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(Review Article)

An assessment of the mass line and mass prism models in terrain correction for regional geoid modelling in low mountainous areas

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Abstract

Terrain correction is a very crucial step in gravity reduction and the reliability of the adopted topographic model plays a significant role in the overall geoid computation process especially in mountainous terrain. Given the cumbersome computation involved in the mass prism model, the mass line topographic model is often utilized in practical computation while implementing terrain correction for most gravity reduction schemes. In this study, an assessment is made of the accuracy of the mass line (ML) model viz a viz the mass prism (MP) model in a low-ranged mountainous area like Ado Ekiti township; with a view to determine the suitability of the continued use of ML in such regions. Both models were implemented in the spatial domain using MATLab codes written from the conventional formulae. Results obtained indicate that the minimum and maximum differences in computed Bouguer anomalies using ML and MP models are 0.0196mgals - 0.0610mgals. Consequently, the choice of model did not have significant effect on the computed geoid models as the derived geoid from both models produced the same RMSE of 83cm when compared with GNSS-Leveling geoid at validation points. The study concludes that for topographic ranges less than 300m, either of both topographic models could be used and similar level of accuracy will be obtained in the resulting geoid.

Keywords: Gravity anomalies; Hammer Charts; Digital Terrain Model; Gravitational Potential; Topographic mass

1. Introduction

The gravity field of the Earth plays a vital role in the determination of size and shape of the Earth. For this reason, measurements and computations towards the realization of the gravity field must take into proper cognisance the size, shape and structure of the Earth. The Earth is fundamentally a 3D surface, therefore, computations for a rigorous realization of the shape of the Earth (especially in mountainous regions) is expected to be highly considerate of the geometry of the Earth (Fotouplous, 2003). Extensive theoretical and numerical investigations indicate that in order to improve the accuracy of calculated gravimetric geoid undulations in mountainous areas, more attention should be paid to the short-wavelength topographic effect, in which the Terrain Correction (TC) has a dominant contribution (Sideris 1994). In other words, the approach used for topographic modelling in terrain correction plays a significant role in the overall accuracy of the computed regional geoid model (Nahavandchi and Sjöberg, 2001).

TC is evaluated either by planar (Figure 1a) or spherical approximation (Figure 1b) of the topography (commonly known as planar or spherical topographic model). Despite the popularity of the planar approximation, it's major setback is that it does not provide a realistic model of the Earth. Besides, with the planar approximation, the TC diminishes

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rapidly with distance from the computation point thereby reducing the distance interval for computation to 50km. Furthermore, Huang *et al* (2001) has shown that in areas with rough topography, it is necessary to compute the TC over a larger radius. Notwithstanding, the planar TC is still the mostly utilized topographic model used for computation of regional gravimetric geoid (Kirby and Featherstone, 2002).

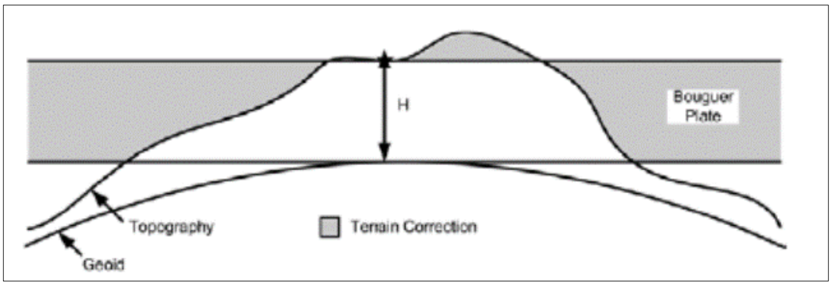


Figure 1a Planar approximation of terrain correction (Bajrachsharya, 2003)

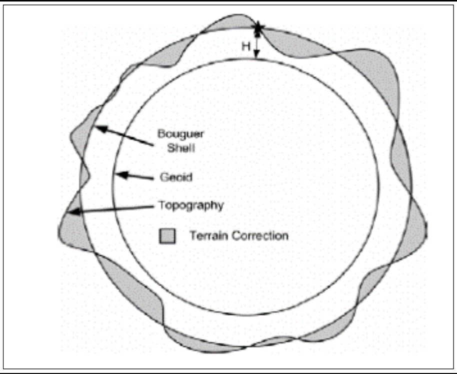


Figure 1(b) Spherical approximation of terrain correction (Amos, 2007)

Within the ambits of implementing TC in the spatial domain, the planar and spherical models are available. However, research has further shown that in regions with height variation less than 2000m, the planar topographic approximation models suffice (Nahavandchi and Sjöberg, 2001). Again, within the context of the planar approximations, two topographic models exist. These are the mass line model and the mass prism model topographic model. Depending on the theoretical assumptions of the behavior of the topographic masses, all gravity reduction schemes, fall into either of these two models. This study therefore presents an assessment of the performance of the two variants of the planar model (Mass line topographic model and mass prism topographic model) over the low mountain region of Ado Ekiti, with a view to identifying topographic considerations for the optimality or otherwise of both models.

2. Topographic models

Usually, the gravitational effects of all topographic masses around the gravity station is divided into three (3), being the density variation, Bouguer plate and terrain correction (Gomez et al, 2013). For computational purposes, the first two are easier to implement and as such have not been too challenging in geodetic discussions. The density variation is usually taken as a constant value of 2670 kg/m^3 (Gaetani et al, 2021), while the Bouguer plate is treated using the standard Bouguer plate formula given in equation 1.

$$A_B = 2\pi G\rho H \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Where;

A_B = Bouguer plate of Uniform thickness
G = Gravitational attraction
ρ = uniform Bouguer plate density
H = Height of gravity station defining thickness of the Bouguer slab

However, terrain correction being a mathematical representation of an hypothetical geometric model (either planar or spherical) that compensates for the actual deviations of the topography from the Bouguer plate is complex to model and its implementation difficult to achieve.

As discussed above, various topographic model approximations are utilized by the different gravimetric reduction schemes for TC computation. The planar topographic model is often realized through the implementation of the mass line and mass prism terrain correction computational models presented below;

2.1. The mass-line Model

In the mass line model, each grid cell is assumed to be concentrated on a vertical mass line situated in the centre of each grid. The mass line model is a discrete model as can be illustrated as seen in Figure 2a.

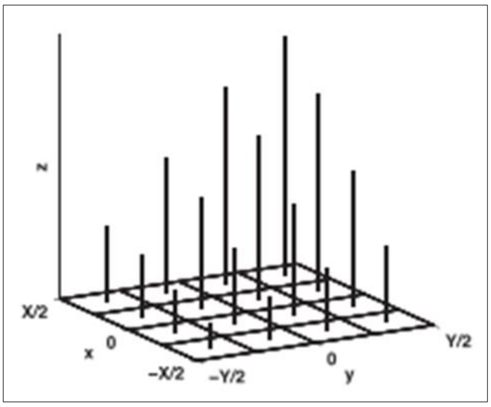


Figure 2(a) The mass line model

The mass line model can be implemented using the mathematical formulae given in equation (2) below; (Li and Sideris, 1994)

$$c(i,j) = G\rho\,\Delta x\,\Delta y \sum_{n=0}^{N-1}\sum_{m=0}^{M-1}\left[\frac{1}{r(x_i - x_n, y_j - y_m, 0)} - \frac{1}{r(x_i - x_n, y_j - y_m, h_{ij} - h_{nm})}\right]. \quad \text{………………} (2)$$

Where;
$c_{(i,j)}$ = terrain correction
G = Gravitational constant
ρ = mean density of the line
x1, y1, z1 = the coordinates of the prism corner in the dummy point
x2, y2, z2 = the coordinates of the prism corner in the integration point
h = Height

2.2. Mass Prism Model

The mass prism model on the other hand considers a surface function wherein each grid is composed of a surface filled with terrain information. This implies that the surface function and the density parameters become step functions.

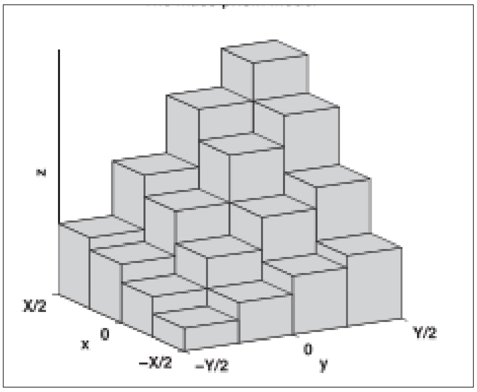


Figure 2(b) The mass prism model

The mass prism model is often implemented by the Nagy prism terrain correction formulae given in equation 3 below (Nagy, 1966). See Figure 3 for graphical illustration of parameters as defined.

$$c = -GD \left| \left| \left| x \bullet \ln(y+R) + y \bullet \ln(x+R) + Z \arctan \frac{Z \bullet R}{x \bullet y} \right|_{X_1}^{X_2} \right|_{Y_1}^{Y_2} \right|_{Z_1}^{Z_2} \quad \text{.............................(3)}$$

Where;
c = terrain correction
G = Gravitational constant
D = mean density of the prism
x1, y1, z1 = the coordinates of the prism corner in the dummy point
x2, y2, z2 = the coordinates of the prism corner in the integration point
Z = H = Height
R = distance between dummy point and integration point

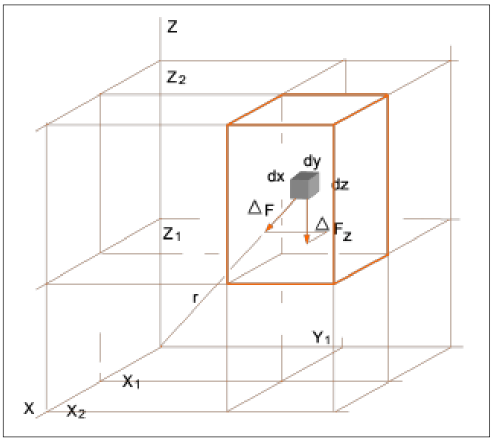


Figure 3 Gravitational attraction of a right rectangular prism (Oasis Montaj publications, 2022)

3. Materials and Methods

568 gravity data points distributed across Ado town were used in this study. The used data comprise of 112 terrestrial points obtained using Scrintrex CG5 gravimeter, 432 points obtained from earlier works using gravity interpolation by Kriging method (Odumosu, 2019) and 24points from Gravity forward modelling approach (Odumosu et al, 2021). The spatial distribution of the data used for this study is presented in Figure 4. The quality estimates of the gravity data is presented in Table 1. The study area (Ado township) is a low-range mountainous area with elevation range of about 261m.

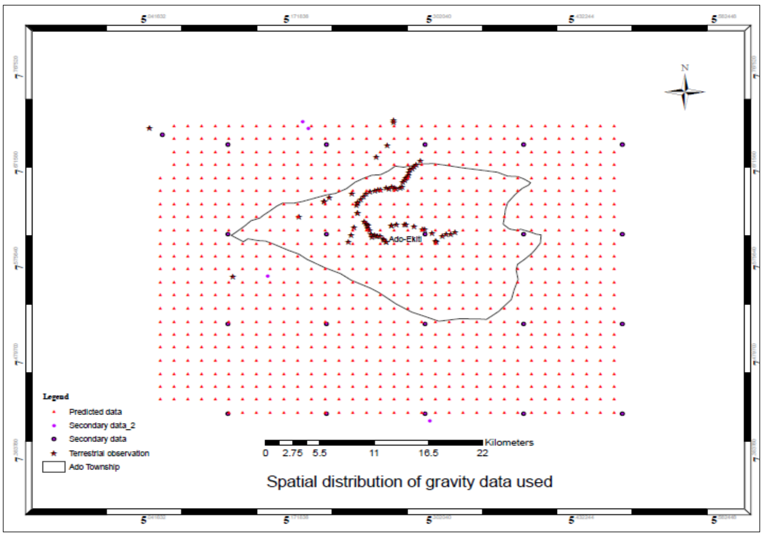


Figure 4 Spatial distribution of gravity data used for the study

Heights obtained from the Shuttle Radar Topographic Mission (SRTM) was used to compute the required terrain correction. The 1 arc seconds SRTM data was used in this study and shown in Figure 5.

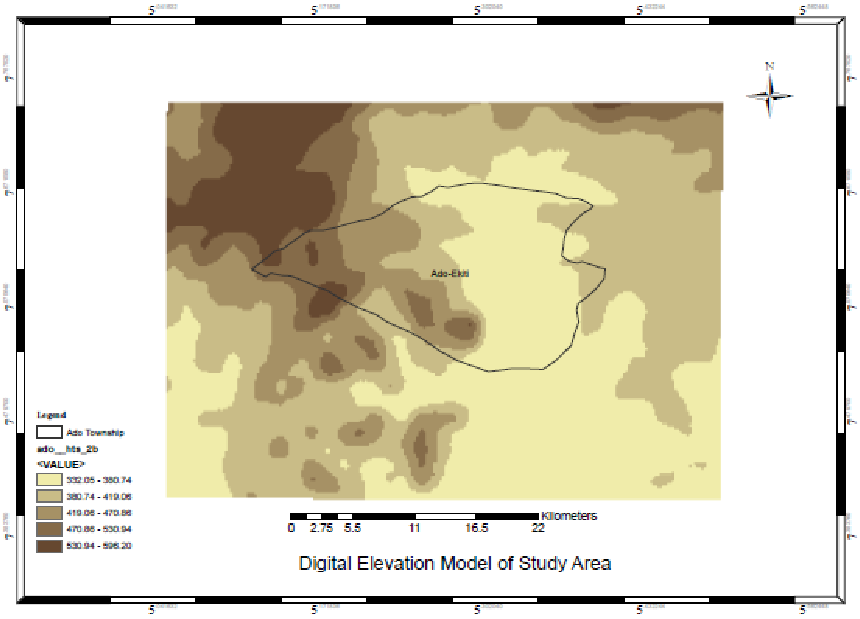


Figure 5 1" SRTM DEM covering Ado Ekiti Township

Table 1 Quality estimates of the gravity data used

S/N	Data Source	Reference ellipsoid for gravity	Type of observation	Observational accuracy	Prediction accuracy
1	Terrestrial data	IGSN71	Profile method	±1.25mgals	Not Applicable
2	Simulated data	IGSN71	Not Applicable	Not Applicable	±4.25mgals
3	Secondary data	IGSN71	Data merging	Not Applicable	±5.57mgals

For the computation of the terrain correction, the mass line and mass prism models as presented in equations (2) and (3) were utilized. MATLAB codes were written to implement both equations after reducing the observed gravity readings to obtain Bouguer anomalies over the region. The reduced Bouguer anomalies having applied terrain correction from the mass line and mass prism methods where then used to compute a regional geoid of the study area using the conventional remove-compute-restore (RCR) techniques. Obtained geoid values were compared with GNSS-Levelling results at selected validation points to identify the most reliable of both methods and identify the degree of reliability of both models in a low-mountainous region like Ado Ekiti township.

4. Results

Extract as well as statistics of the results obtained from the ML and MP topographic models is presented in Tables 2 - 4 respectively. Also, applying the Stokes integral in the Remove Compute Restore (RCR) geoid computation technique, the ML and MP models derived Bouguer reduced gravity anomalies were used to compute the local geoid for Ado town. The obtained geoid models are presented in Figures 6a and b respectively.

Table 2 Extract of results for Bouguer anomalies obtained using the ML and MP models

Bouguer Anomalies		
FA Ano (mgals)	**MP model (mgals)**	**ML model (mgals)**
42.068	41.6613	41.7942
40.093	39.6891	39.6750
42.502	42.0982	42.1182
47.247	46.8438	46.8523
47.074	46.6747	46.6815
40.353	39.9561	39.8178
40.242	39.8453	39.8246
44.000	43.6040	43.6869
38.897	38.5017	38.5134

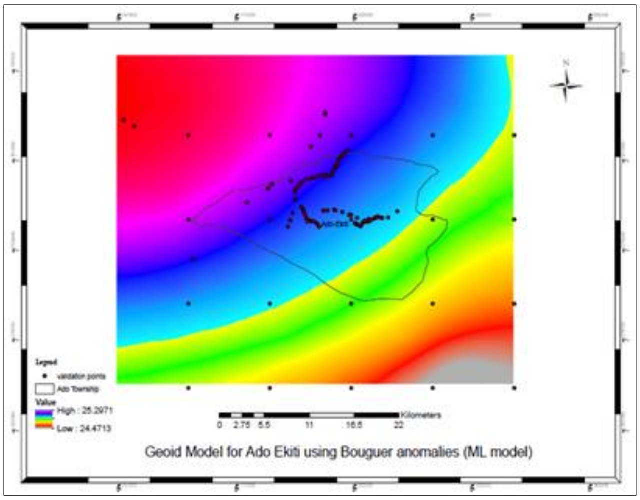
Table 3 Correlation table between ML and MP models

	mass-Prism	**mass-Line**
mass-Prism	1.0000	
mass-Line	0.9960	1.0000

Table 4 Descriptive statistics of the ML and MP reduced Bouguer anomaly

Parameters	**mass-Prism (mgals)**	**mass-Line (mgals)**
Mean	41.9688	41.8498
Standard Error	0.5066	0.4995
Median	40.5858	40.5451
Mode	38.5017	#N/A
Standard Deviation	5.4796	5.4030
Sample Variance	30.0257	29.1927
Kurtosis	-0.2516	-0.1962
Skewness	0.4712	0.4565
Range	27.8621	27.8207
Minimum	33.3771	33.4381
Maximum	61.2392	61.2588

It is observed from Tables 2 and 3 that both the mass line and mass prism models do not show any significant difference between the results obtained from both methods. The minimum and maximum differences in the obtained values of the Bouguer anomalies from both models are 0.0196mgals - 0.0610mgals respectively. This confirms earlier studies by Nahavandchi (2000), that the differences in results obtained in the reduction of gravimetric quantities by the ML and MP planar methods is not significant in areas with mild topographic variation. Given the topographic variation within the study area (593m - 332m i.e > 300m), the maximum observable difference in the choice of either a mass-line or mass-prism model for Bouguer anomaly reduction is 0.0414mgals. This value is insignificant; therefore, for topographic ranges less than 300m, either the mass-line or the mass-prism model could be used without losing much information regarding the gravitational attraction of topography. This is further substantiated with a very strong positive correlation (0.996) as seen in Table 3 between the ML and MP models and very similar descriptive statistics in Table 4. Consequently, the derived geoid model using the ML and MP computed Bouguer anomalies show no significant difference.



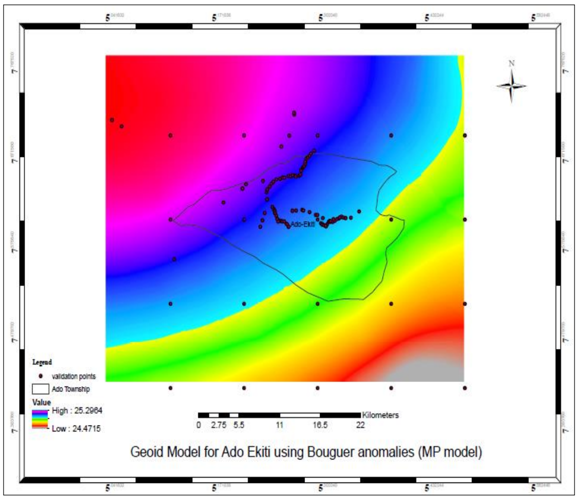


Figure 6(a) Geoid model (ML model) **(b)** Geoid model (MP model)

In same vein, a check of the performance of both geoid models at selected validation points (as shown in Table 5) show that both models had the same value as the RMSE values (0.835). This further stresses that within the topographic range of Ado Ekiti township (and by extension areas with topographic ranges not exceeding 300 m), the choice of ML or MP planar topographic model for computing terrain correction in gravimetric reduction is inconsequential.

5. Conclusion

As seen in results and analysis, the choice of topographic model (i.e either the mass-line or the mass prism) does not make significant difference to the result of Bouguer gravity reduction nor the overall computed geoid within the low-ranged mountainous region of Ado Ekiti. This confirms earlier studies by Nahavandchi (2000). Within the test region, the minimum and maximum elevation values are 332 m and 593 m respectively; and the minimum and maximum differences in the obtained values of the Bouguer anomalies from both models are 0.0196 mgals - 0.0610 mgals. Therefore, the study concludes that for topographic ranges less than 300m, either of both topographic models could be used and similar level of accuracy obtained in the resulting geoid.

Compliance with ethical standards

Disclosure of conflict of interest

No conflict of interest.

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