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Steady blood flow through a porous microchannel with the influence of an inclined magnetic field

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Abstract

This research deals with steady blood flow through a porous microchannel with the influence of an inclined magnetic field. In the research, we formulate time-independent differential equations which represent the blood momentum, energy equation, and mass concentration in the flow governing the flow. The governing equations were scaled to be dimensionless using some prescribed dimensionless parameters in the work. The reduced dimensionless governing equations were further solved using the Frobenius method (FM), where the blood velocity, mass concentration and blood temperature profiles were obtained. Numerical simulation was carried out using Wolfram Mathematica, version 12, to investigate the effect of the variation of the values of pertinent parameters on the flow profiles. In the investigation, it was revealed that the variation of Grashof number, Schmidt number, and porosity parameters increased the blood velocity before decreasing to zero when the boundary layer thickness was at its peak, while the variation of magnetic field and Prandtl number reduced blood velocity. However, the magnetic field angle of inclination initially increased the blood velocity before decreasing. In a similar vein, as the Schmidt number and chemical reaction parameters decrease the mass concentration level in the fluid, the blood temperature increases for the variation of the Schmidt number. In conclusion, the problem was formulated, solved, and simulation was done successfully, where various pertinent parameters were checked. These results are very useful for clinicians and scientists who are interested in investigating the role of some pertinent factors in understanding blood circulation problems theoretically.

Keywords: Blood Flow; Mass Concentration; Magnetic Field; Microchannel; Grashof number; Porosity and Prandtl number

1. Introduction

Blood is a suspension of formed elements in an aqueous solution called plasma; the formed elements are red blood cells (erythrocytes), white blood cells (leukocytes), and platelets in an aqueous electrolyte solution. Plasma contains 90% water and 7% of principal proteins (albumin, globulin, lipoprotein, and fibrinogen). Bunonyo and Amos [1] state that the volumetric fraction of the erythrocytes is 45% of the total volume of blood in normal blood, which defines an important variable called haematocrit. Although blood viscosity is constant in a real physiological system, it may vary in the ratio of haematocrit or depend on temperature and pressure [2]. Bhatti et al. [3] investigated heat transfer in peristaltically induced motion of particle-fluid suspension with variable viscosity using a clot blood model. Elnaqeeb *et al.* [4] investigated the hemodynamic characteristics of gold nanoparticle blood flow through a tapered stenosed vessel with varying nanofluid viscosity. The effect of heat transfer on temperature-dependent viscosity was investigated by Massoudi and Christie [5], Pantokratoras [6], Nadeem and Akbar [7]. Petrofsky et al. [8] studied the effect of the moisture content of the heat source on the blood flow response of the skin through data. [9] developed a mathematical model for bifurcated arteries in order to theoretically investigate the effects of a heat source on magneto-hydrodynamic

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(MHD) blood flow. Akbarzadeh [10] investigated the numerical computation of the effect of periodic body acceleration and periodic body pressure gradient on magneto-hydrodynamic (MHD) blood flow through porous arteries. Bali and Awasthi [11] studied the Newtonian blood flow through the tapered arteries. Chakraborty et al. [12] discussed the suspension model of blood flow through an inclined tube with an axially non-symmetric stenosis. Eldesoky [13] studied the slip effect of an unsteady magnetic field on pulsatile blood flow through a porous medium in an artery under the effect of body acceleration. Korchevskii and Marochnik [14] discussed the possibility of regulating the movement of blood in the human system by applying a magnetic field. In this research, we studied steady blood flow through a porous microchannel with the influence of an inclined magnetic field, considering the effect of mass concentration on the rise in temperature and on blood flow. The investigation would be carried out under well-defined boundary conditions; the resulting nonlinear differential equations have been solved using the Frobenius Method (FM).

2. Mathematical Formulation

We consider blood as a viscous, incompressible and electrically conduction fluid, flowing through an inclined porous microchannel at a velocity with an inclined applied magnetic field. The flow is affected by mass concentration in the fluid and a rise in temperature through a source. It is assumed that the temperature of the fluid is also affected by the mass concentration in the fluid, and the flow could be triggered by the buoyancy and heat generated due to the mass concentration and temperature of the fluid. Following Bunonyo and Ebiwareme [15] and the aforementioned assumptions based on Figure a.

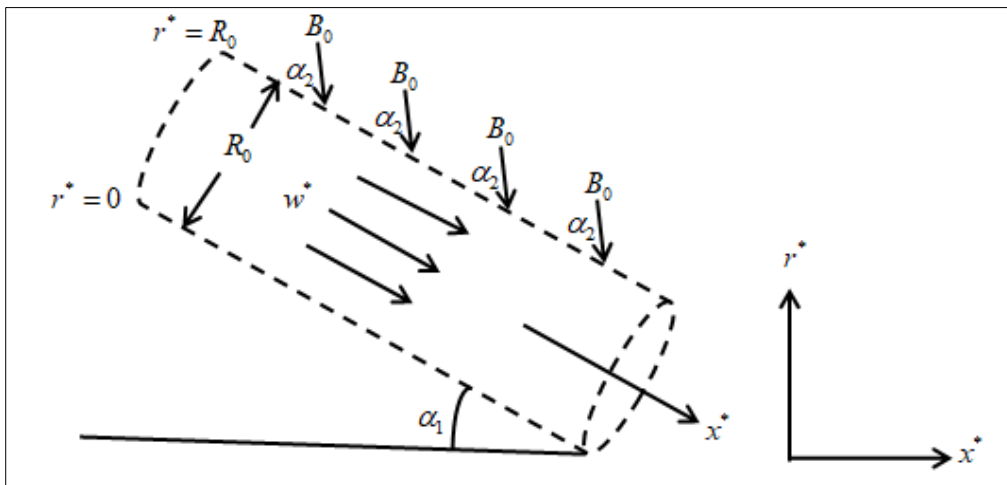


Figure 1 Diagram showing blood flow in a porous vessel with an inclined magnetic field

2.1. Momentum Equation

$$-\frac{\partial P^*}{\partial x^*} + \mu_b \left(\frac{\partial^2 w^*}{\partial r^{*2}} + \frac{1}{r^*} \frac{\partial w^*}{\partial r^*} \right) - \frac{\mu_b \phi}{k^*} w^* - \sigma B_0^2 w^* \cos \alpha_2 + \rho_b g \beta_T (T^* - T_\infty) = 0 \quad \text{-----} \quad (1)$$

2.2. Energy Equation

$$k_{Tb} \left(\frac{\partial^2 T^*}{\partial r^{*2}} + \frac{1}{r^*} \frac{\partial T^*}{\partial r^*} \right) + Q_0 (T^* - T_\infty) - Q_1 (C^* - C_\infty) = 0 \quad \text{-----} \quad (2)$$

2.3. Mass Concentration Equation

$$D_m \left(\frac{\partial^2 C^*}{\partial r^{*2}} + \frac{1}{r^*} \frac{\partial C^*}{\partial r^*} \right) - k_0 (C^* - C_\infty) = 0 \quad \text{-----} \quad (3)$$

The corresponding boundary conditions are

$$\left. \begin{aligned} w^* = 0, T^* = T_w, C^* = C_w \quad \text{at } r^* = R_0 \\ \frac{\partial w^*}{\partial r^*} = 0, \frac{\partial T^*}{\partial r^*} = 0, \frac{\partial C^*}{\partial r^*} = 0 \quad \text{at } r^* = 0 \end{aligned} \right\} \text{-----(4)}$$

2.4. Dimensionless Parameters

$$\left. \begin{aligned} x = \frac{x^*}{\lambda^*}, r = \frac{r^*}{R_0}, t = \frac{t^* \nu_b}{R_0^2}, w = \frac{w^* R_0}{\nu_b}, \theta = \frac{T^* - T_\infty}{T_w - T_\infty}, \phi = \frac{C^* - C_\infty}{C_w - C_\infty}, Rd_1 = \frac{Q_0 R_0^2}{\mu_b c_b}, Rd_3 = \frac{k_0 R_0^2}{\nu_b}, Sc = \frac{\nu_b}{D_m}, \\ Gr = \frac{g \beta_T (T_w - T_\infty) R_0^3}{\nu_b^2}, M = B_0 R_0 \sqrt{\frac{\sigma}{\mu_b}}, \frac{1}{k} = \frac{\phi R_0^2}{k^*}, P = \frac{R_0^3 P^*}{\lambda^* \mu_b \nu_b}, Rd_2 = \frac{Q_1 R_0^2 (C_w - C_\infty)}{k_{Tb} (T_w - T_\infty)} \end{aligned} \right\} \text{-----(5)}$$

$$\frac{d^2 w}{dr^2} + \frac{1}{r} \frac{dw}{dr} - \frac{1}{k} w - M^2 w \cos \alpha_2 + Gr \theta = \frac{\partial P}{\partial x} \text{-----(6)}$$

$$\frac{d^2 \theta}{dr^2} + \frac{1}{r} \frac{d\theta}{dr} - Rd_1 Pr \theta = Rd_2 \phi \text{-----(7)}$$

$$\frac{d^2 \phi}{dr^2} + \frac{1}{r} \frac{d\phi}{dr} + \alpha^2 \phi = 0 \text{-----(8)}$$

Where $\alpha^2 = -Rd_3 Sc$

The corresponding boundary conditions are

$$\left. \begin{aligned} w = 0, \theta = 1, \phi = 1 \quad \text{at } r = 1 \\ \frac{\partial w}{\partial r} = 0, \frac{\partial \theta}{\partial r} = 0, \frac{\partial \phi}{\partial r} = 0 \quad \text{at } r = 0 \end{aligned} \right\} \text{-----(9)}$$

3. Method of Solution

Solving equation (8), we have the solution to be in the form:

$$\phi(r) = A_1 \phi_{01}(r) + A_2 \phi_{02}(r) \text{-----(10)}$$

$$\phi_{01}(r) = \sum_{n=0}^{\infty} a_n r^{n+m} \text{-----(11)}$$

Differentiate equation (11), and substitute the result into equation (8), which is

$$\frac{d\phi_{01}}{dr} = \sum_{n=1}^{\infty} a_n (n+m) r^{n+m-1}, \frac{d^2\phi_{01}}{dr^2} = \sum_{n=2}^{\infty} a_n (n+m)(n+m-1) r^{n+m-2} \text{-----(12)}$$

$$\left(\sum_{n=2}^{\infty} a_n (n+m)(n+m-1)r^{n+m}\right) + \left(\sum_{n=2}^{\infty} a_n (n+m)r^{n+m}\right) + a_1 (m+1)r^{m+1} + \alpha^2 \left(\sum_{n=0}^{\infty} a_n r^{n+m+2}\right) = 0 \quad (13)$$

Let $n = k + 2$ in the lowest degree term of equation (13), so that it can be reduce to

$$\left(\sum_{k=0}^{\infty} a_{k+2} (k+m+2)(k+m+1)r^{k+m+2}\right) + \left(\sum_{k=0}^{\infty} a_{k+2} (k+m+2)r^{k+m+2}\right) + a_1 (m+1)r^{m+1} + \alpha^2 \left(\sum_{n=0}^{\infty} a_n r^{n+m+2}\right) = 0 \quad (14)$$

$$\sum_{k=0}^{\infty} a_{k+2} (k+m+2)^2 r^{k+m+2} + \alpha^2 \sum_{n=0}^{\infty} a_n r^{n+m+2} + a_1 (m+1)r^{m+1} = 0 \quad \text{-----} \quad (15)$$

Simplifying equation (15), we have:

$$a_1 (m+1)r^{m+1} = 0 \quad \text{-----} \quad (16)$$

$$(m+1)r^{m+1} \neq 0, \text{ then } a_1 = 0 \quad \text{-----} \quad (17)$$

$$a_{k+2} = -\frac{\alpha^2 a_k}{(k+m+2)^2} \quad \text{-----} \quad (18)$$

Upon solving for the coefficients for $k = 0, 1, 2, 3, 4$, we have:

$$\forall k : a_{k+2} = -\frac{\alpha^2 a_k}{(m+k+2)^2}, a_1 = 0 \quad \text{-----} \quad (18)$$

Expressing equation (11) fully, we have:

$$\phi_{01}(r) = \sum_{n=0}^{\infty} a_n r^{n+m} = a_0 r^m + a_1 r^{m+1} + a_2 r^{m+2} + a_3 r^{m+3} + a_4 r^{m+4} + a_5 r^{m+5} + a_6 r^{m+6} + a_7 r^{m+7} + \dots \quad (19)$$

Substituting the constants from the series, we have:

$$\phi_{01}(r) = \sum_{n=0}^{\infty} a_n r^{n+m} = \left(a_0 r^m - \left(\frac{\alpha^2 a_0 r^{m+2}}{(m+2)^2} \right) + \left(\frac{\alpha^4 a_0 r^{m+4}}{(m+2)^2 (m+4)^2} \right) - \left(\frac{\alpha^6 a_0 r^{m+6}}{(m+2)^2 (m+4)^2 (m+6)^2} \right) + \dots \right) \quad \text{-----} \quad (20)$$

Simplifying equation (20), we have:

$$\phi_{01}(r) = \sum_{n=0}^{\infty} a_n r^{n+m} = a_0 r^m \left(1 - \left(\frac{\alpha^2 r^2}{(m+2)^2} \right) + \left(\frac{\alpha^4 r^4}{(m+2)^2 (m+4)^2} \right) - \left(\frac{\alpha^6 r^6}{(m+2)^2 (m+4)^2 (m+6)^2} \right) + \dots \right) \quad \text{-----} \quad (21)$$

$$\text{where: } a_2 = -\frac{\alpha^2 a_0}{(m+2)^2}, a_3 = -\frac{\alpha^2 a_1}{(m+3)^2}, a_4 = -\frac{\alpha^2 a_2}{(m+4)^2}, a_4 = \frac{\alpha^4 a_0}{(m+2)^2 (m+4)^2}, a_5 = -\frac{\alpha^2 a_3}{(m+5)^2},$$

$$a_5 = \frac{\alpha^4 a_1}{(m+3)^2 (m+5)^2}, a_6 = -\frac{\alpha^2 a_4}{(m+6)^2}, a_6 = -\frac{\alpha^6 a_0}{(m+2)^2 (m+4)^2 (m+6)^2}, a_7 = -\frac{\alpha^2 a_5}{(m+7)^2},$$

$$a_7 = -\frac{\alpha^6 a_1}{(m+3)^2 (m+5)^2 (m+7)^2}$$

To get the first solution, let $m = 0$ in equation (21), then, we have:

$$\phi_{01}(r) = \sum_{n=0}^{\infty} a_n r^n = a_0 \left(1 - \frac{\alpha^2 r^2}{4} + \frac{\alpha^4 r^4}{4.16} - \frac{\alpha^6 r^6}{4.16.32} + \dots \right) = a_0 J_0(\alpha r) \dots\dots\dots (22)$$

To get the second independent solution, we differentiate equation (21) with respect to m then

$$\phi_{02} = \frac{\partial \phi_{01}}{\partial m} = \frac{\partial}{\partial m} \left[a_0 r^m \left(1 - \frac{\alpha^2 r^2}{(m+2)^2} + \frac{\alpha^4 r^4}{(m+2)^2 (m+4)^2} - \frac{\alpha^6 r^6}{(m+2)^2 (m+4)^2 (m+6)^2} + \dots \right) \right] \dots\dots\dots (23)$$

$$\phi_{02} = \frac{\partial \phi_{01}}{\partial m} = \left[a_0 r^m \left(1 - \frac{\alpha^2 r^2}{(m+2)^2} + \frac{\alpha^4 r^4}{(m+2)^2 (m+4)^2} - \frac{\alpha^6 r^6}{(m+2)^2 (m+4)^2 (m+6)^2} + \dots \right) \log r + a_0 r^m \frac{\partial}{\partial m} \left(1 - \frac{\alpha^2 r^2}{(m+2)^2} + \frac{\alpha^4 r^4}{(m+2)^2 (m+4)^2} - \frac{\alpha^6 r^6}{(m+2)^2 (m+4)^2 (m+6)^2} + \dots \right) \right] \dots\dots\dots (24)$$

$$\phi_{02} = \frac{\partial \phi_{01}}{\partial m} = \left[a_0 r^m \left(1 - \frac{\alpha^2 r^2}{(m+2)^2} + \frac{\alpha^4 r^4}{(m+2)^2 (m+4)^2} - \frac{\alpha^6 r^6}{(m+2)^2 (m+4)^2 (m+6)^2} + \dots \right) \log r + \frac{2\alpha^2 r^2 a_0 r^m \left(1 - \frac{2\alpha^2 r^2 (m+3)}{(m+4)^3} + \frac{\alpha^4 r^4 (44 + 3m(m+8))}{(m+4)^3 (m+6)^3} \right)}{(m+2)^3} \right] \dots\dots\dots (25)$$

If we evaluate $\left. \frac{\partial \phi_{01}}{\partial m} \right|_{m=0}$, then equation (25) becomes:

$$\phi_{02} = \left. \frac{\partial \phi_{01}}{\partial m} \right|_{m=0} = a_0 \left[\left(1 - \frac{\alpha^2 r^2}{4} + \frac{\alpha^4 r^4}{64} - \frac{\alpha^6 r^6}{2304} + \dots \right) \log r + \frac{\alpha^2 r^2}{4} \left(1 - \frac{3\alpha^2 r^2}{32} + \frac{11\alpha^4 r^4}{3456} \right) \right] \dots\dots\dots (26)$$

Simplifying equation (26), we have:

$$\phi_{02} = a_0 \left(J_0(\alpha r) \log r + \frac{\alpha^2 r^2}{4} \left(1 - \frac{3\alpha^2 r^2}{32} + \frac{11\alpha^4 r^4}{3456} \right) \right) \dots\dots\dots (27)$$

Substituting equation (22) and (27) into equation (10), we obtained the general solution as:

$$\phi(r) = A_3 J_0(\alpha r) + A_4 \left(J_0(\alpha r) \log r + \frac{\alpha^2 r^2}{4} \left(1 - \frac{3\alpha^2 r^2}{32} + \frac{11\alpha^4 r^4}{3456} \right) \right) \quad \text{-----} \quad (28)$$

Since the solution is bounded, $A_4 = 0$, then equation (28) reduces to:

$$\phi(r) = A_3 J_0(\alpha r) \quad \text{-----} \quad (29)$$

Solving for the constant coefficient in equation (29) using the boundary condition in equation (9), we have:

$$\phi(r) = \frac{J_0(\alpha r)}{J_0(\alpha)} \quad \text{-----} \quad (30)$$

Where $J_0(\alpha r) = \left(1 - \frac{\alpha^2 r^2}{4} + \frac{\alpha^4 r^4}{64} - \frac{\alpha^6 r^6}{2304} + \dots \right)$ and $J_0(\alpha) = \left(1 - \frac{\alpha^2}{4} + \frac{\alpha^4}{64} - \frac{\alpha^6}{2304} + \dots \right)$

In order to study the effect on concentration on the fluid temperature, we shall substitute equation (30) into equation (7), which is:

$$r^2 \frac{d^2 \theta}{dr^2} + r \frac{d\theta}{dr} - \lambda^2 r^2 \theta = \frac{Rd_2}{J_0(\alpha)} r^2 J_0(\alpha r) \quad \text{-----} \quad (31)$$

Where $\lambda^2 = Rd_1 Pr$

The homogenous solution of equation (31) can be presented in the following form:

$$\theta_h = A_4 \theta_{01}(r) + A_5 \theta_{02}(r) \quad \text{-----} \quad (32)$$

$$\theta_{01}(r) = \sum_{n=0}^{\infty} a_n r^{n+m} \quad \text{-----} \quad (33)$$

Differentiate equation (33) and substitute the result into equation (31), we have:

$$\frac{d\theta_{01}}{dr} = \sum_{n=1}^{\infty} a_n (n+m) r^{n+m-1}, \quad \frac{d^2 \theta_{01}}{dr^2} = \sum_{n=2}^{\infty} a_n (n+m)(n+m-1) r^{n+m-2} \quad \text{-----} \quad (34)$$

Simplifying equation (35), we have:

$$\left(\sum_{n=2}^{\infty} a_n (n+m)(n+m-1) r^{n+m} \right) + \left(\sum_{n=2}^{\infty} a_n (n+m) r^{n+m} \right) + a_1 (m+1) r^{m+1} - \lambda^2 \left(\sum_{n=0}^{\infty} a_n r^{n+m+2} \right) = 0 \quad (35)$$

Let $n = k + 2$ in the lowest degree term of equation (35), and simplify, we have:

$$\sum_{k=0}^{\infty} a_{k+2} (k+m+2)^2 r^{k+m+2} - \lambda^2 \sum_{n=0}^{\infty} a_n r^{n+m+2} + a_1 (m+1) r^{m+1} = 0 \quad \text{-----} \quad (36)$$

Simplifying equation (36), we have:

$$a_{k+2} = \frac{\lambda^2 a_k}{(k+m+2)^2} \tag{37}$$

$$a_1 = 0 \tag{38}$$

Upon solving for the coefficients for $k = 0, 1, 2, 3, 4$, we have:

$$\forall k : a_{k+2} = \frac{\lambda^2 a_k}{(m+k+2)^2}, a_1 = 0 \tag{39}$$

Expressing equation (33) fully, we have:

$$\theta_{01}(r) = \sum_{n=0}^{\infty} a_n r^{n+m} = a_0 r^m + a_1 r^{m+1} + a_2 r^{m+2} + a_3 r^{m+3} + a_4 r^{m+4} + a_5 r^{m+5} + a_6 r^{m+6} + a_7 r^{m+7} + \dots \tag{40}$$

Substituting the constants from the series, we have:

$$\theta_{01}(r) = \sum_{n=0}^{\infty} a_n r^{n+m} = a_0 r^m + \frac{\lambda^2 a_0 r^{m+2}}{(m+2)^2} + \frac{\lambda^4 a_0 r^{m+4}}{(m+2)^2 (m+4)^2} + \frac{\lambda^6 a_0 r^{m+6}}{(m+2)^2 (m+4)^2 (m+6)^2} + \dots \tag{41}$$

Simplifying equation (41), we have:

$$\theta_{01}(r) = \sum_{n=0}^{\infty} a_n r^{n+m} = a_0 r^m \left(1 + \frac{\lambda^2 r^2}{(m+2)^2} + \frac{\lambda^4 r^4}{(m+2)^2 (m+4)^2} + \frac{\lambda^6 r^6}{(m+2)^2 (m+4)^2 (m+6)^2} + \dots \right) \tag{42}$$

$$\text{Where: } a_2 = \frac{\lambda^2 a_0}{(m+2)^2}, a_3 = \frac{\lambda^2 a_1}{(m+3)^2}, a_4 = \frac{\lambda^2 a_2}{(m+4)^2}, a_4 = \frac{\lambda^4 a_0}{(m+2)^2 (m+4)^2}, a_5 = \frac{\lambda^2 a_3}{(m+5)^2},$$

$$a_5 = \frac{\lambda^4 a_1}{(m+3)^2 (m+5)^2}, a_6 = \frac{\lambda^2 a_4}{(m+6)^2}, a_6 = \frac{\lambda^6 a_0}{(m+2)^2 (m+4)^2 (m+6)^2}, a_7 = \frac{\lambda^2 a_5}{(m+7)^2}$$

$$a_7 = \frac{\lambda^6 a_1}{(m+3)^2 (m+5)^2 (m+7)^2}$$

To get the first solution, let $m = 0$ in equation (41), then, we have:

$$\theta_{01}(r) = \sum_{n=0}^{\infty} a_n r^n = a_0 \left(1 + \frac{\lambda^2 r^2}{2^2} + \frac{\lambda^4 r^4}{2^2 \cdot 4^2} + \frac{\lambda^6 r^6}{2^2 \cdot 4^2 \cdot 6^2} + \dots \right) = a_0 I_0(\lambda r) \tag{43}$$

$$\text{Where } I_0(\lambda r) = \left(1 + \frac{\lambda^2 r^2}{4} + \frac{\lambda^4 r^4}{64} + \frac{\lambda^6 r^6}{2304} \right) \text{ and } I_0(\lambda) = \left(1 + \frac{\lambda^2}{4} + \frac{\lambda^4}{64} + \frac{\lambda^6}{2304} \right)$$

To get the second independent solution, we differentiate equation (42) with respect to m then

$$\theta_{02} = \frac{\partial \theta_{01}}{\partial m} = \frac{\partial}{\partial m} \left[a_0 r^m \left(1 + \frac{\lambda^2 r^2}{(m+2)^2} + \frac{\lambda^4 r^4}{(m+2)^2 (m+4)^2} + \frac{\lambda^6 r^6}{(m+2)^2 (m+4)^2 (m+6)^2} + \dots \right) \right] \dots (44)$$

Simplifying equation (44), we have:

$$\theta_{02} = \frac{\partial \theta_{01}}{\partial m} = \left[\begin{array}{l} a_0 r^m \left(1 + \frac{\lambda^2 r^2}{(m+2)^2} + \frac{\lambda^4 r^4}{(m+2)^2 (m+4)^2} + \frac{\lambda^6 r^6}{(m+2)^2 (m+4)^2 (m+6)^2} + \dots \right) \log r \\ -2a_0 \lambda^2 r^2 r^m \frac{\left(1 + \frac{2(m+3)r^2 \lambda^2}{(m+4)^3} + \frac{\lambda^4 r^4 (44 + 3m(m+8))}{(m+4)^3 (m+6)^3} \right)}{(m+2)^3} \end{array} \right] (45)$$

If we evaluate $\left. \frac{\partial \theta_{01}}{\partial m} \right|_{m=0}$, then equation (45) becomes:

$$\theta_{02} = \left. \frac{\partial \theta_{01}}{\partial m} \right|_{m=0} = \left[a_0 \left(1 + \frac{\lambda^2 r^2}{4} + \frac{\lambda^4 r^4}{64} + \frac{\lambda^6 r^6}{2304} + \dots \right) \log r - \frac{a_0 \lambda^2 r^2}{4} \left(1 + \frac{6r^2 \lambda^2}{64} + \frac{44\lambda^4 r^4}{13824} \right) \right] \dots (46)$$

Simplifying equation (46), we have:

$$\theta_{02} = a_0 \left(I_0(\lambda r) \log r - \frac{\lambda^2 r^2}{4} \left(1 + \frac{3r^2 \lambda^2}{32} + \frac{11\lambda^4 r^4}{3456} \right) \right) \dots (47)$$

Substituting equation (43) and (47) into equation (32), we obtained the homogenous solution as:

$$\theta_h(r) = A_6 I_0(\lambda r) + A_7 \left(I_0(\lambda r) \log r - \frac{\lambda^2 r^2}{4} \left(1 + \frac{3r^2 \lambda^2}{32} + \frac{11\lambda^4 r^4}{3456} \right) \right) \dots (48)$$

Since the solution is bounded, $A_7 = 0$, then equation (48) reduces to:

$$\theta_h(r) = A_6 I_0(\lambda r) \dots (49)$$

$$\text{where } J_0(\alpha r) = \left(1 - \frac{\alpha^2 r^2}{4} + \frac{\alpha^4 r^4}{64} - \frac{\alpha^6 r^6}{2304} + \dots \right) \text{ and } J_0(\alpha) = \left(1 - \frac{\alpha^2}{4} + \frac{\alpha^4}{64} - \frac{\alpha^6}{2304} + \dots \right)$$

The particular solution of equation (31) can be represented as:

$$\theta_{0p} = -\frac{Rd_2}{\lambda^2 J_0(\alpha)} J_0(\alpha r) + \dots (50)$$

The solution of equation (31) is:

$$\theta(r) = A_6 I_0(\lambda r) - \frac{Rd_2}{\lambda^2 J_0(\alpha)} J_0(\alpha r) \quad \text{-----} \quad (51)$$

Solving for the constant coefficient in equation (51) using the boundary condition in equation (9), we have:

$$\theta(r) = \frac{Rd_2}{\lambda^2} \left(\frac{I_0(\lambda r)}{I_0(\lambda)} - \frac{J_0(\alpha r)}{J_0(\alpha)} \right) \quad \text{-----} \quad (52)$$

To investigate the impact of the fluid temperature on blood flow, we shall substitute equation (52) into equation (6), which is:

$$r^2 \frac{d^2 w}{dr^2} + r \frac{dw}{dr} - M_1^2 r^2 w = r^2 \left[\frac{\partial P}{\partial x} - \frac{Rd_2 Gr}{\lambda^2} \left(\frac{I_0(\lambda r)}{I_0(\lambda)} - \frac{J_0(\alpha r)}{J_0(\alpha)} \right) \right] \quad \text{-----} \quad (53)$$

where $M_1^2 = \left(\frac{1}{k} + M^2 \cos \alpha_2 \right)$

The homogenous solutions of equations (53) can be express in the form of power series:

$$w_{01}(r) = \sum_{n=0}^{\infty} a_n r^{n+m} \quad \text{-----} \quad (53)$$

Solving the homogenous part of equation (52) using equation (53), we have:

$$\left(\sum_{n=2}^{\infty} a_n (n+m)(n+m-1) r^{n+m} \right) + \left(\sum_{n=2}^{\infty} a_n (n+m) r^{n+m} \right) + a_1 (m+1) r^{m+1} - M_1^2 \left(\sum_{n=0}^{\infty} a_n r^{n+m+2} \right) = 0 \quad (54)$$

Let $n = k + 2$ in the lowest degree term of equation (54), so that it can be reduce to

$$\sum_{k=0}^{\infty} a_{k+2} (k+m+2)^2 r^{k+m+2} - M_1^2 \sum_{n=0}^{\infty} a_n r^{n+m+2} + a_1 (m+1) r^{m+1} = 0 \quad \text{-----} \quad (55)$$

Simplifying equation (55), we have:

$$a_{k+2} = \frac{M_1^2 a_k}{(k+m+2)^2} \quad \text{-----} \quad \text{-----} \quad (56)$$

$$a_1 = 0 \quad \text{-----} \quad (57)$$

Expressing equation (53) fully, we have:

$$w_{01}(r) = \sum_{n=0}^{\infty} a_n r^{n+m} = a_0 r^m + a_1 r^{m+1} + a_2 r^{m+2} + a_3 r^{m+3} + a_4 r^{m+4} + a_5 r^{m+5} + a_6 r^{m+6} + a_7 r^{m+7} + \dots \quad (58)$$

Upon solving for the coefficients in equation (56) for $k = 0, 1, 2, 3, 4$, we have:

$$\text{where: } a_2 = \frac{M_1^2 a_0}{(m+2)^2}, a_3 = \frac{M_1^2 a_1}{(m+3)^2}, a_4 = \frac{M_1^2 a_2}{(m+4)^2}, a_4 = \frac{M_1^4 a_0}{(m+2)^2 (m+4)^2}, a_5 = \frac{M_1^2 a_3}{(m+5)^2},$$

$$a_5 = \frac{M_1^4 a_1}{(m+3)^2 (m+5)^2}, a_6 = \frac{M_1^2 a_4}{(m+6)^2}, a_6 = \frac{M_1^6 a_0}{(m+2)^2 (m+4)^2 (m+6)^2}, a_7 = \frac{M_1^2 a_5}{(m+7)^2},$$

$$a_7 = \frac{M_1^6 a_1}{(m+3)^2 (m+5)^2 (m+7)^2}$$

Substituting the constants from the series constant coefficients into equation (58), we have:

$$w_{01}(r) = \sum_{n=0}^{\infty} a_n r^{n+m} = a_0 r^m \left(1 + \frac{M_1^2 r^2}{(m+2)^2} + \frac{M_1^4 r^4}{(m+2)^2 (m+4)^2} + \frac{M_1^6 r^6}{(m+2)^2 (m+4)^2 (m+6)^2} + \dots \right) \quad (59)$$

To get the first solution, let $m = 0$ in equation (59), we have:

$$w_{01}(r) = \sum_{n=0}^{\infty} a_n r^n = a_0 \left(1 + \frac{M_1^2 r^2}{2^2} + \frac{M_1^4 r^4}{2^2 \cdot 4^2} + \frac{M_1^6 r^6}{2^2 \cdot 4^2 \cdot 6^2} + \dots \right) = a_0 I_0(M_1 r) \quad (60)$$

$$\text{where } I_0(M_1 r) = \left(1 + \frac{M_1^2 r^2}{4} + \frac{M_1^4 r^4}{64} + \frac{M_1^6 r^6}{2304} \right) \text{ and } I_0(M_1) = \left(1 + \frac{M_1^2}{4} + \frac{M_1^4}{64} + \frac{M_1^6}{2304} \right)$$

To get the second independent solution, we differentiate equation (59) with respect to m then

$$w_{02} = \frac{\partial w_{01}}{\partial m} = \frac{\partial}{\partial m} \left[a_0 r^m \left(1 + \frac{M_1^2 r^2}{(m+2)^2} + \frac{M_1^4 r^4}{(m+2)^2 (m+4)^2} + \frac{M_1^6 r^6}{(m+2)^2 (m+4)^2 (m+6)^2} + \dots \right) \right] \quad (61)$$

Simplifying equation (61), we have:

$$w_{02} = \frac{\partial w_{01}}{\partial m} = \left[a_0 r^m \left(1 + \frac{M_1^2 r^2}{(m+2)^2} + \frac{M_1^4 r^4}{(m+2)^2 (m+4)^2} + \frac{M_1^6 r^6}{(m+2)^2 (m+4)^2 (m+6)^2} + \dots \right) \log r \right. \\ \left. - 2a_0 M_1^2 r^2 r^m \frac{\left(1 + \frac{2(m+3)r^2 M_1^2}{(m+4)^3} + \frac{M_1^4 r^4 (44 + 3m(m+8))}{(m+4)^3 (m+6)^3} \right)}{(m+2)^3} \right] \quad (62)$$

If we evaluate $\left. \frac{\partial w_{01}}{\partial m} \right|_{m=0}$, then equation (62) becomes:

$$w_{02} = a_0 \left(I_0(M_1 r) \log r - \frac{M_1^2 r^2}{4} \left(1 + \frac{3r^2 M_1^2}{32} + \frac{11M_1^4 r^4}{3456} \right) \right) \quad (63)$$

The homogenous solution is the sum of equations (63) and (60), which is

$$w_h(r) = A_8 I_0(M_1 r) + A_9 \left(I_0(M_1 r) \log r - \frac{M_1^2 r^2}{4} \left(1 + \frac{3r^2 M_1^2}{32} + \frac{11M_1^4 r^4}{3456} \right) \right) \dots\dots\dots (64)$$

The particular solution of equation (53) can be represented as

$$w_{0p} = B_1 + B_8 I_0(\lambda r) + B_9 J_0(\alpha r) + \dots \dots\dots (65)$$

Simplifying equation (65), we have:

$$w_{0p} = -\frac{\partial P}{M_1^2 \partial x} + \frac{Rd_2 Gr}{M_1^2 \lambda^2 I_0(\lambda)} I_0(\lambda r) - \frac{Rd_2 Gr}{M_1^2 \lambda^2 J_0(\alpha)} J_0(\alpha r) + \dots \dots\dots (66)$$

$$\text{where: } B_1 = -\frac{\partial P}{M_1^2 \partial x}, B_8 = \frac{Rd_2 Gr}{M_1^2 \lambda^2 I_0(\lambda)}, B_9 = -\frac{Rd_2 Gr}{M_1^2 \lambda^2 J_0(\alpha)}$$

The solution of equation (53) is the sum of equations (66) and (64), which is:

$$w(r) = A_8 I_0(M_1 r) - \frac{\partial P}{M_1^2 \partial x} + \frac{Rd_2 Gr}{M_1^2 \lambda^2 I_0(\lambda)} I_0(\lambda r) - \frac{Rd_2 Gr}{M_1^2 \lambda^2 J_0(\alpha)} J_0(\alpha r) \dots\dots\dots (67)$$

In order to solve for the constant coefficient in equation (67), we use the boundary conditions in equation (9), so that:

$$w(r) = \frac{P}{M_1^2} \left(\frac{I_0(M_1 r)}{I_0(M_1)} - 1 \right) + \frac{Rd_2 Gr}{M_1^2 \lambda^2} \left(\frac{I_0(\lambda r)}{I_0(\lambda)} - \frac{J_0(\alpha r)}{J_0(\alpha)} \right) \dots\dots\dots (68)$$

4. Results

The results presented here are classified into three sections, the first section displays the plots for lipid concentration profile, the second section displays plots for the blood temperature profile, and the third section deals with plots for blood velocity profiles for different values of the pertinent entering parameters, such as: chemical reaction parameter, Schmidt number, Prandtl number, magnetic field parameter, Grashof number, porosity parameter respectively.

4.1. Effect of chemical reaction and Schmidt number on concentration profile

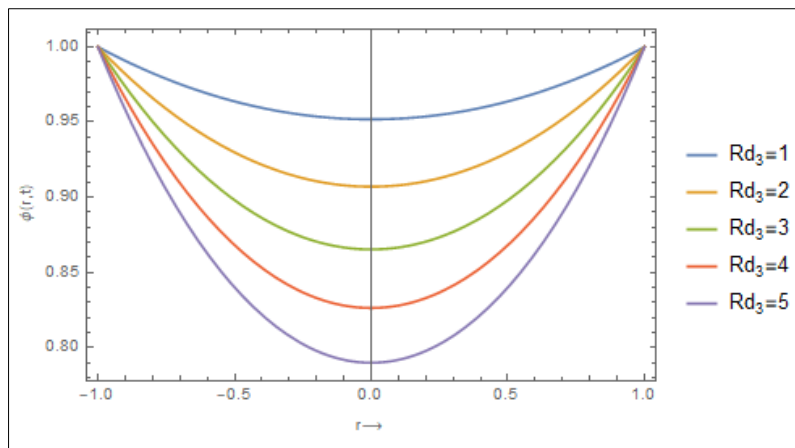


Figure 2 Effect of chemical reaction on concentration profile with other parameter value $Sc = 0.2$

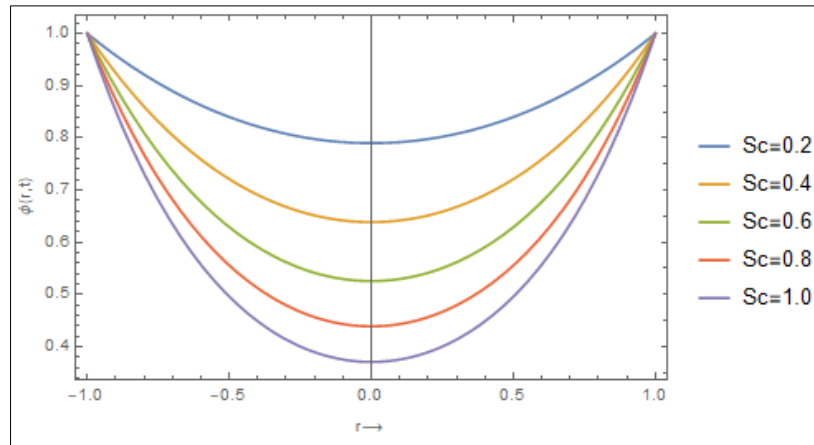


Figure 3 Effect of Schmidt number on concentration profile with other parameter value $Rd_2 = 5$

4.2. Effect of Prandtl number and Schmidt number on blood temperature profile

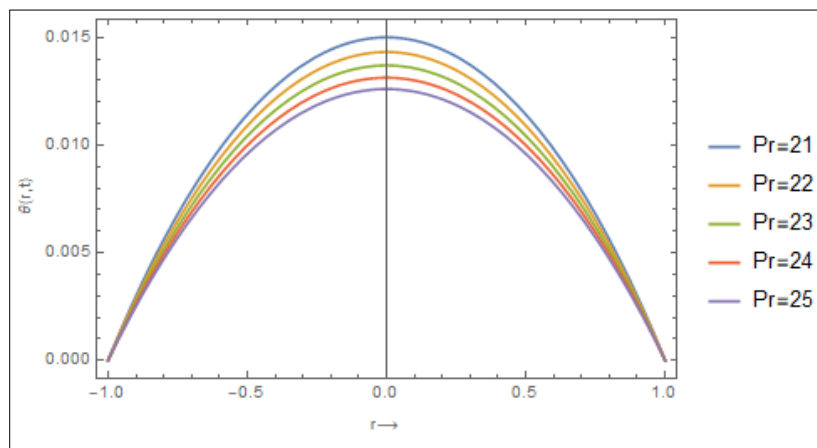


Figure 4 Effect of Prandtl number on blood temperature profile with other parameter values $Sc = 0.2, Rd_1 = 2, Rd_2 = 3, Rd_3 = 5$

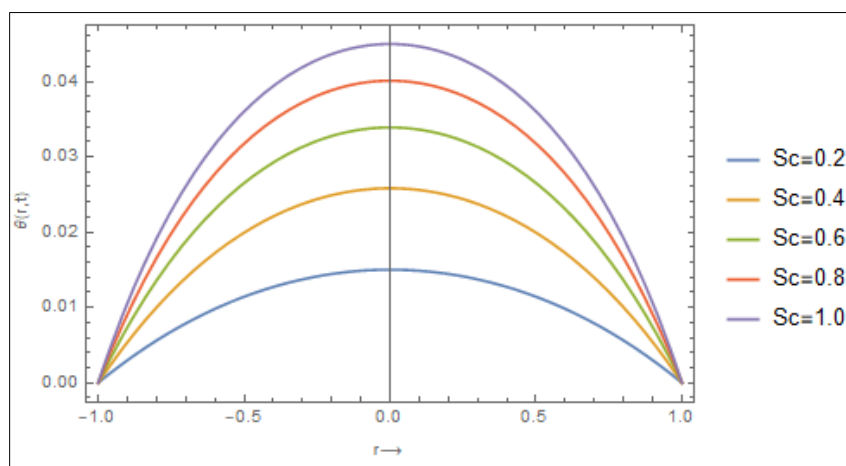


Figure 5 Effect of Schmidt number on blood temperature profile with other parameter values $Pr = 21, Rd_1 = 2, Rd_2 = 3, Rd_3 = 5$

4.3. Effect of Prandtl number, angle of inclination, Schmidt number, magnetic field parameter, Grashof number and porosity parameter on blood velocity profile

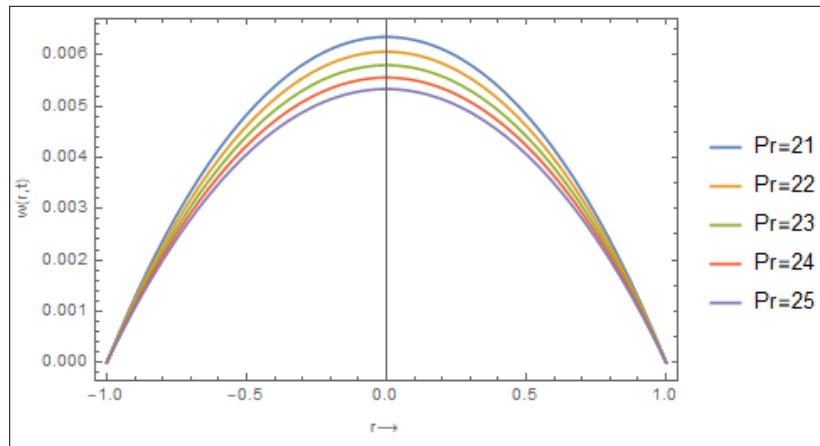


Figure 6 Effect of Prandtl number on blood velocity profile with other parameter values $Sc = 0.2, Gr = 15, M = 5, k = 0.05, \alpha = 30, Rd_1 = 2, Rd_2 = 3, Rd_3 = 5$

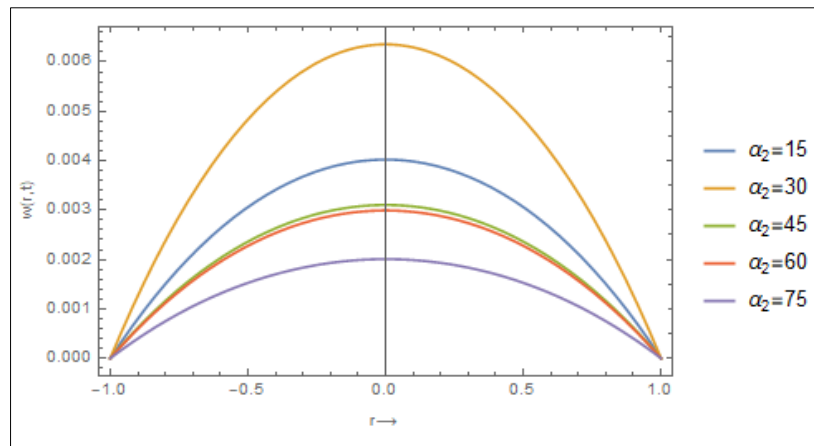


Figure 7 Effect of angle of inclination on blood velocity profile with other parameter values $Sc = 0.2, Pr = 21, Gr = 15, M = 5, k = 0.05, Rd_1 = 2, Rd_2 = 3, Rd_3 = 5$

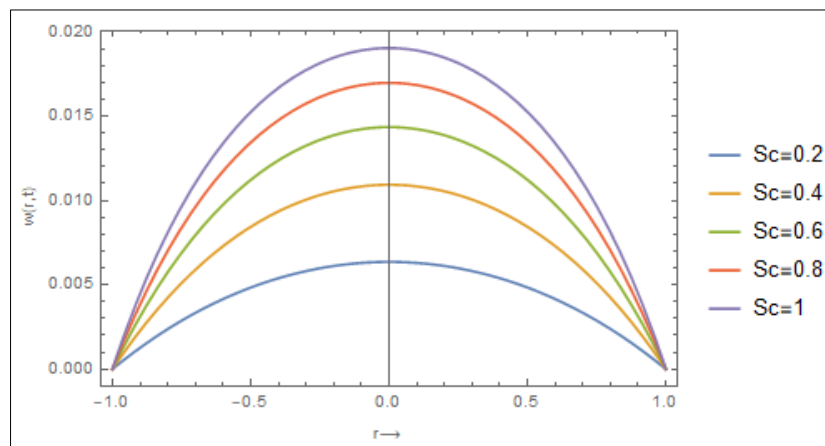


Figure 8 Effect of Schmidt number on blood velocity profile with other parameter values $Sc = 0.2, Pr = 21, Gr = 15, M = 5, k = 0.05, \alpha = 30, Rd_1 = 2, Rd_2 = 3$

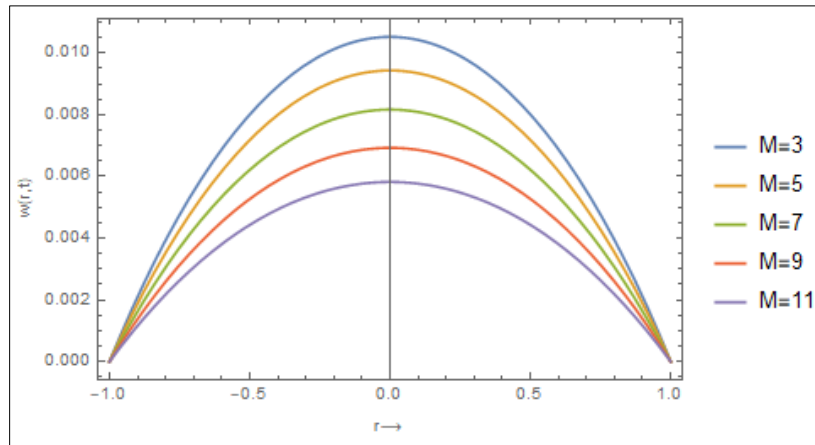


Figure 9 Effect of magnetic field on blood velocity profile with other parameter values $Sc = 0.2, Pr = 21, Gr = 15, M = 5, k = 0.05, \alpha = 30, Rd_1 = 2, Rd_2 = 3$

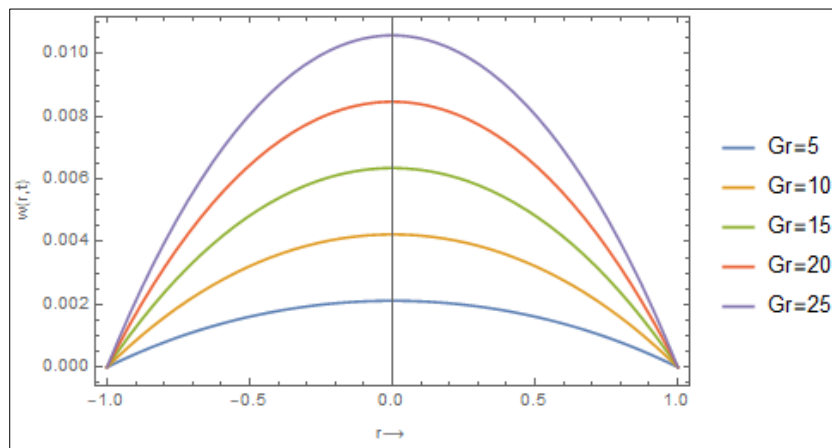


Figure 10 Effect of Grashof number on blood velocity profile with other parameter values $Sc = 0.2, Pr = 21, Gr = 15, M = 5, k = 0.05, \alpha = 30, Rd_1 = 2, Rd_2 = 3$

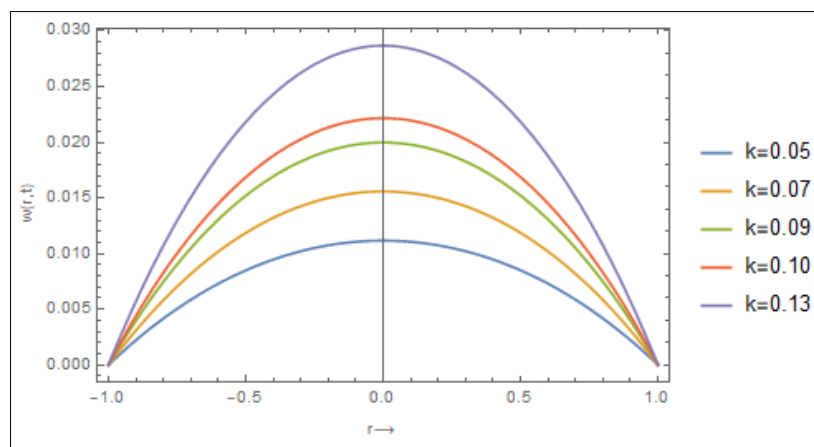


Figure 11 Effect of porosity on blood velocity profile with other parameter values $Sc = 0.2, Pr = 21, Gr = 15, M = 5, k = 0.05, \alpha = 30, Rd_1 = 2, Rd_2 = 3$

5. Discussion

In this section, we shall discuss the effect of various pertinent parameters on the concentration profile independently before discussing the parameters derived as a result of dimensional transformation on the blood temperature and blood velocity profiles. The discussion is as follows: Figure 2 shows the effect of chemical reaction parameters on mass concentration in the fluid with the value of the Schmidt number. This result supports the view that the mass concentration decreases with increasing values of the mass concentration. The concentration attained different minimums at 0.951811, 0.907006, 0.865262, 0.826293, and 0.789848 before they converged at 1 when the boundary thickness is 1. However, with the chemical reaction level observed at units, this increase attained a different minimum increase as seen in Figure 3. Figure 4 shows the effect of Prandtl number on the blood temperature profile. The result indicates that the blood temperature decreases with different values of the Prandtl number. However, this result was assisted by the following parameter values $Sc = 0.2, Rd_1 = 2, Rd_2 = 3, Rd_3 = 5$: Figure 5 shows the effect of the Schmidt number on the blood temperature profile with other parameter values. This result supports the view that the blood temperature increases with an increase in Schmidt number. This increase attains different maximums of 0.0150108, 0.0258189, 0.0338921, 0.0400946, and 0.044967 for the different increases in Schmidt number before converging to zero. In Figure 6, we notice that the blood velocity decreases with a different value increase in Prandtl number while other parameters remain constant. The effect of the angle of inclination $\alpha_2 = 15, 30, 45, 60, 75$ was investigated, and the result in Figure 7 shows that the blood velocity increases for the first two units before decreasing for the remaining units, with other parameter values. Figure 8 shows the effect of Schmidt number on blood velocity with the other contributing parameter values. The figure shows that blood velocity increases for different values of Schmidt's number. It has been observed in Figure 9, that the increase in magnetic field causes the flow to reduce. We noticed different velocities such as 0.0105274, 0.00943829, 0.0081704, 0.00692928, and 0.00582351 for the magnetic field intensity. Thereafter, each of the maximum velocities begins to reduce until it gets to zero when the boundary thickness becomes maximum. The reduction in velocity was due to the interaction between the applied magnetic field and the electrically conducting fluid, which resulted in an opposing force called the Lorentz force. Figure 10 shows the effect of Grashof number on blood velocity, while the other parameter values are thereby contributing. The results show that an increase in Grashof number $Gr = 5, 10, 15, 20, 25$ results in an increase in blood velocity. However, the fluid velocity was lowest from the starting value of the Grashof number but increased to a maximum velocity at the centre of blood velocity when the Grashof number became the highest. Thereafter, the fluid velocity decreases as the boundary thickness continuously increases. Figure 11 shows the effect of porosity on blood flow through a vessel with other parameter values. The figure indicates that the velocity of the fluid increases in the vessel as the porosity increases $k = 0.05, 0.06, 0.07, 0.08, 0.09$, that is, as the fluid velocity converges to zero when the boundary thickness gets to the peak value.

Nomenclature

x^*	Dimensional coordinate along the channel
r^*	Dimensional coordinate perpendicular to the channel
R_0	Radius of normal channel
P_0	Systolic pressure
Q_0	Dimensional heat source
Q_1	Dimensional concentration source
Rd_1	Radiation parameter
Rd_2	Heat due to mass concentration
Rd_3	Chemical reaction parameter
k_{Tb}	Blood thermal conductivity
w^*	Dimensional velocity profile
w	Dimensionless velocity profile
w_0	Perturbed velocity profile
C^*	Dimensional lipid particle concentration
C_∞	Far field cholesterol particle concentration

c_{bp}	The specific heat capacity of blood
T	Temperature of the fluid
T_{∞}^*	Far field temperature
T_w^*	Temperature at the wall
B_0	Magnetic field
M	Magnetic field parameter
α_2	Magnetic angle of inclination

Greek Symbols

ν_b	Kinematic viscosity of blood
μ_b	Dynamic viscosity of blood
Pr	Prandtl number for blood
g	Acceleration due to gravity
σ_e	Electrical conductivity
ω	Oscillatory frequency
θ	Dimensionless blood temperature
ϕ	Dimensionless cholesterol particle concentration
θ_0	Perturbed blood temperature profile
ϕ_0	Perturbed lipid concentration
ρ_b	Blood density

Subscripts

w	Wall
b	Blood
e	Electrical
T	Thermal
∞	Far field

6. Conclusion

In this research, we have investigated the steady blood flow through a porous microchannel with the influence of an inclined magnetic field mathematical model. The formulated models were solved using the Frobenius method (FM), which is a series-defined method. Considerations were made concerning the blood velocity, mass concentration, and blood temperature of the flowing fluid in the blood vessel. However, numerical simulation was carried out using the data obtained from Bunonyo and Amos [1] and Bunonyo and Ebiwareme [16]. Wolfram Mathematica, version 12 was used to perform the computation by varying the pertinent parameters within some specific range. The study enables us to conclude as follows:

- Grashof's number increases blood velocity significantly if found in the blood flow domain.
- The magnetic field increase retards blood velocity due to the formation of the Lorentz force.
- Blood velocity reduces because of the rise in Prandtl number. The decrease is due to the high level of dynamic viscosity over the thermal conductivity of blood.
- The blood velocity increases with an increase in Schmidt's number. The velocity was due to a greater kinematic viscosity over the molecular diffusivity of the fluid.
- The greater the level of the porosity, the greater the speed of the fluid passing through the blood vessel.
- The increase in magnetic angle of inclination caused the fluid velocity to increase before decreasing at its peak, though they all converged to zero when the boundary layer thickness was 1.

- The mass concentration in the blood decreases as the chemical reaction and Schmidt number increase. Hence, they both reduce mass concentration.
- The Prandtl number decreases the blood temperature while the Schmidt number increases the temperature of the fluid.

Compliance with ethical standards

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Disclosure of conflict of interest

We declare that there is no conflict of interest.

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