past an accelerated infinite vertical plate with variable temperature and uniform mass diffusion. Radiation and mass transfer effects on MHD free convection flow past an exponentially accelerated vertical plate with variable temperature was studied [9]. Radiation effects on MHD flow through a porous medium with variable temperature or variable mass diffusion was studied by Rajesh and Verma [9]. The present study considered an unsteady MHD flow past an accelerated vertical plate with the effects of thermal radiation and chemical reaction.

2. Formulation of the problem

The unsteady flow of an incompressible viscous fluid which is initially at rest past an infinite vertical plate with variable temperature in the presence of MHD and thermal Radiation is considered. The flow is assumed to be in x-direction which takes vertical plate in the upward direction.The y-axis is taken to be normal to the plate. Initially the plate and the fluid are in same temperature $\}$ with the same concentration level $C^{'}$ at all points. At time $t^{'}$ > 0 the plate accelerated with velocity $U = \frac{t^{'} u_{\circ}^{2}}{v}$ in its own plane. The plate temperature is raised to $T_{w}^{'}$ and the level of concentration near the plate is raised to $C_{w}^{'}$ linearly with the time t. By Boussinesq's approximation the unsteady flow is governed by the following equation.

$$\frac{\partial u}{\partial t^{'}} = g\beta(T - T_{\infty}) + g\beta^{*}(C^{'} - C^{'}) + v\frac{\partial^{2} u}{\partial y^{2}} - \frac{\sigma\beta_{\circ}^{2} u^{'} u_{\circ}^{2}}{\rho v} \quad \text{.....................} (1)$$

$$\rho C_{P}\frac{\partial T}{\partial t^{'}} = k\frac{\partial^{2} T}{\partial y^{2}} - \frac{\partial q_{r}}{\partial y^{'}} \quad \text{.....................} (2)$$

$$\frac{\partial C^{'}}{\partial t^{'}} = D\frac{\partial^{2} C^{'}}{\partial y^{2}} - \gamma\left(C - C_{\infty}\right) \quad \text{.....................} (3)$$

Where (1) is the momentum equation, (2) is the energy equation and (3) is the mass concentration equation.

Where U is the velocity of the fluid, T is the fluid temperature, C' is the concentration g is gravitational constant, β and β^{*} are the thermal expansions of fluid and concentration, t' is the time, ρ is the fluid density, C ρ is the specific heat capacity v is the viscosity of the fluid, k is the thermal conductivity of the fluid.

The initial and boundary condition are;

$$u = 0, \quad T = T_{\infty,} \; C^{'} = C_{\infty}^{'}, \qquad \text{For all } y, t^{'} \leq 0$$

$$t^{'} \geq 0: \; u = u_{\circ} t, T = T_{\infty} + (T_{w} - T_{\infty}) \text{ At } C^{'} = C_{w}, \text{at } y = 0$$

$$u \to 0, T \to T_{\infty,} C^{'} \to C^{'}_{\infty}, at y \to \infty \quad \text{.....................} (4)$$

Where $A = \left(\frac{u_{\circ}^{2}}{v}\right)^{\frac{1}{3}}$, A is a constant

$$\frac{\partial q_{r}}{\partial y^{'}} = -4a^{*}\sigma\left(T^{'4}_{\infty} - T^{'4}\right) \quad \text{.....................} (5)$$

It is assumed that the temperature differences within the flow are sufficiently small such that T'^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T'^4 in a Taylor series about T'_∞ and neglecting higher order terms, thus;

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty \quad \text{.....................} (6)$$

By using equations (5) and (6), equation (2) reduces to

$$\rho C_p \frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial y'^2} + 16a^*\sigma T'^3_\infty \left(T'_\infty - T'\right) \quad \text{.....................} (7)$$

The non-dimensional quantities are:

$$U = \left(\frac{u'}{u_\circ}\right), t = \left(\frac{t' u_\circ^2}{v}\right), Y = \left(\frac{y u_\circ}{v}\right), \quad \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, Gr = \frac{g\beta v\left(T_w - T_\infty\right)}{u_\circ^3}$$

$$C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, Gc = g\beta^* v\frac{\left(C'_w - C'_\infty\right)}{u_\circ^3}, \ \Pr = \frac{\mu c_\rho}{k}, Sc = \frac{v}{D}, M = \frac{\sigma\beta_\circ^2 u'}{\rho},$$

$$R = \frac{16a^* v^2 \sigma T'^3_\infty}{k u'^2_\circ}, \omega' = \frac{\omega v}{U_\circ^2}, \alpha = \frac{\gamma v}{U_\circ^2} \quad \text{.....................} (8)$$

The non-dimensional quantities of equation (9) which analyzed (1) to (4) and they lead to the dimensionless equations are as follows;

$$\frac{\partial U}{\partial t} = Gr\theta + GcC + \frac{\partial^2 U}{\partial Y^2} - MU \quad \text{.....................} (9)$$

$$\frac{1}{\Pr}\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial Y^2} + R\theta \quad \text{.....................} (10)$$

$$Sc\frac{\partial \phi}{\partial t} = \frac{\partial^2 \phi}{\partial Y^2} - \alpha Sc\phi \quad \text{.....................} (11)$$

Where Sc is the Schmidt number, Pr is the Prandtl number and Gr and Gc are the Grashof numbers.

The initial and boundary conditions in dimensionless form are:

$$U = 0, \theta = 0, C = 0, \ forall Y, t \le 0$$

$$t > 0 : U = 1, \theta = 1, \phi = 1, at Y = 0$$

$$U \to 0, \theta \to 0, C \to 0, as, Y \to \infty \quad \text{.....................} (12)$$

3. Method of solution

Equation (9) to (11) are solved subject to the boundary conditions of (12) and the solutions are obtained for velocity, temperature and concentration field in terms of exponential and complementary error function using Laplace transform technique as follows;

$$\phi = \frac{1}{2}\left\{e^{2\eta\sqrt{Sc\alpha t}} erfc\left(\eta\sqrt{Sc}+\sqrt{\alpha t}\right)+e^{-2\eta\sqrt{Sc\alpha t}} erfc\left(\eta\sqrt{Sc}-\sqrt{\alpha t}\right)\right\} \text{ .... (13)}$$

$$\theta = \frac{1}{2}\left\{e^{2\eta\sqrt{Rt}} erfc\left(\eta\sqrt{\Pr}+\sqrt{Rt}\right)+e^{-2\eta\sqrt{Rt}} erfc\left(\eta\sqrt{\Pr}-\sqrt{Rt}\right)\right\} \text{ .... (14)}$$

$$U = \frac{1}{2}\left\{e^{2\eta\sqrt{mt}} erfc\left(\eta+\sqrt{Mt}\right)+e^{-2\eta\sqrt{mt}} erfc\left(\eta-\sqrt{Mt}\right)\right\}+\frac{Grt}{2a}\left[e^{2\eta\sqrt{mt}} erfc\left(\eta+\sqrt{Mt}\right)+e^{-2\eta\sqrt{mt}} erfc\left(\eta-\sqrt{Mt}\right)\right]-$$

$$\frac{\eta\sqrt{t}}{\sqrt{M}}\left[e^{2\eta\sqrt{mt}} erfc\left(\eta+\sqrt{Mt}\right)-e^{-2\eta\sqrt{mt}} erfc\left(\eta-\sqrt{Mt}\right)\right]+\frac{Gr}{2f}\left\{e^{2\eta\sqrt{mt}} erfc\left(\eta+\sqrt{Mt}\right)+e^{-2\eta\sqrt{mt}} erfc\left(\eta-\sqrt{Mt}\right)\right\}+$$

$$\frac{Gct}{2h}\left[e^{2\eta\sqrt{mt}} erfc\left(\eta+\sqrt{Mt}\right)+e^{-2\eta\sqrt{mt}} erfc\left(\eta-\sqrt{Mt}\right)\right]-\frac{\eta\sqrt{t}}{\sqrt{M}}\left[e^{2\eta\sqrt{mt}} erfc\left(\eta+\sqrt{Mt}\right)-e^{-2\eta\sqrt{mt}} erfc\left(\eta-\sqrt{Mt}\right)\right]+$$

$$\frac{Gc}{2b}\left\{e^{2\eta\sqrt{mt}} erfc\left(\eta+\sqrt{Mt}\right)+e^{-2\eta\sqrt{mt}} erfc\left(\eta-\sqrt{Mt}\right)\right\}-\frac{Grt}{2a}\left[e^{-2\eta\sqrt{\Pr Rt}} erfc\left(\eta\sqrt{\Pr}-\sqrt{Rt}\right)+e^{2\eta\sqrt{\Pr Rt}} erfc\left(\eta\sqrt{\Pr}+\sqrt{Rt}\right)\right]-$$

$$\frac{\eta\sqrt{\Pr t}}{\sqrt{R}}\left[e^{-2\eta\sqrt{\Pr Rt}} erfc\left(\eta\sqrt{\Pr}-\sqrt{Rt}\right)-e^{2\eta\sqrt{\Pr Rt}} erfc\left(\eta\sqrt{\Pr}+\sqrt{Rt}\right)\right]+\frac{Gr}{2f}\left\{e^{2\eta\sqrt{Rt}} erfc\left(\eta\sqrt{\Pr}+\sqrt{Rt}\right)+e^{-2\eta\sqrt{Rt}} erfc\left(\eta\sqrt{\Pr}-\sqrt{Rt}\right)\right\}-$$

$$\frac{Gct}{2h}\left[e^{-2\eta\sqrt{Sc\alpha t}} erfc\left(\eta\sqrt{Sc}-\sqrt{\alpha t}\right)+e^{2\eta\sqrt{Sc\alpha t}} erfc\left(\eta\sqrt{Sc}+\sqrt{\alpha t}\right)\right]-\frac{\eta\sqrt{Sct}}{\sqrt{\alpha}}\left[e^{-2\eta\sqrt{Sc\alpha t}} erfc\left(\eta\sqrt{Sc}-\sqrt{\alpha t}\right)-e^{2\eta\sqrt{Sc\alpha t}} erfc\left(\eta\sqrt{Sc}+\sqrt{\alpha t}\right)\right]+$$

$$\frac{Gc}{2b}\left\{e^{2\eta\sqrt{Sc\alpha t}} erfc\left(\eta\sqrt{Sc}+\sqrt{\alpha t}\right)+e^{-2\eta\sqrt{Sc\alpha t}} erfc\left(\eta\sqrt{Sc}-\sqrt{\alpha t}\right)\right\} \text{ .... (15)}$$

Where $a=(\Pr-1),\ h=(Sc-1),\ b=(Sc\alpha-M),\ f=\frac{Gr}{R-M},\ \eta=\frac{y}{2\sqrt{t}}$

4. Results and discussion

The results are discussed graphically and qualitatively. The numerical computations are carried out for different physical parameters like Gr, Gc, Sc, alpha, R, M and t upon the nature of the flow and transport. The value of the Schmidt number Sc is taken to be 0.57, which corresponds to water-vapor. The value of the Prandtl number Pr is chosen such that it represents air (Pr = 0.71). The numerical values of the velocity, temperature and concentration are computed for the above mentioned parameters. The numerical results reveal that the radiation induces a rise in both the velocity and temperature, and a decrease in the concentration.

The effects of velocity profiles for different values of time t is presented in figure 1. It shows that the velocity is increases with the increasing values of t. The effects of velocity profiles for different values of Schmidt number Sc. is presented in figure 2. It shows that the velocity is increases with the increasing values of Sc. The effects of velocity profiles for different values of mass Grashof number Gc is presented in figure 3. It shows that the velocity is increases with the increasing values of Gc. The effects of velocity profiles for different values of magnetic field parameter M is presented in figure 4. It shows that the velocity is increases with the increasing values of M. The effects of velocity profiles for different values of radiation parameter R is presented in figure 5. It shows that the velocity is increases with the increasing values of R. The effects of velocity profiles for different values of Prandtl number Pr is presented in figure 6. It shows that the velocity is increases with the increasing values of Pr. The effects of velocity profiles for different values of thermal Grashof number Gr is presented in figure 7. It shows that the velocity is increases with the increasing values of Gr.

The effects of temperature profiles for different values of radiation parameter R is presented in figure 8. It shows that the temperature is increases with the increasing values of R. The effects of temperature profiles for different values of time t is presented in figure 9. It shows that the temperature is increases with the increasing values of t. The effects of temperature profiles for different values of Prandtl number Pr is presented in figure 10. It shows that the temperature is decreases with the decreasing values of Pr.

The effects of concentration field profiles for different values of alpha is presented in figure 11. It shows that the concentration field is increases with the increasing values of alpha. The effects of concentration field profiles for different values of time t is presented in figure 12. It shows that the concentration field is increases with the increasing values of t. The effects of concentration field profiles for different values of Schmidt number Sc is presented in figure 13. It shows that the concentration field is decreases with the decreasing values of Sc.

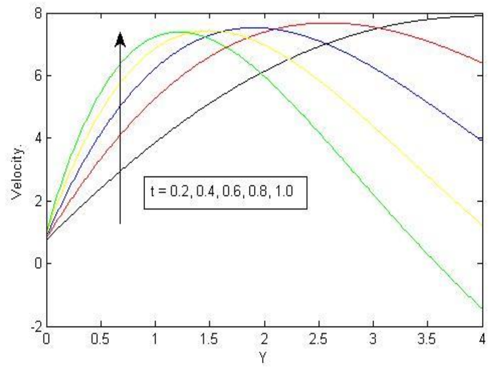

Figure 1 Velocity profile for different values of t.

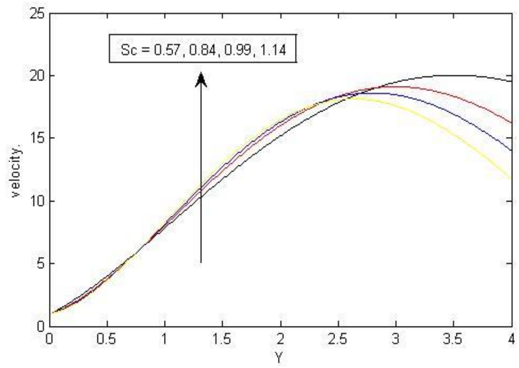

Figure 2 Velocity profile for different values of Sc

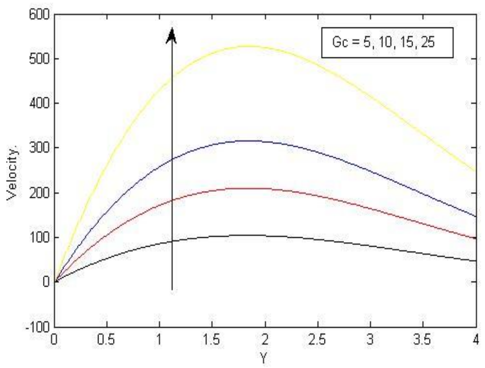

Figure 3 Velocity profile for different values of Gc

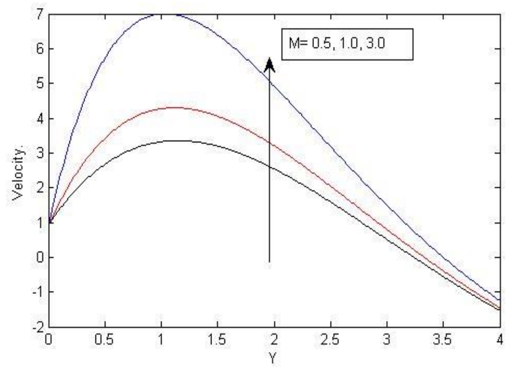

Figure 4 Velocity profile for different values of M

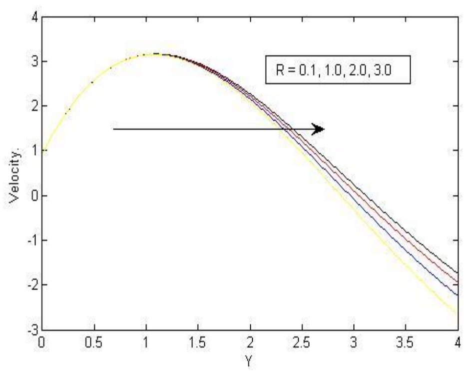

Figure 5 Velocity profile for different values of R

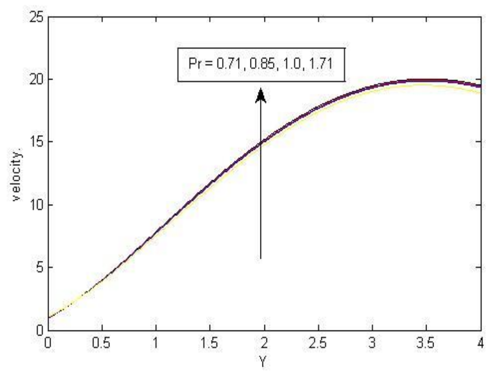

Figure 6 Velocity profile for different values of Pr

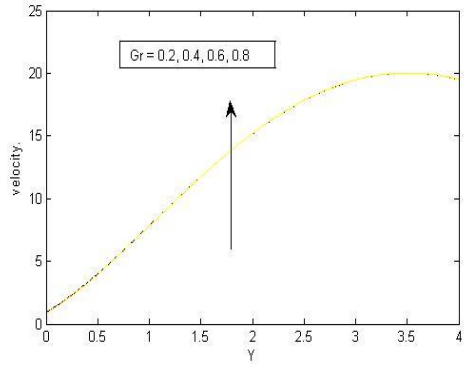

Figure 7 Velocity profile for different values of Gr**.**

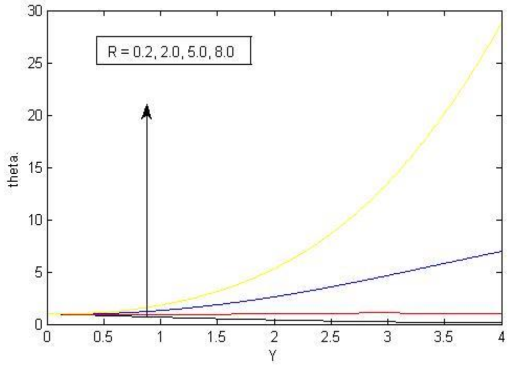

Figure 8 Temperature profile for different values of R

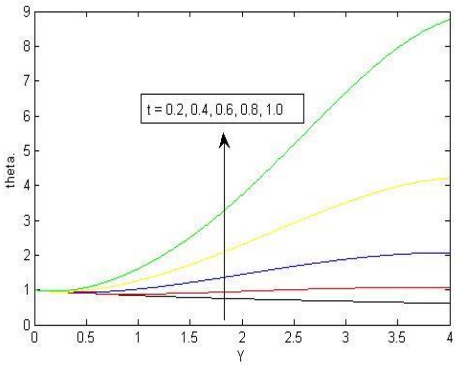

Figure 9 Temperature profile for different values of t.

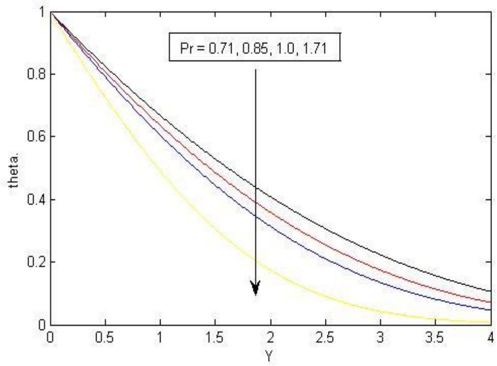

Figure 10 Temperature profile for different values of Pr

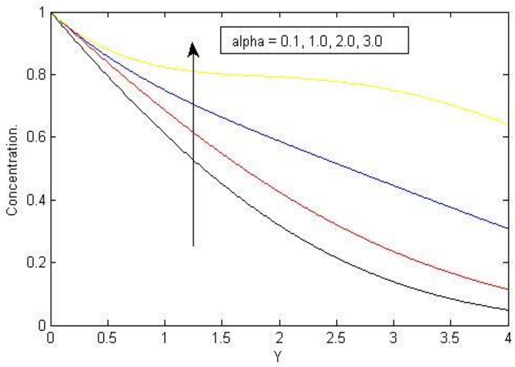

Figure 11 Concentration profile for different values of alpha.

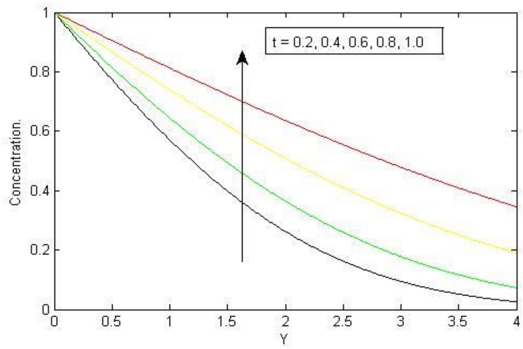

Figure 12 Concentration profile for different values of t.

5. Conclusion

The unsteady magnetohydrodynamic flow past an accelerated vertical plate with the effects of thermal radiation and chemical reaction have been studied, the dimensional governing equations are solved by Laplace-transform technique and computed for different parameters using MATLAB. The effect of different parameters such as Schmidt number, Prandtl number, thermal Grashof number, mass Grashof number, chemical reaction parameter, radiation parameter, magnetic field parameter, alpha and t are studied graphically. It was observed that the velocity profiles increases with the increasing parameters like t, Sc, Gc, M, R, Pr, and Gr. It also observed that the temperature increases with the increasing in R and t, while decreases with the decreasing value of Pr. The concentration field increases with the increasing in alpha and t, while decreases with the decreasing value of Sc.

Compliance with ethical standards

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Disclosure of conflict of interest

The authors have contributed in the preparation of this manuscript and declared no conflict of interest in preparing this article.

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