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On the Approximation of the Bay of Bengal Domain to be Compatible for the Implementation of Finite Element Method

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Abstract

In this study, the Bay of Bengal domain has been approximated using triangular mesh so that the finite element method (FEM) can be employed on it. The area between 15° N and 23° N Latitudes and 85° E and 95° E Longitudes is considered as the physical domain. A MATLAB routine and the cubic spline interpolation have been used to extract the coordinates of the points on the boundary of the whole domain and the points on the boundary of the islands from a colour image of the domain. A C++ routine is used to generate an edp file for triangular mesh using the extracted coordinates. Then FreeFem++ is used to create triangular mesh for the whole domain from the generated edp file. All the major islands are also incorporated in the final mesh. The obtained triangular mesh can be used to develop a storm surge prediction model for the Bay of Bengal region implementing FEM.

Keywords: Approximation of Geometric Domain; Bay of Bengal; Triangular Mesh; Finite Element Method

1. Introduction

The coast of Bangladesh is affected by storm surge causing a tremendous loss of lives and properties every year. As in [4], on an average 5–6 storms form in this region every year, causing 80% of global casualties. The coastal belt of Bangladesh is very vulnerable. Thickly populated low-lying islands, highly curved coastal and island boundaries, river discharge are few major factors behind the vulnerability (see [1]). A proper warning system for the region can mitigate the sufferings of its people and live stocks resulting from these storm surges. Thus, an effective storm surge prediction model is highly desirable for the coastal region of Bangladesh. To develop an effective storm surge prediction model, the shallow water equations are the main mathematical equations which are solved numerically on a coastal geometry with some suitable initial and boundary conditions considering all the possible major factors which are responsible for high surge. It is of interest to note here that shallow water equations are solved on water only. Thus, the area of the geometric domain where water is present should be identified. For this reason, the costal and island boundaries should be identified properly. It is very important to note here that there are many big and small rivers as well as islands along the coastal region of Bangladesh (see [4]). To get an accurate result, the geometric domain should be approximated properly. Considering this fact into account, the Bay of Bengal domain was approximated in Cartesian coordinates and in polar coordinates in [2] and [6], respectively, for the implementation of finite difference method (FDM). It is known that the FDM is suitable for a domain of rectangular or square shape, but the Bay of Bengal domain is highly curved. It is also known that the FEM is more appropriate for a domain of irregular shape. It is to be noted here that both rectangular and triangular mesh are used for FEM, but triangular mesh is more suitable for a domain of complex shape. To consider the effect of islands and rivers, a higher mesh resolution is required near the coastal region compared to the deep water. In FDM, nesting methods are employed to overcome this problem. But it requires different approximation for different resolution and for each nesting the model equations should be solved separately, which is very time consuming. In FEM, higher mesh resolution can be considered anywhere within the domain in the same model. Considering this fact into account, the Bay of Bengal domain has been approximated roughly in [3] with the triangular

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mesh for the implementation of FEM without considering the islands and rivers. But to get an accurate result, islands should be considered (see [4]). Considering this fact into account the Bay of Bengal domain has been approximated in [5] with the triangular mesh for the implementation of FEM considering few major islands. But to get more appropriate result, all the major islands should be considered (see [4]). For this reason, we have intended to approximate the Bay of Bengal domain taking all the major islands into account with the triangular mesh which can be used to develop a storm surge prediction model for the Bay of Bengal region implementing FEM.

Figure 1 Colour image of the Bay of Bengal Domain (between Latitudes **15**°N and **23**°N and Longitudes **85**°E and **95**°E)

2. Methodology

A colour picture of the Bay of Bengal domain between Latitudes 15°N and 23°N and Longitudes 85°E and 95°E (see **Figure 1**) is generated through ArcGIS software. Since the earth is almost spherical and the radius of earth is $R = 6371.22\ km$, the actual distance between Latitudes 15°N and 23°N is $\left\{(23-15)\times\frac{\pi}{180}\times \mathrm{R}\right\} km = (8\times 0.0174532\times 6371.22)\ km\ = 889.59\ km$ (approximately) and the actual distance between Longitudes 85°E and 95°E is $\left\{(95-85)\times\frac{\pi}{180}\times \mathrm{R}\times\cos\left(\frac{19\pi}{180}\right)\right\} km = (10\times 0.0174532\times 6371.22\times 0.9455186)\ km\ = 1051.40\ km$ (approximately). To employ finite element method in solving equations on a domain of irregular shape it should be approximated with rectangular or triangular mesh. As triangular mesh gives more accurate result compared to rectangular mesh, in this study we have approximated the domain with triangular mesh. For this, the Paint software is firstly used to smooth the map (see **Figure 2**). To create triangular mesh for a domain we need coordinates of the points on the boundary of the domain as an input data. Thus, keeping the boundary, all parts of the figure have been omitted (see **Figure 3**). Set origin (0,0) at (15°N, 85°E). Then the domain is extended from 0 to 1051.40 km along x-axis and from 0 to 889.59 km along y-axis. Following [7], a MATLAB code has been written to extract the coordinates of the points on the boundary with the above setting of axes. The cubic spline interpolation is used in that code to make the boundary lines continuous. To employ the code a portion of the boundary line should be taken so that the curve represents a function. Let $x_0 < x_1 < \cdots < x_n$ be some given points where $y_i = f(x_i)$, $(i = 0, \dots, n)$ are known via cubic spline.

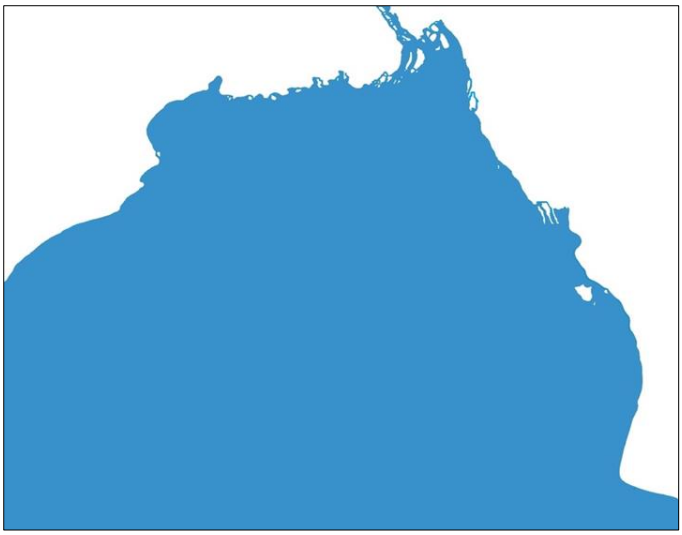

Figure 2 The smoothed map of Bay of Bengal created from **Figure 1**

Figure 3 Boundary of the Bay of Bengal domain

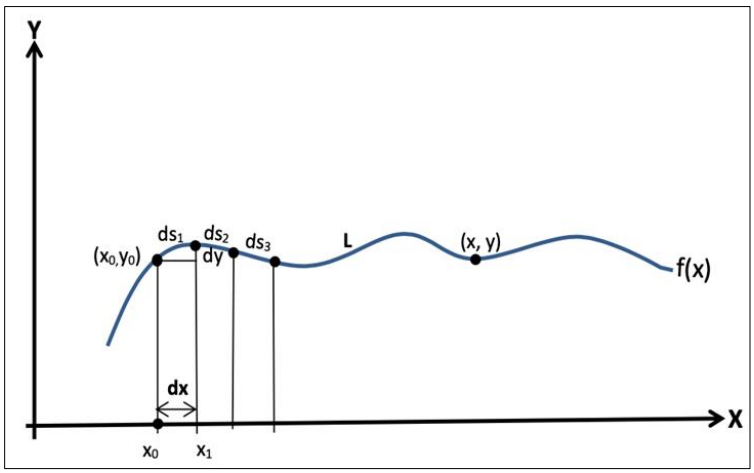

Figure 4 Computation of equidistant point along a curve

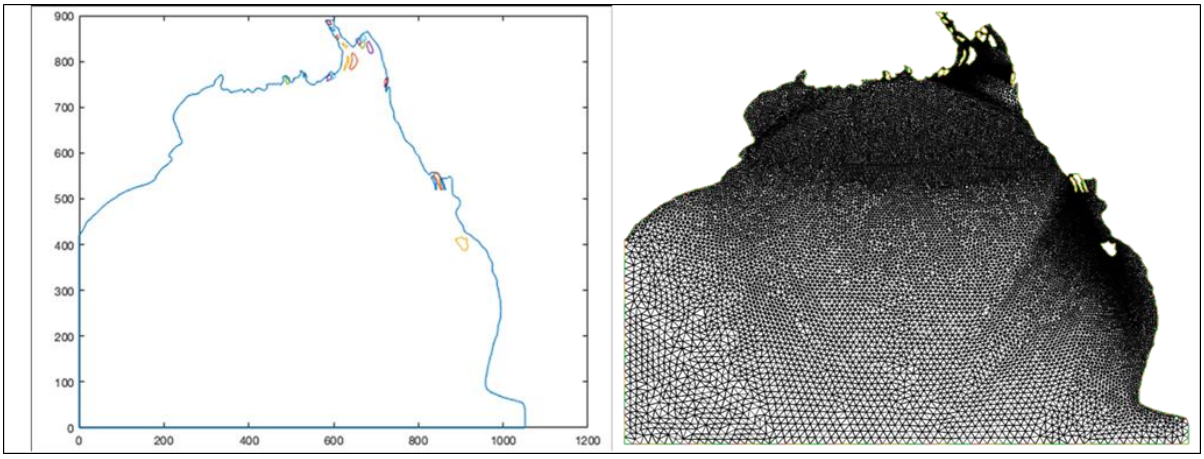

Figure 5 The whole domain without (left) and with (right) triangular mesh

interpolation. For any $x \in [x_0, x_1]$, $y = f(x)$ can be found. To create nice triangular mesh coordinates of equidistant points on the boundary should be given in anti-clockwise direction. For this reason, we need to find some coordinates of equidistant points on the curve. This can be done using the following technique. $y = f(x)$ be a given function and let (x_0, y_0) be a point on the given function. We need to find another point (x, y) on the function such that distance between (x_0, y_0) and (x, y) along the curve is L (see **Figure 4**). It is easy to calculate the Euclidean distance between (x_0, y_0) and (x, y) which is not equal to L. To find length along the curve we need to calculate arc length between those points. Let (x_1, y_1) be a point on the curve which is very close to (x_0, y_0) and let $dx_1 = x_1 - x_0$, $dy_1 = y_1 - y_0$ and ds_1 be the arc length between (x_0, y_0) and (x_1, y_1) (see **Figure 4**). Since ds_1 is very small, then

$ds_1 = \sqrt{({dx_1}^2 + {dy_1}^2)} = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}$. To find the point (x, y) choose very small dx and find $ds_1, ds_2, \ldots$ to get $L = ds_1 + ds_2 + \cdots ds_n$.

Algorithm

- Step-1: $dx = 0.0001$.
- Step-2: Calculate dy with the formulae $dy = f(x_0 + dx) - f(x_0)$.
- Step-3: Calculate $ds_1 = \sqrt{(dx^2 + dy^2)}$.
- Step-4: Update (x_0, y_0) as $x_0 = x_0 + dx$ and $y_0 = f(x_0 + dx)$
- Step-5: Save (x_0, y_0) as a new point at ds_1 distance.
- Step-6: Repeat the above process.
- Step-7: Condition: If $L - \varepsilon \leq ds_1 \leq L + \varepsilon$; $\varepsilon = 0.000001$, then stop.

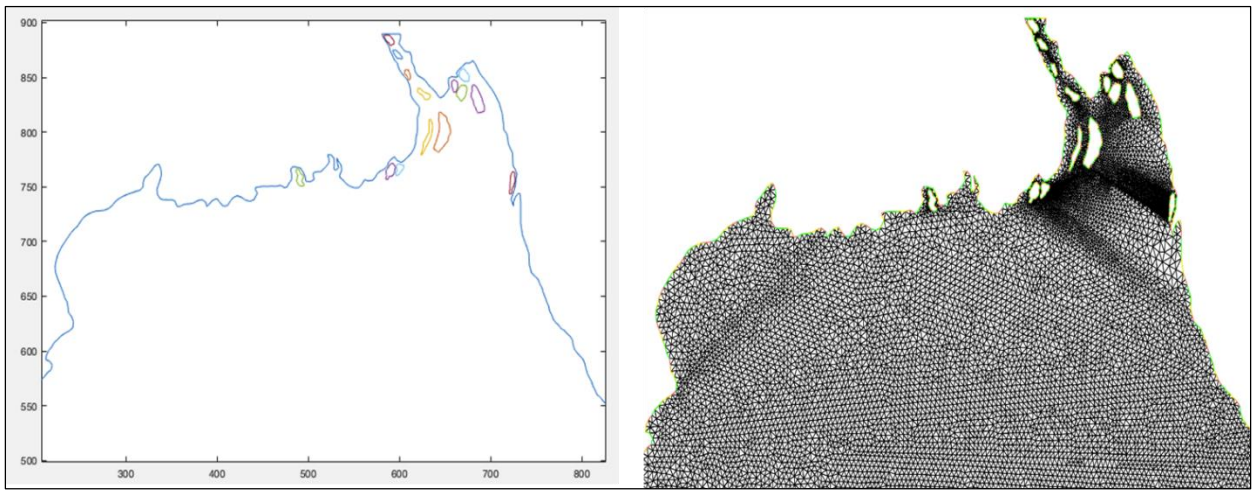

Figure 6 A part of the domain without (left) and with (right) triangular mesh

In a similar way the coordinates of the pointes on the boundary of islands are also extracted. It is to be noted here that the coordinates of the pointes on the boundary of islands should be given in clockwise direction to exclude islands from the mesh. A C++ code is written and used to generate an edp file for triangular mesh using the extracted coordinates. Then FreeFem++ is used to create triangular mesh for the whole domain from the generated edp file. Few major islands are also incorporated in the final mesh. The obtained triangular mesh can be used to develop a storm surge prediction model for the Bay of Bengal region implementing FEM.

3. Results and discussion

Figure 5 shows the whole domain before and after the approximation. It seems that the coastal and island boundaries are approximated properly. **Figure 6** and **Figure 8** show the coastal region of Bangladesh before and after the approximation. It is found from the figures that all the major islands have been excluded from the computational domain properly. **Figure 7** shows the eastern part of the Bay of Bengal domain before and after the approximation. It is found from the figure that the approximation has been done properly. Thus, the results show that the Bay of Bengal domain has been approximated by triangular mesh properly.

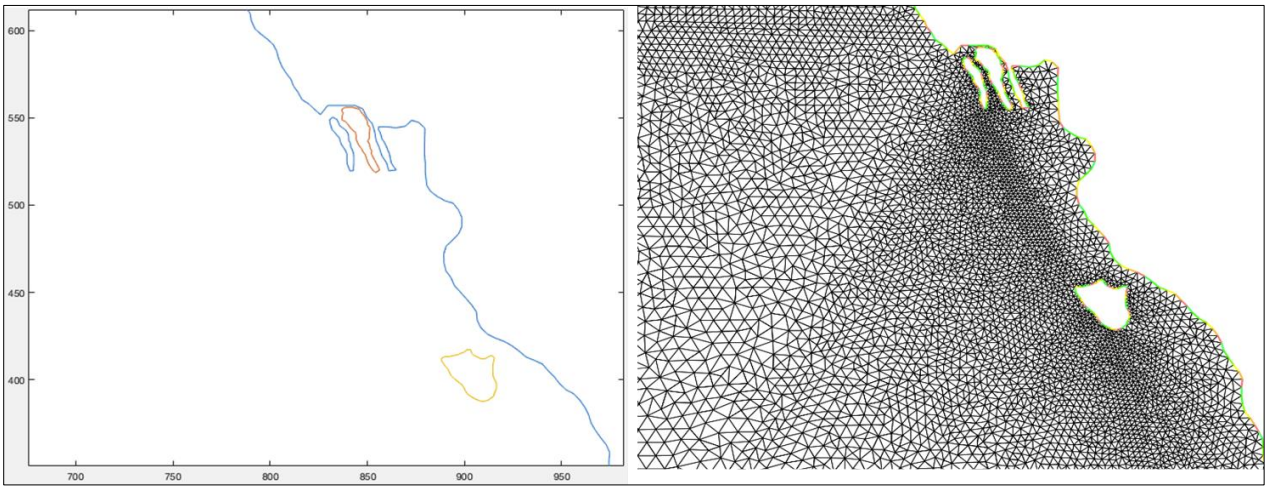

Figure 7 A part of the domain without (left) and with (right) triangular mesh

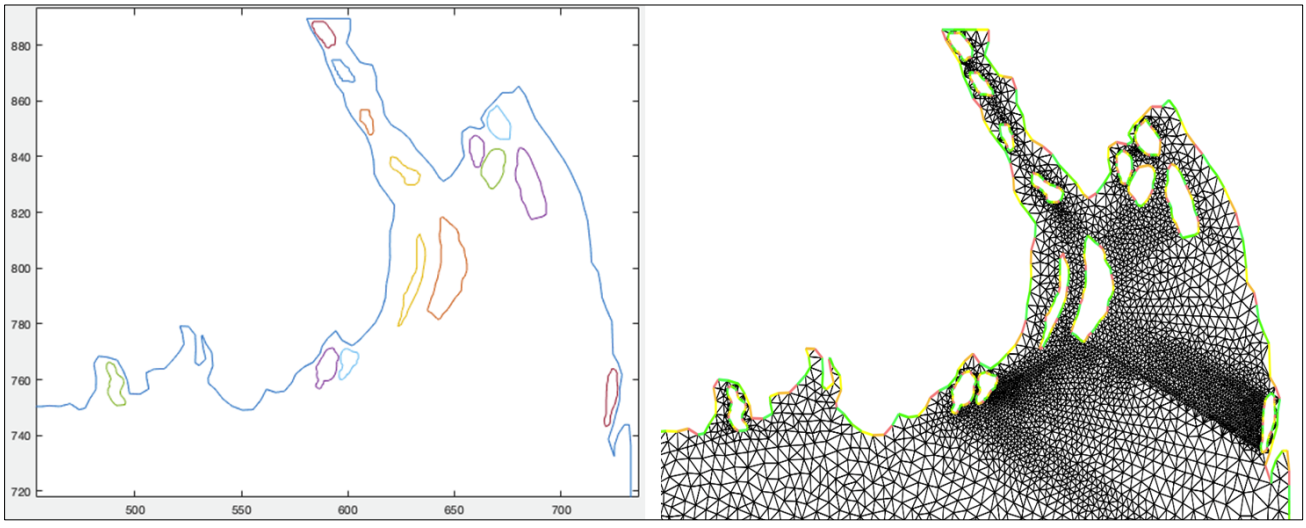

Figure 8 A part of the domain without (left) and with (right) triangular mesh

4. Conclusion

The approximation of the Bay of Bengal domain taking all the major islands into account with the triangular mesh has been done in this work. The results can be used to develop a storm surge prediction model for the Bay of Bengal region implementing FEM. This will help to develop a proper warning system for the coastal region of Bangladesh and can mitigate the sufferings of its people and live stocks resulting from storm surges.

Compliance with ethical standards

Acknowledgments

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Disclosure of conflict of interest

The authors declare no conflicts of interest regarding the publication of this paper.

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