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(Review Article)

Random noise test of ADCs with a sinusoidal stimulus

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Abstract

The recommended random noise test for analogue to digital converters (ADCs), as stated in the IEEE 1057 Standard for Digitizing Waveform Recorders, suggests the utilization of a triangular signal as the stimulus for the ADC under examination. However, we will demonstrate that an alternative option of employing a sinusoidal stimulus signal is equally feasible. This substitution provides enhanced flexibility during the testing process and enables the utilization of sine fitting algorithms.

Keywords: Analogue to Digital Converter; Random Noise Test; Sine Wave

1. Introduction

The measurement of the standard deviation of random noise in an analogue-to-digital converter (ADC) holds significant importance as it serves as a crucial parameter for evaluating ADC performance [1]-[4] and selecting the appropriate ADC for a specific application. Additionally, understanding the noise standard deviation in a test configuration is necessary for conducting other ADC tests, such as the Standard Static Test [1], the Standard Histogram Test [5]-[10], Small-wave Histogram Test [11] or the Sine-fitting Test [12]. These tests are used to estimate the error and precision of ADC parameters, and accurate knowledge of the noise standard deviation is instrumental in obtaining reliable results from these tests. There are other tests that focus on other non-ideal phenomena like jitter [13]-[14] and harmonic distortion.

The precision of the estimates derived from this test was analysed in [15]. Furthermore, [15] proposed an expression to determine the minimum number of samples necessary to ensure a specific level of uncertainty in the results. This calculation is crucial for minimizing the test duration as it enables the determination of the optimal number of samples required.

The test described in section 8.6.2 of [1] involves the synchronous acquisition of two sets of samples, each comprising a certain number (M) of samples. The noise standard deviation (σ) is then estimated by calculating the root mean square of the difference (*msd*) between the output codes of these two sets. If the noise standard deviation is sufficiently high, the test can be performed using a null input voltage. However, if the noise standard deviation is lower, a triangular stimulus signal should be employed instead.

Here it is claimed that a sinusoidal stimulus signal can also be used, which introduces enhanced flexibility to the test methodology. The primary advantage of this approach lies in its compatibility with conventional sine fitting algorithms, enabling the determination of the initial phase of the two data records. It is important to note that two records must be acquired for subsequent subtraction of sample values, aiming to eliminate systematic errors such as ADC non-linearity, gain and offset error, and stimulus signal distortion. Consequently, only random effects like random additive noise

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remain. It is worth mentioning that other random errors, such as amplitude or phase noise in the stimulus signal or jitter in the ADC, will contribute to the final test result. As a result, the value estimated by this test serves as an upper bound on the level of additive random noise present in the ADC under examination.

For systematic errors to be nullified, perfect alignment of the two records is crucial, requiring the acquisition to start precisely at the same point relative to the stimulus signal period. The IEEE 1057 standard recommends triggering the start of acquisition based on the stimulus signal voltage. However, the presence of amplitude noise affects this triggering process, as the instant of the first sample depends not only on the ideal stimulus signal value, as it should, but also on the random voltage noise present at that moment.

It is worth noting that, in principle, curve fitting could be applied to a triangular signal as well. However, it is important to acknowledge that triangular fitting procedures are not as straightforward as sine fitting methods.

The objective of this paper is to demonstrate that a sinusoidal stimulus signal can be effectively utilized, using the same estimator employed for the triangular stimulus signal. This serves as the main goal of the research. However, the intricate process of aligning the records using the sine fitting information, as well as a comprehensive assessment of the improvements offered by this approach compared to the traditional method, will be explored and detailed in a forthcoming publication.

Section 2 of the paper focuses on the analysis of the variance of the ADC output codes, taking into account three different types of stimuli: continuous (DC), triangular, and sinusoidal. The goal is to examine the impact of these stimuli on the variability of the ADC output.

Moving on to section 3, the paper proceeds to derive the estimator specifically for the sinusoidal stimulus case. This estimator is then compared to the estimators derived for the DC and triangular stimulus cases. By comparing these estimators, the researchers aim to assess the performance and effectiveness of the sinusoidal stimulus in terms of estimating the ADC output.

Lastly, in section 4, the paper presents the conclusions drawn from the analysis conducted in the previous sections. These conclusions summarize the findings of the study and highlight the key insights regarding the variance of ADC output codes under different stimulus types. The researchers may discuss the advantages and limitations of each stimulus type and provide recommendations for practical applications based on their conclusions.

2. Variance of the a DC Output Codes

2.1. DC Stimulus Signal

The random additive noise observed in analogue-to-digital converters (ADCs), as described in [1], refers to a non-deterministic fluctuation in the ADC output characterized by its frequency spectrum and statistical properties. Typically, the noise is assumed to be white, meaning it has a flat frequency spectrum, exhibits a stationary probability density function, and is independent of the stimulus signal.

In the presence of random noise at the ADC input, the output code (k) can be regarded as a discrete random variable capable of assuming any value between 0 and ($2^{n_b}-1$) for an n_b-bit ADC.

When the standard deviation of the additive noise exceeds the ADC's ideal code bin width (Q), the recommended approach in [1] involves short-circuiting the ADC input and acquiring two sets of samples (A and B). By subtracting the obtained codes from each other, fixed errors originating from the ADC can be eliminated, while the inherent random nature of the output codes is preserved.

In this context, normalized voltages will be employed, denoted in units of least significant bits (LSB), achieved by dividing the voltages with the ideal code bin width Q of the ADC. The normalized stimulus signal voltage will be denoted as y, while the normalized random noise voltage will be represented as r. Consequently, the normalized sampled voltage at a specific instant t_j can be expressed as follows:

$$u(t_j) = y(t_j) + r(t_j) \text{ ....(1)}$$

Given that the normalized additive noise has a mean of zero and a standard deviation represented by σ_r (σ/Q), the sampled voltage, which is also a random variable, has

$$\mu_u = y \quad \text{and} \quad \sigma_u = \sigma_r \text{ ....(2)}$$

Since the additive noise is considered here to be normally distributed, the sampled voltage probability density function is [16]

$$f_u(u \mid y) = \frac{1}{\sqrt{2\pi}\sigma_r} \cdot e^{-\frac{(u-y)^2}{2\sigma_r^2}} \text{ ....(3)}$$

and its distribution function is [10]

$$F_u(U \mid y) = \int_{-\infty}^{U} f_u(u) \cdot du = \frac{1}{2} + \frac{1}{2} \cdot erf\left(\frac{U-y}{\sqrt{2} \cdot \sigma_r}\right). \text{ ....(4)}$$

By subtracting the codes obtained from the two sample sets, the ADC can be viewed as exhibiting ideal behavior, as any fixed errors have been eliminated through the subtraction process. Consequently, random errors can be attributed to the presence of noise in the stimulus signal input.

The probability, denoted as p_k, of a sample having an output code equal to k is determined by the probability of the sampled voltage being equal to or lower than the transition voltage $T[k+1]$, while simultaneously being equal to or greater than the transition voltage $T[k]$. This applies specifically to the middle codes,

$$p_k = P\{U[k] \le u \le U[k+1]\} \quad , \quad k = 1, \ldots, 2^{n_b} - 2 \text{, ...(5)}$$

where we the normalized transition voltage $U[k]=T[k]/Q$ was used. The probability p_k can thus be expressed with the help of the sampled voltage distribution function:

$$p_k(y) = \begin{cases} F_u(U[1] \mid y) & , \quad k = 0 \\ F_u(U[k+1] \mid y) - F_u(U[k] \mid y) & , \quad k = 1, 2, \ldots, 2^{n_b} - 2 \\ 1 - F_u(U[2^{n_b} - 1] \mid y) & , \quad k = 2^{n_b} - 1 \end{cases} \text{ ....(6)}$$

The mean, second moment and variance of the output codes are, by definition [10],

$$\mu_{k|y} = \sum_{k=0}^{2^{n_b}-1} k \cdot p_k(y), \; m_{2k|y} = \sum_{k=0}^{2^{n_b}-1} k^2 \cdot p_k(y) \text{ , } \sigma_{k|y}^2 = m_{2k|y} - \mu_{k|y}^2 \text{ ....(7)}$$

2.2. Triangular Stimulus Signal

If the level of random noise in an ADC is relatively small compared to the ADC's ideal code bin width, the IEEE 1057 standard [1] recommends using a triangular stimulus signal that encompasses a range of approximately ten ADC codes. By analyzing the amplitude distribution, f_y, of this triangular stimulus signal [10], it becomes possible to calculate the variance of the output codes.

$$\sigma_k^2 = \int_{-\infty}^{\infty} \sigma_{k|y}^2(y) \cdot f_y(y) dy \text{ ....(8)}$$

For a triangular shaped stimulus signal, with an amplitude A and an offset C, which are both normalized by the ideal code bin width ($A_Q=A/Q$ and $C_Q=C/Q$) the amplitude distribution is

$$f_y(y)=\begin{cases}\dfrac{1}{2A_Q} & , \ |y-C_Q|<A_Q \\ 0 & , \ \text{otherwise}\end{cases} \quad \text{..........(9)}$$

The output codes' variance is consequently equal to

$$\sigma_k^2=\frac{1}{2A_Q}\int_{C_Q-A_Q}^{C_Q+A_Q}\sigma_{k|y}^2(y)dy \quad \text{..........(10)}$$

2.3. Sinusoidal Stimulus Signal

In the scenario where a sinusoidal stimulus signal is employed, the variance of the output codes can be determined in a similar manner as described in the previous paragraph, with the calculation relying on [16].

$$f_y(y)=\begin{cases}\dfrac{1}{\pi\sqrt{A_Q^2-(y-C_Q)^2}} & , \ |y-C_Q|<A_Q \\ 0 & , \ \text{otherwise}\end{cases} \quad \text{..........(11)}$$

Inserting it into (8) leads to the expression

$$\sigma_k^2=\int_{C_Q-A_Q}^{C_Q+A_Q}\sigma_{k|y}^2(y)\frac{1}{\pi\sqrt{A_Q^2-(y-C_Q)^2}}dy \quad \text{..........(12)}$$

3. Random Noise Estimators

The estimator for the random noise standard deviation is computed from the mean square difference:

$$msd=\frac{1}{M}\sum_{j=0}^{M-1}(ka_j-kb_j)^2 \quad \text{..........(13)}$$

The mean square difference obtained from the two sets possesses twice the variance of each individual set, as they are independent of one another. As stated in [16], the expected value of the mean square difference, as determined by equation (13), is twice the variance of the output codes:

$$E\{msd\}=\sigma_{ka}^2+\sigma_{kb}^2=2\sigma_k^2 \quad \text{..........(14)}$$

Taken this into account, and considering that the variance of the output codes is equal to that of the additive noise, the estimated variance of the noise is just given by

$$\sigma_r=\sqrt{\frac{msd}{2}} \quad \text{..........(15)}$$

The expected value of the estimated noise standard deviation can be approximated by utilizing equation (15).

$$\mathrm{E}\{\sigma_r\}\approx\sqrt{\frac{\mathrm{E}\{msd\}}{2}} \quad \text{..........(16)}$$

This approximation allows for the estimation of the noise standard deviation based on the calculated mean square difference. Inserting equation (14) leads to

$$\mathrm{E}\{\sigma_r\} \approx \sigma_k \,,\ldots\ldots\ldots(17)$$

where σ_k is given by equation (7).

Figure 1 illustrates that for small values of the random noise standard deviation, the expected value of the estimator significantly deviates from the actual value of the noise standard deviation. The dashed and dotted curves demonstrate this discrepancy.

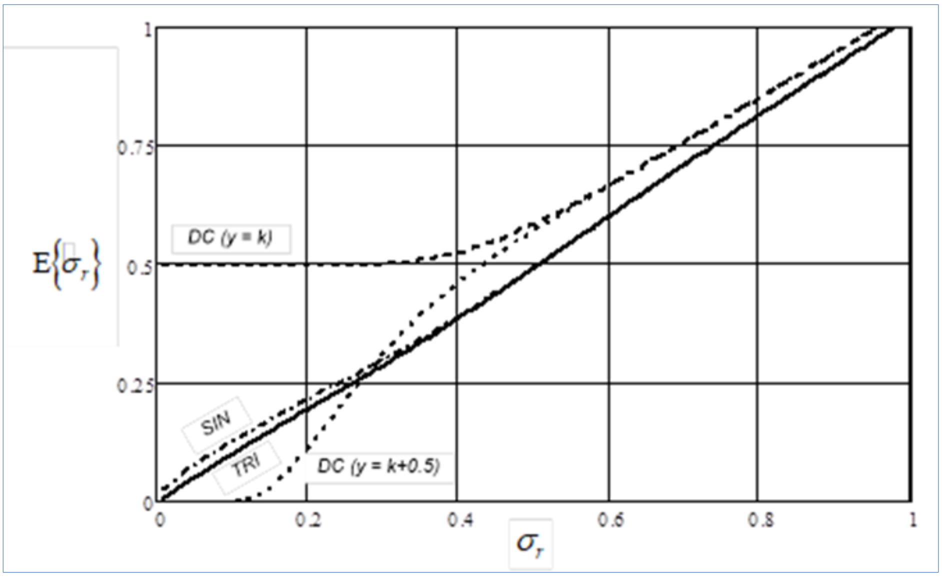

Figure 1 The graph displays the expected value of the estimated random noise as a function of the actual standard deviation of the random noise. It showcases three different stimulus types: a DC stimulus (represented by dashed and dotted lines), a triangular stimulus (represented by a solid line), and a sinusoidal stimulus (represented by a dash-dotted line). In the case of the DC stimulus, two scenarios are depicted: one where the DC value is equal to one of the ADC transition voltages (shown by the dashed line), and another where the DC value is precisely between two consecutive ADC transition voltages. Both the triangular and sinusoidal stimulus signals have an amplitude of 5 LSB, denoting their respective behaviors in the graph

In the case of a triangular stimulus signal, the estimator recommended in the IEEE 1057 standard is employed for estimating the noise standard deviation. This estimator is designed to analyze the differences between the output codes obtained from two sets of samples.

The process involves acquiring two sets of samples, labelled as A and B, during the test. The output codes from these two sets are subtracted from each other, effectively cancelling out any fixed errors introduced by the ADC. The remaining differences primarily reflect the random effects, such as random additive noise.

The estimator then calculates the mean square difference (*msd*) between the output codes of the two sets. This mean square difference serves as the basis for estimating the noise standard deviation. By using appropriate formulas and statistical analysis, the estimator provides an estimate of the noise standard deviation for the ADC under test.

It is worth noting that the accuracy and effectiveness of this estimator for the triangular stimulus signal have been validated and endorsed by the IEEE 1057 standard, making it a reliable method for assessing the noise performance of ADCs.

$$\sigma_r = \left[\left(\sqrt{\frac{msd}{2}}\right)^{-4} + \left(\frac{\sqrt{\pi}}{2}msd\right)^{-4}\right]^{\frac{-1}{4}} \ldots\ldots\ldots(18)$$

In the case of small random noise, the expression is determined by equations (6) and (7), taking into account that the stimulus signal spans only two ADC output codes. Under these conditions, one can observe the following:

- The mean square difference (*msd*) is approximately equal to the variance of the output codes (σ^2) when the random noise is negligible. This can be attributed to the fact that the dominant factor affecting the *msd* is the inherent variation in the ADC output codes.
- The *msd* is inversely proportional to the number of samples (*M*) when the random noise is small. This relationship arises due to the reduced contribution of random noise in the presence of a larger number of samples. As *M* increases, the impact of random noise on the *msd* diminishes, resulting in a more accurate estimation of the noise standard deviation.

By considering these extreme cases and analysing the behaviour of the *msd*, an approximate expression for the expected value of the estimator for the noise standard deviation can be obtained. Although heuristic in nature, this expression provides a practical means of estimating the noise standard deviation based on the observed *msd* and the characteristics of the stimulus signal.

$$\sigma_k^2 = \frac{1}{2A_Q}\int_{C_Q-A_Q}^{C_Q+A_Q}\left[\frac{1}{4}-\frac{1}{4}\cdot erf^2\left(\frac{U[k]-y}{\sqrt{2}\cdot\sigma_r}\right)\right]dy \quad , \quad \sigma_r \ll 1 \text{..........(19)}$$

Inserting (19) into (10) leads to

$$\lim_{\substack{\sigma_r\to 0\\ A_Q\to\infty}} \sigma_k^2 = \frac{1}{\sqrt{\pi}}\sigma_r \text{ . ...(20)}$$

Taking into account equations (14) and (20), a potential estimator suitable for scenarios involving a low level of random noise can be expressed as follows:

$$\sigma_r = \frac{\sqrt{\pi}}{2}msd \quad , \quad \sigma_r << 1 \text{..........(21)}$$

It is the combination of (15) and (21) that leads to (18). The expected value of (18) can be approximated by

$$\mathrm{E}\{\sigma_r\} \approx \left[\left(\sqrt{\frac{\mathrm{E}\{msd\}}{2}}\right)^{-4} + \left(\frac{\sqrt{\pi}}{2}\mathrm{E}\{msd\}\right)^{-4}\right]^{\frac{-1}{4}} \text{..........(22)}$$

Inserting (14) leads to

$$\mathrm{E}\{\sigma_r\} \approx \left[(\sigma_k)^{-4} + \left(\sqrt{\pi}\sigma_k^2\right)^{-4}\right]^{\frac{-1}{4}} \text{,..........(23)}$$

Where σ_k is given by (7). This expected value is shown in Figure 1 (solid curve) which proves that it is a good estimator of σ_r.

In order to utilize a sinusoidal stimulus signal, an estimator can be derived using a similar approach to that of the triangular stimulus signal. Instead of relying on equations (9) and (10), equations (11) and (12) would be employed. Remarkably, for the extreme scenario involving a small standard deviation of random noise and a large amplitude of the stimulus signal, the resulting expression remains the same as that derived for the triangular stimulus signal, namely equation (20).

Consequently, the estimator obtained through heuristic analysis for the sinusoidal stimulus case aligns with the estimator used for the triangular stimulus, specifically equation (18). The expected value of this estimator for the sinusoidal case is depicted in Figure 1 as a dash-dotted curve, demonstrating its similarity to the solid curve representing the triangular case.

In Figure 2 the error of the estimators, defined as

$$e_{\sigma_r} \triangleq \mathrm{E}\{\sigma_r\} - \sigma_r \text{ ,.........} \qquad (24)$$

is represented. When dealing with small quantities of random noise, both the triangular and sinusoidal estimators exhibit significantly lower errors compared to the DC estimator. In this case, the triangular and sinusoidal estimators prove to be more accurate and reliable in estimating the noise standard deviation.

However, when confronted with substantial levels of random noise, all three estimators perform equally well, displaying comparable levels of accuracy and reliability. In such situations, the choice between the triangular, sinusoidal, or DC estimator may not significantly impact the accuracy of the noise standard deviation estimation.

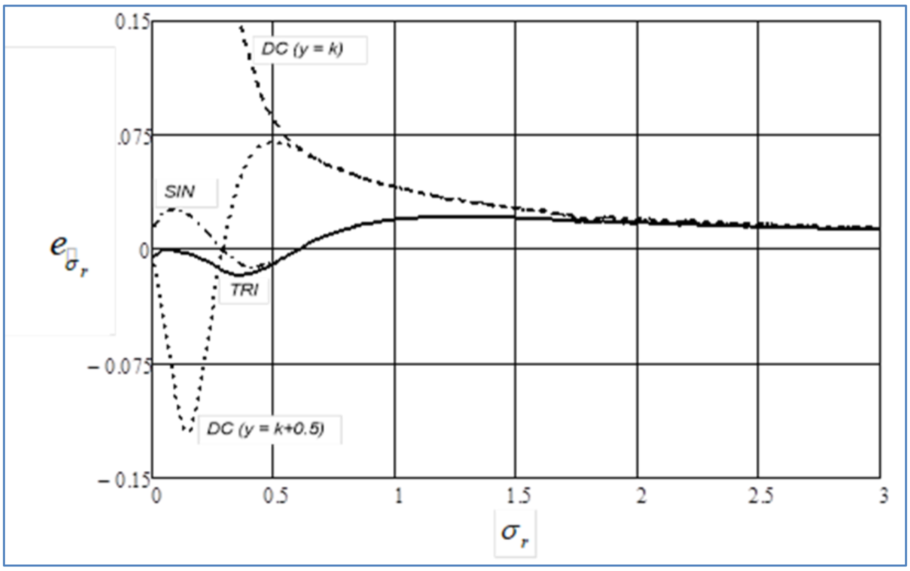

Figure 2 The graph illustrates the error of the estimators employed to determine the random noise standard deviation, plotted as a function of the actual standard deviation of the noise. The estimators are utilized for three different stimulus types: a DC stimulus (represented by dashed and dotted lines), a triangular stimulus (represented by a solid line), and a sinusoidal stimulus (represented by a dash-dotted line). Within the context of the DC stimulus, two scenarios are depicted: one where the DC value is equal to one of the ADC transition voltages (shown by the dashed line), and another where the DC value is precisely between two consecutive ADC transition voltages. Both the triangular and sinusoidal stimulus signals exhibit an amplitude of 5 LSB, reflecting their respective behaviors on the graph. The error of the estimators for each stimulus type provides insights into their performance and accuracy in estimating the random noise standard deviation

4. Conclusion

The research presented shows that stimulating an ADC with a sinusoidal signal is a viable alternative approach for estimating the random noise standard deviation in ADCs. Remarkably, the estimator expression proposed is the same as the one recommended by the IEEE 1057 standard. This proposed approach not only offers increased flexibility during the testing process but also introduces the possibility of incorporating sine fitting algorithms and record alignment techniques.

By employing a sinusoidal stimulus signal, the need for record triggering, which is essential when using the triangular stimulus signal, can be potentially eliminated. This presents an opportunity to alleviate a source of uncertainty in the estimation process. The application of sine fitting algorithms and record alignment techniques allows for the precise alignment of data records, leading to more accurate and reliable estimations of the random noise standard deviation.

By considering these advancements and adopting the proposed sinusoidal stimulus signal approach, it is possible to enhance the overall accuracy and precision of the random noise standard deviation estimation in ADCs, thereby reducing potential sources of uncertainty in the testing process.

Compliance with ethical standards

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Disclosure of conflict of interest

There is no conflict of interest.

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