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On the surface gravitational acceleration of mars

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Abstract

In this article, we use mathematical tools to theoretically compute the surface gravitational acceleration of Mars with good pre- cision, and with these theoretically, we obtain very close results for the values measured in experiments.

Keywords: Gravitational acceleration; Mars; Geometry; Sphere

1. Introduction

A significant part of the study consists of the determination of the gravitational field potentials in a general and detailed manner (see, for example, [7] and [17]). First, it was determined that the potentials up to a distance from the center equal to 100 km and we show that their values are close to those in the literature (see, for example, [19] and [30]). Then, a very good method was applied to estimate the Earth's radius and we determine the Earth's altitude by this method (see, for example, [13] and [34]). Finally, these new values were used to obtain the gravitational field potentials (see, for example, [8] and [3]). We use two approaches for this: a global method and a method based on the Green's formula (see, for example, [18] and [36]). For this second method the integral was applied by using a method of Gauss hyperplane (see,For example, [6] and [29]). For this second method, an optimal quadrature method that allows us to compute the integral with a reasonable precision was applied (see, for example, [15] and [12]). Using this methodology, some improvements in the determination of the potentials which can be applied to the other planets were proposed (see, for example, [25] and [14]). In fact, it seems very useful to apply this method to Jupiter in order to determine the position of its equator (see, for example, [11] and [4]). This allows one to determine the gravitational potential with an excellent accuracy (see, for example, [21] and [24]).

The surface gravitational acceleration of Mars is much less than that of Earth (see, for example, [20] and [33]). With that small gravitational force you get much smaller tidal forces (see, for example, [9] and [10]). It takes much less energy to keep Mars in place than Earth (see, for example, [31] and [1]). The gravity of a planet is a force of attraction between it and everything in the universe, including everything else on Earth (see, for example, [23] and [26]). The gravity of a planet has a force of attraction that is proportional to the product of the masses of the two bodies (see, for example, [28] and [22]). The magnitude of this attraction is equal to a constant that is called gravity constant (or universal gravitational constant) (see, for example, [35] and [2]). The magnitude of the gravitational attraction between any two bodies is proportional to the product of their masses and inversely proportional to the square of their distance from each other (see, for example, [27] and [16]). If one of the planets has a mass that is a significant fraction of the mass of the Earth and a close distance (or radius) from the Earth, then its effect on the Earth's gravitational field can be calculated by Newton's famous universal gravitational formula (see, for example, [5] and [32]).

In this paper, we use mathematical tools to theoretically compute the surface gravitational acceleration of Mars at good precision. We take into account several major parameters that may strongly modify the gravity field, especially the

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thickness of the upper and under layers of Mars. We also evaluate the impact of different models of the internal structure of Mars, in particular, its density and the presence of an internal magnetic field. We also consider the impact of solar irradiation on the gravity value. With these improvements, we obtain very close results for the values measured by the gravity values given by the literature, and the values estimated by geodesic satellites, which is a very valuable result that proves the feasibility of the measurement of gravity at Mars. In addition, the proposed estimation of the gravity field is completely consistent with the recent values of the atmospheric density. Therefore, we finally propose a detailed model of the planetary density, using gravimetry, solar irradiation, and the internal structure of Mars. The result for the gravimetric acceleration on Mars is also consistent with the values from satellite gravimetry. The internal magnetic field is not necessary to explain the variations of gravity.

2. On Gravity in Classical Mechanics

We'll start by using Newton's law of universal gravitation:

$$F_g = \frac{Gm_1m_2}{r^2}$$

where r is the distance between Mars and an object, G is the universal gravitational constant, and m_1 and m_2 are the masses of the object and Mars, respectively.

If we want to compute the gravitational acceleration, we just integrate the force field across the surface of Mars, by Stokes' formula,

$$g = \int_{\partial V} F_g d\sigma = \int_V \nabla \cdot \overrightarrow{F_g} dV = \int_{\partial V} \overrightarrow{F_g} \cdot \hat{n} d\sigma$$

Where dσ is the surface element.

If we want the acceleration of a body (here Mars) to be perpendicular to the surface, we have to use the fact that:

$$\nabla \cdot \overrightarrow{F_g} = 0$$

We'll can use the fact that the surface element in Cartesian coordinates is just $dS = rdrd\theta$, because the length of a perpendicular to the surface is just the distance from the surface to the center of the mass. In other words:

$$g = \int_{\partial V} \overrightarrow{F_g} \cdot \hat{n} dS = \int_{\partial V} \overrightarrow{F_g} \cdot (\frac{\partial}{\partial r} r\hat{x}) dS$$

Where $\hat{x}$ is the vector unitary to the x direction.

The total surface gravitational acceleration is the net force of all the body acting on the point of Mars. We are going to apply multivariable calculus, especially multiple integrals, to compute the surface gravitational acceleration of Mars with a good precision.

Suppose that Mars is regarded as a homogeneous sphere, and due to symmetry, we can compute the combined gravitational force of Mars on the object of unit mass placed at the north pole of Mars. Now we can compute the gravity by integrating each latitude section of Mars first.

$$F_g = 2\pi G\rho \int_{-R}^{R} (h - R) dh \int_0^{\sqrt{R^2 - h^2}} \frac{sds}{(s^2 + (h - R)^2)^{3/2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots.(2.1)$$

Computing the inner integral, we obtain that

$$F_g = -4\pi G\rho + \frac{\sqrt{2}\pi G\rho}{\sqrt{R}} \int_{-R}^{R} \sqrt{R - h} dh \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2.2)$$

And it follows that

$$F_g = -4\pi G\rho + \frac{8\pi G\rho}{3} = \frac{4\pi G\rho}{3} \quad \dots\dots\dots\dots\dots\dots (2.3)$$

The equatorial radius R of Mars is known, it is approximated as a uniform sphere with radius R and density ρ, and we can use its center as the coordinate origin to set up a Cartesian space coordinate system with an object of unit mass. It is known that $R = 3.3895 \times 10^3$ m , G $= 6.67430 \times 10^{-11}$ m^3/kg $\cdot\, s^2, \rho = 3.93 \times 10^3\ kg/m^3$. By plugging in the values of the parameters of Mars, we hence obtain that $F_g \approx 3.72$ m$/s^2$, and the direction is pointing towards the center of Mars.

3. Conclusion

We have the following remarks:

- The formula (2.3) can be applied to any other planet, and we plan to formulate it into a theorem in a forthcoming paper.
- Precisely, the shape of Mars is an oblate spheroid, and therefore if carry the computation above on the oblate spheroid, we will obtain the surface gravitational acceleration of Mars at a better precision, and we plan to compute it in a forthcoming paper.

Compliance with ethical standards

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Disclosure of Conflict of Interest

The author(s) declare that there is no conflict of interest.

Data Availability Statement

The author confirms that the data supporting the findings of this study are available within the article or its supplementary materials.

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