



(REVIEW ARTICLE)



The power flow analysis for electric power network: A tutorial article

Anthony Hill*, Penrose Cofie, John Fuller, Justin Foreman, Kelvin Kirby, Emmanuel Dada, Olatunde Adeoye and Adeyemi Taylor.

Department of Electrical and Computer Engineering, Prairie View University, Prairie View, United States of America.

World Journal of Advanced Engineering Technology and Sciences, 2023, 10(02), 223–238

Publication history: Received on 06 November 2023; revised on 17 December 2023; accepted on 20 December 2023

Article DOI: <https://doi.org/10.30574/wjaets.2023.10.2.0305>

Abstract

The power system network is complex. This makes it more challenging to obtain valuable information because of its hundreds of buses and transmission lines. In addition, solving the steady-state equations of the power system network requires substantial mathematical computations. The power flow analysis is a study of the electrical power system. Engineers and utility companies use it for the design, control, planning, and future improvement of the electrical power network. Power flow analysis calculates the voltage magnitude, phase angles, active and reactive power flows, losses of the power system network under various loads, and generation conditions. This paper is a tutorial article aimed to present the power flow analysis techniques of Gauss-Seidel, Newton-Raphson, and Fast-Decoupled approaches. The Power World Simulator is used to demonstrate the software implementation of the approaches. The results show that Newton Raphson and Fast Decoupled have a faster convergence. The Newton Raphson method is the popular technique used for the analysis. An illustrative example for an IEEE 3-bus system on each technique is also presented in the article.

Keywords: Power Flow; Iteration; Convergence; Real Power; Reactive Power; Gauss-Seidel; Newton Raphson; Fast Decouple

1. Introduction

The power flow is also known as load flow, and both terms are interchangeably utilized. In a typical electric power system, power flows from the generation station through the transmission lines to the loads. The two main types of power that flow in a power system network are real and reactive power [1]. The load flow study is an essential part of the power system analysis that enables the engineers to plan and determine the steady-state solution of the entire power system network. Under the steady-state conditions, the load flow analysis offers an organized mathematical technique for computing the magnitude of the voltages and the angles at each bus or node, the real and reactive power at the reference bus or node, the phase angle of the voltage and the reactive power at the generator bus or node. In addition, the modeling of the power system is achieved by the electrical circuit of generators, transmission lines, and distribution networks [2, 3]. The load flow equations are expressed in terms of power that is nonlinear and are to be computed by iterative processes using numerical methods. The numerical methods employed in the technique involve solving mathematical problems using arithmetic operations that give only approximate solutions as the outcome.

In recent years, many mathematical analysis approaches have been used to solve power flow or load flow analysis problems [4]. The most generally used iterative techniques are Gauss-Seidel, Newton-Raphson, and Fast Decoupled methods [5]. The continuous increase in power demand leads to a vast increase in the size of the power system and the dimension of the power flow equations. This abrupt increase makes it more challenging for numerical mathematical approaches to converge to the correct solutions without the iteration process. The challenge faced nowadays by all engineers involved in the electrical designs of industrial and commercial power systems are determining the most appropriate method suitable for the power system analysis. In addition, since reliable convergence is the most critical

* Corresponding author: Hill Anthony

criterion for load flow analysis methods, high accuracy in terms of reliability and faster result time in terms of computational speed, determine the best acceptable method. The use of digital computers in load flow studies began in the mid-1950s, and before this time, the solutions obtained from using analog boards were inaccurate compared to practical applications [6]. Subsequently, several techniques for calculating load flow have since emerged. The load flow analysis methods have witnessed tremendous improvements as the most efficient computational procedures in power systems planning, design, and operation because their calculations result in solutions of some nonlinear equations. Aside from this, manual solutions are impractical except for calculating some examples of circuit operating characteristics. The advent of digital computers makes load flow analysis studies in power systems efficient and reliable compared to the past because modern large-scale computers have sufficient speed and size to handle power system network calculations. In addition, solving network equations is more efficient with the development of hardware algorithms. The improved performance of these algorithms makes load flow analysis solutions easier to achieve in terms of high computational speed, simplicity, flexibility, low computer storage, and reliability. Thus, the application of the numerical methods using the computer remains the most consistent technique for arriving at the best solution.

This paper is a tutorial article aimed to present the power flow analysis techniques of Gauss-Seidel, Newton-Raphson, and Fast-Decoupled approaches using the Power World Simulator to demonstrate an example of software implementation of the approaches. Also, hand calculations of a typical IEEE 3-bus system present numerical solutions of the various methods.

1.1. Fundamentals of power network analysis.

The complexity of the power system network, indicates that power flows through many paths or branches of hundreds of buses or nodes and transmission links. Hence, such systems form networks of series and parallel paths. Based on Kirchoff's laws, the anyalsis of the power network requires that the electric power flow in the systems divides between the branches proportionately. In network analysis, there is a need for electrically distinct reference points called nodes or buses. The buses are nodes with one or more lines or loads connected to the generators. Figure 1 illustrates a simple 3-bus power network for anylasis.

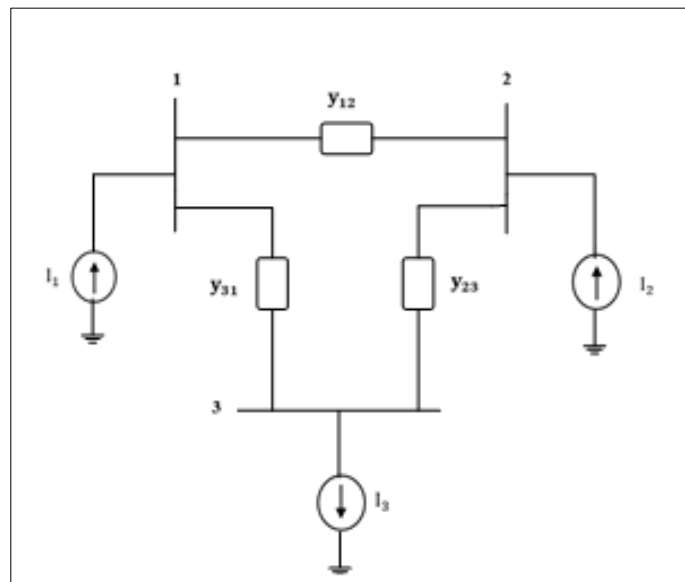


Figure 1 A 3-bus system

With reference to Figure 1, applying Kirchoff's current law and taking current injection into a bus as postive, the equations are as follows:

$$\text{Node 1: } I_1 = y_{12} (V_1 - V_2) + y_{13}(V_1 - V_3) \dots\dots\dots(1)$$

With $y_{12} = y_{21}$, $y_{23} = y_{32}$, and $y_{13} = y_{31}$ representing the admittance of the lines.

$$\text{Node 2: } I_2 = y_{12} (V_2 - V_1) + y_{23}(V_2 - V_3) \dots\dots\dots (2)$$

$$\text{Node 3: } -I_3 = y_{31} (V_3 - V_1) + y_{23}(V_3 - V_2) \dots\dots\dots (3)$$

Rearranging Equations (1), (2), and (3) gives,

$$I_1 = V_1(y_{12} + y_{13}) + V_2(-y_{12}) + V_3(-y_{13}) \dots\dots\dots(4)$$

$$I_2 = V_1(-y_{21}) + V_2(y_{21} + y_{23}) + V_3(-y_{23}) \dots\dots\dots (5)$$

$$-I_3 = V_1(-y_{31}) + V_2(-y_{32}) + V_3(y_{31} + y_{23}) \dots\dots\dots (6)$$

In general, for an n-bus power network the node equations at bus (*i*) is expressed as,

$$I_i = \sum_{j=1}^n Y_{ij} V_j \dots\dots\dots (7)$$

Where Y_{ij} is expressed using both, Equations (8) and (9).

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} y_{12} + y_{13} & -y_{12} & -y_{13} \\ -y_{21} & y_{21} + y_{23} & -y_{23} \\ y_{31} & y_{32} & -(y_{31} + y_{23}) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \dots\dots\dots(8)$$

The power flow nodal equations for a 3-bus system in matrix form are as follows,

$$\begin{bmatrix} I_1 \\ I_2 \\ -I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \dots\dots\dots (9)$$

$$\begin{aligned} \text{In Equation 9, } Y_{11} &= y_{12} + y_{13}, Y_{12} = -y_{12}, Y_{13} = -y_{13} \\ Y_{21} &= -y_{21}, Y_{22} = y_{21} + y_{23}, Y_{23} = -y_{23} \\ Y_{31} &= y_{31}, Y_{32} = y_{32}, Y_{33} = -(y_{31} + y_{23}) \end{aligned}$$

Thus, Equation 9 can generally be expressed as $Y_{bus}V = I \dots\dots\dots(10)$

1.2. Power Flow Equations

In the power systems, the loads and generations are specified in terms of power as in Equation (11).

$$S = P + jQ = VI^* \dots\dots\dots(11)$$

Where S is the complex power with P and Q being the real and reactive components respectively.

Therefore, the current, I in Equation (8) can be replaced from Equation (11) as

$$I = \frac{P-jQ}{V^*} \dots\dots\dots(12)$$

Equation (8) then becomes,

$$\begin{bmatrix} \frac{P_1-jQ_1}{V_1^*} \\ \frac{P_2-jQ_2}{V_2^*} \\ \frac{P_3-jQ_3}{V_3^*} \end{bmatrix} = \begin{bmatrix} y_{12} + y_{13} & -y_{12} & -y_{13} \\ -y_{21} & y_{21} + y_{23} & -y_{23} \\ y_{31} & y_{32} & -(y_{31} + y_{23}) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \dots\dots\dots(13)$$

1.3. General form of solution Procedures

In Equation (13), the branches of networks can be represented by their respective branch admittance, and the voltages and currents in the network are related by the general form as follows,

$$\begin{bmatrix} \frac{P-jQ}{V^*} \end{bmatrix} = [Y][V] \dots\dots\dots(14)$$

The general form of Equation (13) is expressed in Equation (14); this constitutes a nonlinear system of equations that cannot be simplified using closed-form matrix procedures but requires methods involving numerical and iterative approaches.

For an n bus system, the nodal equation in general form for the i^{th} bus is as follows,

$$I_i = \sum_{j=1}^n Y_{ij}V_j \text{ for } i = 1,2,3 \dots n \dots\dots\dots(15)$$

2. Classification of buses

In power systems, the three classifications of buses are slack (swing), generation, and load buses. Each of these buses has known and unknown variables of four quantities namely real power P , reactive power Q , voltage magnitude $|V|$, and the voltage phase angle δ [7, 8]. The unknown variables are determined by writing systems of equations of the form shown in Equation 13 with one equation for each node and solving the equation using the numerical method [9]. The buses classification is based on the specified and unspecified variables as shown in Table 1. Figure 1 can be compared to Figure 2, which shows generator slack bus 1, generator PV bus 3, and the PQ load bus 2.

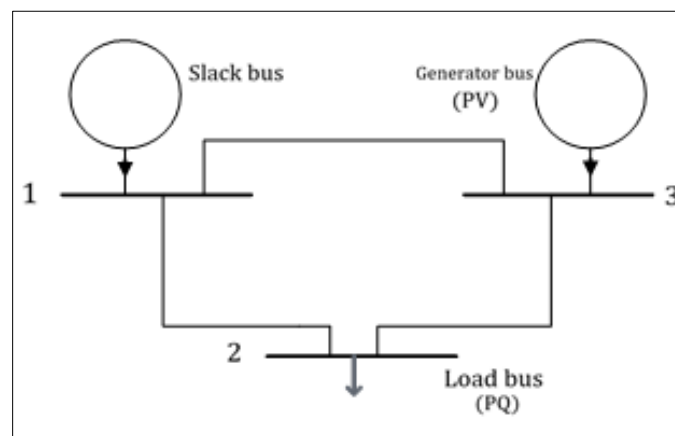


Figure 2 3-bus power system

2.1. Generator Bus (PV)

This is a regulated bus. The real power P and the voltage magnitude $|V|$ are known. For the regulated bus, it is required to determine the reactive power Q and the phase angle of the voltage δ because both are unknown. See bus 3 in Figure 2.

2.2. Slack Bus

This is a swing bus and is generally considered as the reference bus. The slack bus provides the mismatch between the scheduled generation and the net system load with the losses and net generation. The voltage magnitude $|V|$, and voltage angle δ are specified, while the real power P and the reactive power Q are calculated using the load flow voltages solution substituted in Equations 18 and 19, [4, 7, 9]. This bus is indicated as bus 1 in Figure 2.

2.3. Load Bus (PQ)

The bus 3 in Figure 2 is also referred to as a PQ bus without any connected generator, and in practical power systems, most buses are load buses. The real power P and the reactive power Q are given at this bus, and both the voltage magnitude $|V|$, and the phase angle of the voltage δ are calculated because they are unknown.

Table 1 Classification of Busses

No	Type of bus	Known Variables	Unknown Variables
1	Slack/Swing	$\delta, [V]$	P, Q
2	Generator/Regulated (PV)	$P, [V]$	Q, δ
3	Load (PQ)	P, Q	$\delta, [V]$

3. Load Flow Analysis

The load flow or power flow analysis is fundamental in interconnected power system analysis because its mathematical calculations and the simultaneous algebraic equations are the basis for the solution of the performance equations in computer-aided electrical power systems analyses [10]. The load flow analysis results have applications in economic scheduling and design of power systems, verification of system overloading conditions, optimal operation of every single generator, and ensuring uninterrupted system operation and security during systems maintenance.

3.1. Gauss-Seidel Technique

This technique is the most straightforward iterative technique to solve load flow problems. This technique involves an initial guess of the voltage value to get a calculated value of a specific variable. After that, the guess value is substituted by the computed value. The repetition of this process continues till the iteration solutions converge. This technique has poor convergence characteristics but has applications in load flow with fewer buses [11]. Load flow problems are solved by applying the iterative approach to Equation (16) [3, 12].

$$\frac{P_i - jQ}{V_i^*} = V_i \sum_{j=1}^n Y_{ij} - \sum_{j=1}^n y_{ij} V_j \quad j \neq i \dots\dots\dots(16)$$

Applying the Gauss-Seidel iterative technique to Equation (16) for the value of the voltage magnitude V_i yields Equation (17), where both real power and reactive power are scheduled for the load buses, in iterative sequence.

$$V_i^{(k+1)} = \frac{\frac{P_i^{sch} - jQ_i^{sch}}{V_i^*} + \sum y_{ij} V_j^{(k)}}{\sum y_{ij}} \quad j \neq i \dots\dots\dots(17)$$

Therefore, applying Kirchhoff's current law to Equation (17) and injecting a positive current into bus i makes the real and reactive power delivered into the generator buses P_i^{sch} and Q_i^{sch} have positive values. The load buses P_i^{sch} and Q_i^{sch} have negative values because of the real and reactive power flow out of the load buses. The calculated P_i and Q_i using Equation (16) is given in Equations (18-20) [13, 14].

$$P_i^{(k+1)} = \text{Real} [V_i^{*(k)} \{ \sum_{i=0}^n y_{ij} - \sum_{ji} V_i^{(k)} \}] \quad j \neq i \dots\dots\dots(18)$$

$$Q_i^{(k+1)} = \text{Imaginary} [V_i^{*(k)} \{ \sum_{j=1}^n y_{ij} - \sum_{ji} V_i^{(k)} \}] \quad j \neq i \dots\dots\dots(19)$$

The power flow equation is usually stated in terms of the bus admittance matrix, and applying the matrix's nondiagonal elements and the bus admittance's diagonal elements to Equation (17) gives Equations (20), (21), and (22) [3, 13].

$$V_i^{(k+1)} = \frac{\frac{P_i^{sch} - jQ_i^{sch}}{V_i^*} - \sum y_{ij} V_j^{(k)}}{Y_{ii}} \dots\dots\dots(20)$$

$$P_i^{(k+1)} = \text{Real} [V_i^{*(k)} \{ V_i^{*(k)} Y_{ii} + \sum_{i=1, j=1}^n y_{ij} V_j^{(k)} \}] \quad j \neq i \dots\dots\dots(21)$$

$$Q_i^{(k+1)} = \text{Imaginary} [V_i^{*(k)} \{ V_i^{*(k)} Y_{ii} + \sum_{i=1, j=1}^n y_{ij} V_j^{(k)} \}] \quad j \neq i \dots\dots\dots(22)$$

In this iterative technique, *PQ* buses iteration is performed before proceeding with the *PV* buses iteration. In addition, *P* and *V* are specified at the *PV* bus, whereas *Q* and δ are computed because they are not specified. And, after every iteration, the values of *Q* and δ are updated.

3.2. Newton-Raphson Technique

This iterative technique uses Taylor's series expansion approximation to obtain a set of linear simultaneous equations from a set of nonlinear simultaneous equations [3, 11]. The technique is widely used to solve cases that lead to divergence due to its high-speed convergence characteristics and excellent reliability compared to other iterative approaches used in power flow analysis [11] [12]. The Newton-Raphson iterative results are fast, provided the guess value is near the solution, while the results take longer if the guess value is far from the solution [13]. Due to its various applications, the Newton-Raphson technique is effective and practical for massive power systems. The following Equations develop the Newton Raphson technique. Re-writing Equation (15) in polar form results in Equation (23) [13, 14].

$$I_i = \sum_{j=1}^n |Y_{ij}| |V_j| (\theta_{ij} + \delta_j) \dots\dots\dots(23)$$

The complex power at bus *i* is as follows,

$$P_i - jQ_i = V_i^* I_i \dots\dots\dots(24)$$

We obtain Equation (25) by substituting Equation (23) into Equation (24)

$$P_i - jQ_i = |V_i| < -\delta_i \sum_{j=1}^n |Y_{ij}| |V_j| (\theta_{ij} + \delta_j) \dots\dots\dots(25)$$

Equation (26) and Equation (27) are the result of the separation of real and reactive powers.

$$P_i = \sum_{j=1}^n |V_i| |V_j| |Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \dots\dots\dots(26)$$

$$Q_i = - \sum_{j=1}^n |V_i| |V_j| |Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \dots\dots\dots(27)$$

Equations (26) and (27) are approximated in Taylor's series since they are a set of nonlinear algebraic equations and neglecting all the higher-order terms gives a set of linear equations of Equation (28) [3].

$$\begin{bmatrix} \Delta P_2^{(k)} \\ \vdots \\ \Delta P_n^{(k)} \\ \Delta Q_2^{(k)} \\ \vdots \\ \Delta Q_n^{(k)} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2^{(k)}}{\partial \delta_2} & \dots & \frac{\partial P_2^{(k)}}{\partial \delta_n} & \frac{\partial P_2^{(k)}}{\partial |V_2|} & \dots & \frac{\partial P_2^{(k)}}{\partial |V_n|} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial P_n^{(k)}}{\partial \delta_2} & \dots & \frac{\partial P_n^{(k)}}{\partial \delta_n} & \frac{\partial P_n^{(k)}}{\partial |V_2|} & \dots & \frac{\partial P_n^{(k)}}{\partial |V_n|} \\ \frac{\partial Q_2^{(k)}}{\partial \delta_2} & \dots & \frac{\partial Q_2^{(k)}}{\partial \delta_n} & \frac{\partial Q_2^{(k)}}{\partial |V_2|} & \dots & \frac{\partial Q_2^{(k)}}{\partial |V_n|} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial Q_n^{(k)}}{\partial \delta_2} & \dots & \frac{\partial Q_n^{(k)}}{\partial \delta_n} & \frac{\partial Q_n^{(k)}}{\partial |V_2|} & \dots & \frac{\partial Q_n^{(k)}}{\partial |V_n|} \end{bmatrix} \begin{bmatrix} \Delta P_2^{(k)} \\ \vdots \\ \Delta P_n^{(k)} \\ \Delta Q_2^{(k)} \\ \vdots \\ \Delta Q_n^{(k)} \end{bmatrix} \dots\dots\dots(28)$$

In Equation (28), the slack bus variable voltage magnitude and angle elements are not included because they are already specified. Taking the partial derivatives of Equations (26) and (27) gives the element of the Jacobian matrix of Equation (29), which explains the small changes in voltage magnitude and voltage angle [14].

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_3 \\ J_2 & J_4 \end{bmatrix} = \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix} \dots\dots\dots(29)$$

where J_1, J_2, J_3, J_4 are the elements of the Jacobian matrix. The power residuals, $\Delta P_i^{(k)}$ and $\Delta Q_i^{(k)}$, are the difference between the scheduled and calculated powers [14] are as follows:

$$\Delta P_i^{(k)} = P_i^{sch} - P_i^{(k)} \dots\dots\dots(30)$$

$$\Delta Q_i^{(k)} = P_i^{sch} - Q_i^{(k)} \dots\dots\dots(31)$$

Equations (32) and (33) are the approximation of the angles and magnitudes of the bus voltages respectively.

$$\delta_i^{(k+1)} = \delta_i^{(k)} - \Delta\delta_i^{(k)} \dots\dots\dots(32)$$

$$|V_i^{(k+1)}| = |V_i^{(k)}| + \Delta|V_i^{(k)}| \dots\dots\dots(33)$$

In Equation (33), $|V_i^{(k)}|$ and $\delta_i^{(k)}$ are assumed to be 1, and $\angle 0^\circ$ respectively for near-solution results.

3.3. Fast-Decoupled Technique

This technique is the most advanced method for solving load flow analysis. It is a simplified version of the Newton-Raphson technique. It involves decoupling the calculations of real and reactive powers and directly obtaining the elements of the Jacobian matrix from the Y-bus [8]. The Fast-Decoupled is the most utilized technique in load flow analysis because of its fast convergence. In some cases, the fast-decoupled technique convergence is less effective because of the heavy loading on some buses or the high reactance-to-resistance ratio [14]. In addition, it uses assumptions to simplify the Jacobian matrix because it requires more iterations than the Newton-Raphson technique. However, to overcome the weak coupling between the real power and voltage magnitude in the power network, and reactive power and the voltage angle, the Jacobian matrix of Equation (29) is reduced in half by eliminating the element of J_2 and J_4 [12] [13] [14] to obtain Equation (34).

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & 0 \\ 0 & J_4 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix} \dots\dots\dots(34)$$

And, Equation (34) is simplified to Equations (35), (36), (37), and (38) as follows:

$$\Delta P = J_1 \Delta\delta = \left[\frac{\partial P}{\partial \delta} \right] \Delta\delta \dots\dots\dots(35)$$

$$\Delta Q = J_4 \Delta|V| = \left[\frac{\partial P}{\partial |V|} \right] \Delta|V| \dots\dots\dots(36)$$

$$\frac{\partial P}{\partial \delta} = -B' \Delta\delta \dots\dots\dots(37)$$

$$\frac{\partial Q}{\partial |V|} = -B'' \Delta|V| \dots\dots\dots(38)$$

J_1 and J_4 are derived from the Y-bus results in Equations (39) and (40).

$$\Delta Q = [B']^{-1} \frac{\Delta P}{|V|} \dots\dots\dots(39)$$

$$\Delta|V| = [B'']^{-1} \frac{\Delta Q}{|V|} \dots\dots\dots(40)$$

where $\Delta|V|$ and ΔQ are the changes in voltage magnitude and phase angle. In addition, B' and B'' are the imaginary parts of the Y-bus admittance.

3.4. Slack bus Power and losses calculations

The slack bus real and reactive power are computed using Equations 26 and 27. The complex power losses in the system is $S_{Losses} = S_{GEN} - S_{Load}$. The loss in the line from bus i to bus j, $S_{Loss ij} = S_{ij} + S_{ji}$; where $S_{ij} = V_i I_{ij}^*$ and $S_{ji} = V_j I_{ji}^*$.

Also the current in the line $I_{ij} = (V_i - V_j)y_{ij}$.

4. Illustrative Examples [15].

Shown in Figure 3 is an IEEE 3-bus system that is used to illustrate the 3 techniques outlined in the previous sections of this paper.

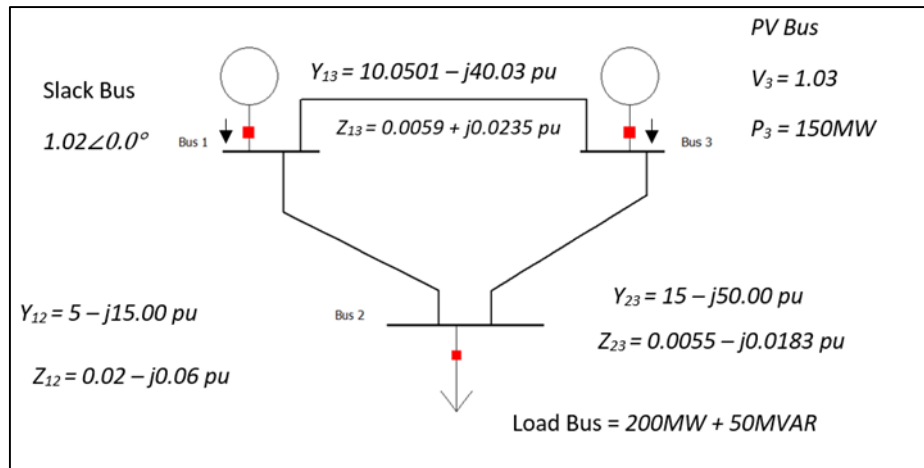


Figure 3 IEEE 3-Bus System [15]

4.1. Gauss-Seidel Approach

The Admittance matrix:

$$Y_{BUS} = \begin{bmatrix} 15.0501 - j55.03 & -5 + j15 & -10.0501 + j40.03 \\ -5 + j15 & 20.00 - j65.00 & -15.00 + j50.00 \\ -10.0501 + j40.03 & -15.00 - j50.00 & 25.0 - j90.03 \end{bmatrix}$$

The admittance matrix in polar form:

$$Y_{BUS} = \begin{bmatrix} 57.01\angle-74.74^\circ & 15.81\angle108.43^\circ & 41.23\angle104.04^\circ \\ 15.81\angle108.43^\circ & 68.01\angle-72.90^\circ & 52.20\angle106.70^\circ \\ 41.23\angle104.04^\circ & 52.20\angle106.70^\circ & 93.41\angle-74.48^\circ \end{bmatrix}$$

First Iteration: assume $V_2^{[0]} = 1.00\angle 0^\circ$ and $V_3^{[0]} = 1.03\angle 0^\circ$.

$$\begin{aligned} V_2^{[1]} &= \frac{1}{Y_{11}} \left(\frac{P_2^{sch} - jQ_2^{sch}}{V_2^{[0]*}} + y_{21}V_1 + y_{23}V_3^{[0]} \right) \\ &= \frac{1}{57.0509\angle-74.7043^\circ} \left(\frac{-0.2 + j0.5}{1.00\angle 0^\circ} + (5 - j15)1.02\angle 0^\circ + (-15.00 + j50)1.00\angle 0^\circ \right) \\ V_2^{[1]} &= \mathbf{1.0120 - j0.0260 pu = 1.0123\angle -1.4717^\circ} \end{aligned}$$

Q_3 is unknown. Hence, it can be calculated using the equation below:

$$\begin{aligned} Q_3^{[1]} &= -\text{Im} \left\{ V_3^{*[0]} \left(V_3^{[0]} (y_{31} + y_{32}) - [y_{31}V_1 + y_{32}V_2^{[1]}] \right) \right\} \\ Q_3^{[1]} &= -\text{Im} \{ 1.03\angle 0^\circ (1.03\angle 0^\circ (10 - j40 + 15.00 - j50) - [(10 - j40)1.02\angle 0^\circ \\ &\quad + (15.00 - j50)(1.0120 - j0.0260)] \} \\ &= -\text{Im} \{ 1.7201 - j0.9373 \} = 0.9373 pu \end{aligned}$$

Therefore, the voltage $V_3^{[1]}$ can be calculated as follows:

$$\begin{aligned}
 V_3^{[1]} &= \frac{1}{Y_{33}} \left(\frac{P_3^{\text{sch}} - jQ_3^{[1]}}{V_3^{[0]*}} + y_{31}V_1 + y_{32}V_2^{[1]} \right) \\
 &= \frac{1}{10 - j40 + 15.00 - j50} \left(\frac{1.5 - j0.9373}{1.03 \angle 0^\circ} + (10 - j40)1.02 \angle 0^\circ \right. \\
 &\quad \left. + (15.00 - j50)(1.0120 - j0.0260) \right) \\
 &= \mathbf{1.0294 - j0.0022 pu} = \mathbf{1.0294 \angle -1.12245^\circ}
 \end{aligned}$$

4.11 Line Flow and Power Loss

Based on the explanation on section 3.4 of this paper, from bus 1 to 2, $I_{12} = (V_1 - V_2) y_{12} = (1.02 \angle 0^\circ - 1.0123 \angle -1.4717^\circ) (5 - j15) = 15.7385 \angle 103.409^\circ \text{ pu}$

$$S_{12} = V_1 I_{12}^* = (1.02 \angle 0^\circ) (15.7385 \angle -103.409^\circ) = 16.0533 \angle -103.409^\circ \text{ pu}$$

16.0533 ∠ - 103.409° pu.

$$\text{Also, } S_{21} = V_2 I_{21}^* = (1.0123 \angle -1.4717^\circ) (-15.7385 \angle 103.409^\circ)^* = \mathbf{(15.9320 \angle 75.1193) pu.}$$

Power loss in **Line**₁₂, $S_{L12} = S_{12} + S_{21} = \mathbf{0.3687 - 0.2179j pu.}$

4.2. Newton-Raphson Approach

From Figure 1, we have 3 unknowns: angles (,) of bus 2 and 3 and bus 2 voltage.

Step 1 - Using the admittance matrix in polar form:

Step 2 - Obtain the Jacobian matrix elements using the following formulas:

$$P_2 = |V_2||V_1||Y_{21}| \cos(\theta_{21} - \delta_2 + \delta_1) + |V_2|^2|Y_{22}| \cos(\theta_{22}) + |V_2||V_3||Y_{23}| \cos(\theta_{23} - \delta_2 + \delta_3) \dots\dots\dots(41)$$

$$\frac{\partial P_2}{\partial \delta_2} = |V_2||V_1||Y_{21}| \sin(\theta_{21} - \delta_2 + \delta_1) + |V_2||V_3||Y_{23}| \sin(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial P_2}{\partial \delta_3} = -|V_2||V_3||Y_{23}| \sin(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial P_2}{\partial |V_2|} = |V_1||Y_{21}| \cos(\theta_{21} - \delta_2 + \delta_1) + 2|V_2||Y_{22}| \cos(\theta_{22}) + |V_3||Y_{23}| \cos(\theta_{23} - \delta_2 + \delta_3)$$

$$P_3 = |V_3||V_1||Y_{31}| \cos(\theta_{31} - \delta_3 + \delta_1) + |V_3||V_2||Y_{32}| \cos(\theta_{32} - \delta_3 + \delta_2) + |V_3|^2|Y_{33}| \cos(\theta_{33}) \dots\dots\dots(42)$$

$$\frac{\partial P_3}{\partial \delta_2} = -|V_3||V_2||Y_{32}| \sin(\theta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial P_3}{\partial \delta_3} = |V_3||V_1||Y_{31}| \sin(\theta_{31} - \delta_3 + \delta_1) + |V_3||V_2||Y_{32}| \sin(\theta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial P_3}{\partial |V_3|} = |V_3||Y_{33}| \cos(\theta_{33} - \delta_3 + \delta_2)$$

$$Q_2 = -|V_2||V_1||Y_{21}| \sin(\theta_{21} - \delta_2 + \delta_1) - |V_2|^2|Y_{22}| \sin(\theta_{22}) - |V_2||V_3||Y_{23}| \sin(\theta_{23} - \delta_2 + \delta_3) \dots\dots\dots(43)$$

$$\frac{\partial Q_2}{\partial \delta_2} = |V_2||V_1||Y_{21}| \cos(\theta_{21} - \delta_2 + \delta_1) + |V_2||V_3||Y_{23}| \cos(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial Q_2}{\partial \delta_3} = -|V_2||V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial Q_2}{\partial |V_2|} = -|V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1) - 2|V_2||Y_{22}|\sin(\theta_{22}) - |V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3)$$

Single iteration: assume $V_2^{[0]} = 1.00\angle 0^\circ$ and $V_3^{[0]} = 1.03\angle 0^\circ$.

Calculated values:
$$\begin{bmatrix} P_2^{[1]} \\ P_3^{[1]} \\ Q_2^{[1]} \end{bmatrix} = \begin{bmatrix} -0.5500 \\ 0.5665 \\ -1.8000 \end{bmatrix}$$

Given values (Figure 3):
$$\begin{bmatrix} P_2^{[0]} \\ P_3^{[0]} \\ Q_2^{[0]} \end{bmatrix} = \begin{bmatrix} -2 \\ 1.5 \\ -0.5 \end{bmatrix}$$

Therefore, the power and Jacobian Matrix becomes:

$$\begin{bmatrix} \Delta P_2^{[1]} \\ \Delta P_3^{[1]} \\ \Delta Q_2^{[1]} \end{bmatrix} = \begin{bmatrix} -2 \\ 1.5 \\ -0.5 \end{bmatrix} - \begin{bmatrix} -0.5500 \\ 0.5665 \\ -1.8000 \end{bmatrix} = \begin{bmatrix} -1.4500 \\ 0.9335 \\ 1.3000 \end{bmatrix} - \text{Power Matrix}$$

$$\text{Jacobian Matrix, } J = \begin{bmatrix} 66.8 & -51.5 & 19.45 \\ -51.5 & 93.52 & -15.45 \\ -20.55 & 15.45 & 63.2 \end{bmatrix}$$

Step 3: Newton-Raphson expression:

$$\begin{bmatrix} \Delta \delta_2^{[1]} \\ \Delta \delta_3^{[1]} \\ \Delta |V_2|^{[1]} \end{bmatrix} = \begin{bmatrix} 66.8 & -51.5 & 19.45 \\ 20.19 & 93.52 & -15.45 \\ 20.55 & 15.45 & 63.2 \end{bmatrix}^{-1} \begin{bmatrix} -1.4500 \\ -0.9335 \\ 1.3000 \end{bmatrix} = \begin{bmatrix} -0.0279 \text{ rad} \\ -0.0033 \text{ rad} \\ 0.0123 \text{ pu} \end{bmatrix}$$

$$\begin{bmatrix} \delta_2^{[1]} \\ \delta_3^{[1]} \\ |V_2|^{[1]} \end{bmatrix} = \begin{bmatrix} \delta_2^{[0]} \\ \delta_3^{[0]} \\ |V_2|^{[0]} \end{bmatrix} + \begin{bmatrix} \Delta \delta_2^{[1]} \\ \Delta \delta_3^{[1]} \\ \Delta |V_2|^{[1]} \end{bmatrix}$$

$$\begin{bmatrix} \delta_2^{[1]} \\ \delta_3^{[1]} \\ |V_2|^{[1]} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1.00 \end{bmatrix} + \begin{bmatrix} -0.0279 \text{ rad} \\ -0.033 \text{ rad} \\ 0.0123 \text{ pu} \end{bmatrix} = \begin{bmatrix} -0.0279 \text{ rad} \\ -0.0033 \text{ rad} \\ 1.0123 \text{ pu} \end{bmatrix} = \begin{bmatrix} -1.5986^\circ \\ -0.1891^\circ \\ 1.0123 \text{ pu} \end{bmatrix}$$

4.3. Fast-Decoupled Approach

$$[B'] = \begin{bmatrix} -65 & 50 \\ 50 & -90 \end{bmatrix}$$

Using values from the same power matrix in Newton-Raphson method, the Fast Decoupled method can be expressed as:

$$\begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \end{bmatrix} = -[B']^{-1} \begin{bmatrix} \frac{\Delta P_2}{|V_2|} \\ \frac{\Delta P_3}{|V_3|} \end{bmatrix} \text{ and } \Delta |V_2| = -[B'']^{-1} \frac{\Delta Q_2}{|V_2|} \dots\dots\dots(44)$$

Where $[B''] = [-65]$ because of the unknown V_2

Therefore,

$$\begin{aligned} \begin{bmatrix} \Delta\delta_2^{[1]} \\ \Delta\delta_3^{[1]} \end{bmatrix} &= -[B']^{-1} \begin{bmatrix} \Delta P_2^{[1]} \\ |V_2|^{[0]} \\ \Delta P_3^{[1]} \\ |V_3|^{[0]} \end{bmatrix} = - \begin{bmatrix} -65 & 50 \\ 50 & -90 \end{bmatrix}^{-1} \begin{bmatrix} -1.4500 \\ 1.00 \\ -0.9335 \\ 1.03 \end{bmatrix} \\ &= \begin{bmatrix} -0.0254 \text{ rad} \\ -0.0041 \text{ rad} \end{bmatrix} = \begin{bmatrix} -1.4569^\circ \\ -0.2324^\circ \end{bmatrix} \\ \begin{bmatrix} \delta_2^{[1]} \\ \delta_3^{[1]} \end{bmatrix} &= \begin{bmatrix} \delta_2^{[0]} \\ \delta_3^{[0]} \end{bmatrix} + \begin{bmatrix} \Delta\delta_2^{[1]} \\ \Delta\delta_3^{[1]} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -1.49569^\circ \\ -0.2324^\circ \end{bmatrix} = \begin{bmatrix} -1.4569^\circ \\ -0.2324^\circ \end{bmatrix} \\ \Delta|V_2|^{[1]} &= -[-19.6]^{-1} \begin{bmatrix} 1.3 \\ 1.00 \end{bmatrix} = 0.0200 \\ |V_2|^{[1]} &= |V_2|^{[0]} + \Delta|V_2|^{[1]} = 1.00 + 0.0200 = 1.020 \text{ pu} \end{aligned}$$

5. Simulations

Power World software is used in simulating the IEEE 3-bus system to check against the illustrative results and an IEEE 9-bus system. The software platform provides a user-friendly interface and a wide range of tools for analysis, visualization, and optimization of the power system. It is widely used by engineers and researchers in the field of electrical power systems to study and analyze the behavior of electric grids. It is a powerful tool for power system analysis and planning.

5.1. Newton Raphson Method - Simulation

The iteration solution for the IEEE 3-bus system using the software, for Newton Raphson technique is displayed in Table 2. The corresponding simulation diagram is also shown in Figure 4.

Table 2 Newton Raphson Method 5 iterations

Iteration	V_2^k	V_3^k
1	1.0123 \angle - 1.4717°	1.03 \angle -0.1226°
2	1.0119 \angle - 1.5273°	1.03 \angle -0.1644°
3	1.0119 \angle - 1.5598°	1.03 \angle -0.1846°
4	1.0119 \angle - 1.5750°	1.03 \angle -0.1941°
5	1.0118 \angle - 1.5823°	1.03 \angle - 0.1986°

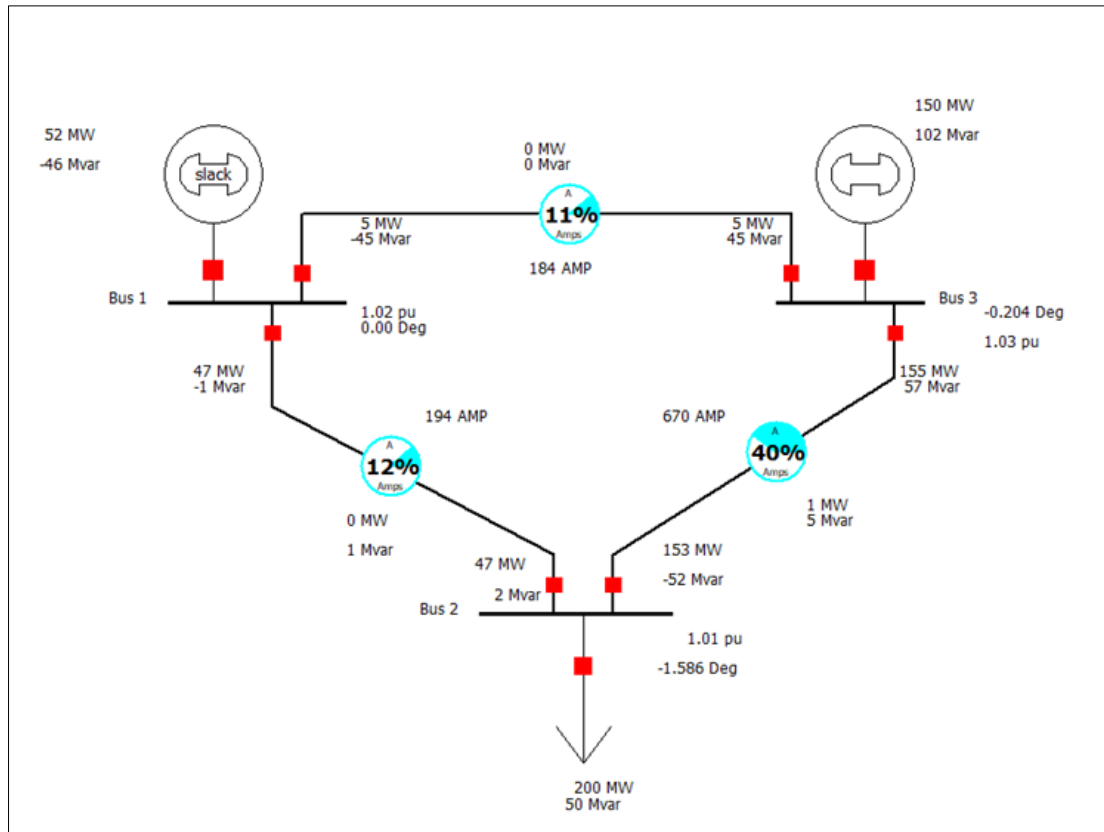


Figure 4 Newton Raphson Method – Simulation

5.2. Gauss-Seidel Method - Simulation

The iteration solution for the IEEE 3-bus system using the software, for Gauss-Seidel technique is displayed in Table 3. The corresponding simulation diagram is also shown in Figure 5.

Table 3 Gauss-Seidel Method 5 iterations

Iteration	V_2^k	V_3^k
1	$1.0123 \angle -1.4717^\circ$	$1.03 \angle -0.1226^\circ$
2	$1.0119 \angle -1.5273^\circ$	$1.03 \angle -0.1644^\circ$
3	$1.0119 \angle -1.5598^\circ$	$1.03 \angle -0.1846^\circ$
4	$1.0119 \angle -1.5750^\circ$	$1.03 \angle -0.1941^\circ$
5	$1.0118 \angle -1.5823^\circ$	$1.03 \angle -0.1986^\circ$

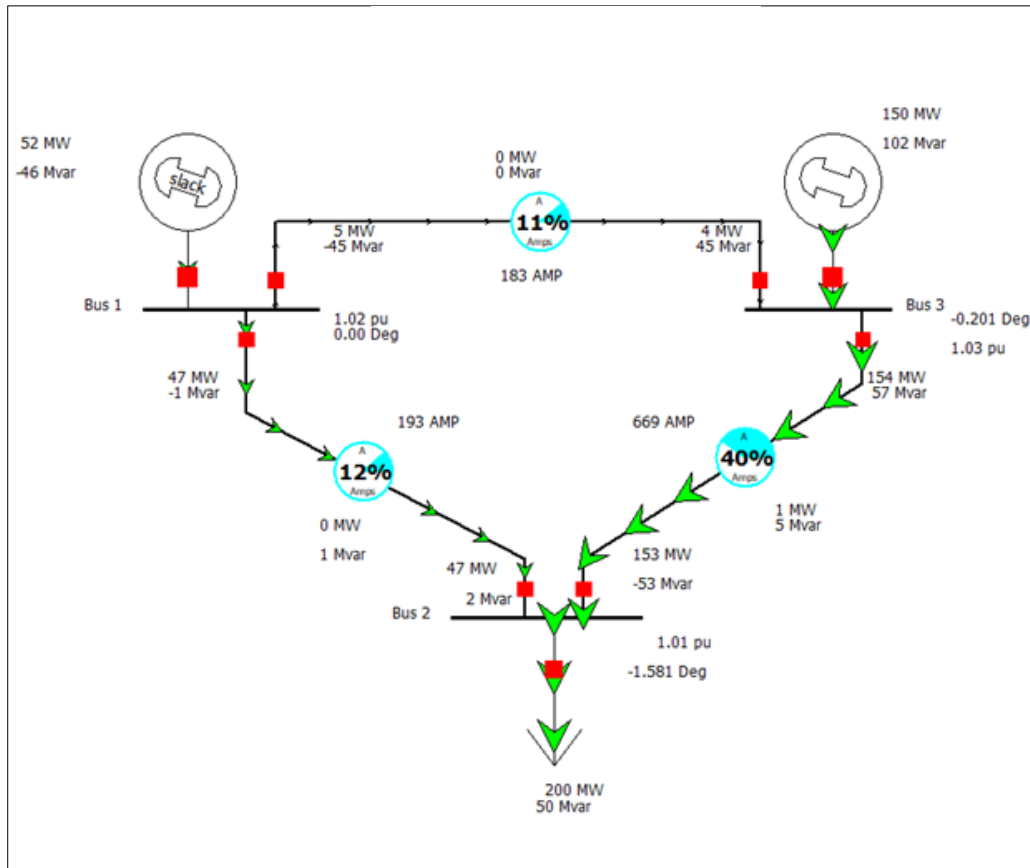


Figure 5 Gauss-Seidel Method-Simulation

5.3. Fast Decoupled Method – Simulation

The iteration solution for the IEEE 3-bus system using this software for Fast Decoupled technique is displayed in Table 4. The corresponding simulation diagram is also shown in Figure 5.

Table 4 Fast Decoupled Method 5 iterations

Iteration	V_2^k	V_3^k
1	1.0123 \angle - 1.4717°	1.03 \angle -0.1226°
2	1.0119 \angle - 1.5273°	1.03 \angle -0.1644°
3	1.0119 \angle - 1.5598°	1.03 \angle -0.1846°
4	1.0119 \angle - 1.5750°	1.03 \angle -0.1941°
5	1.0118 \angle - 1.5823°	1.03 \angle - 0.1986°

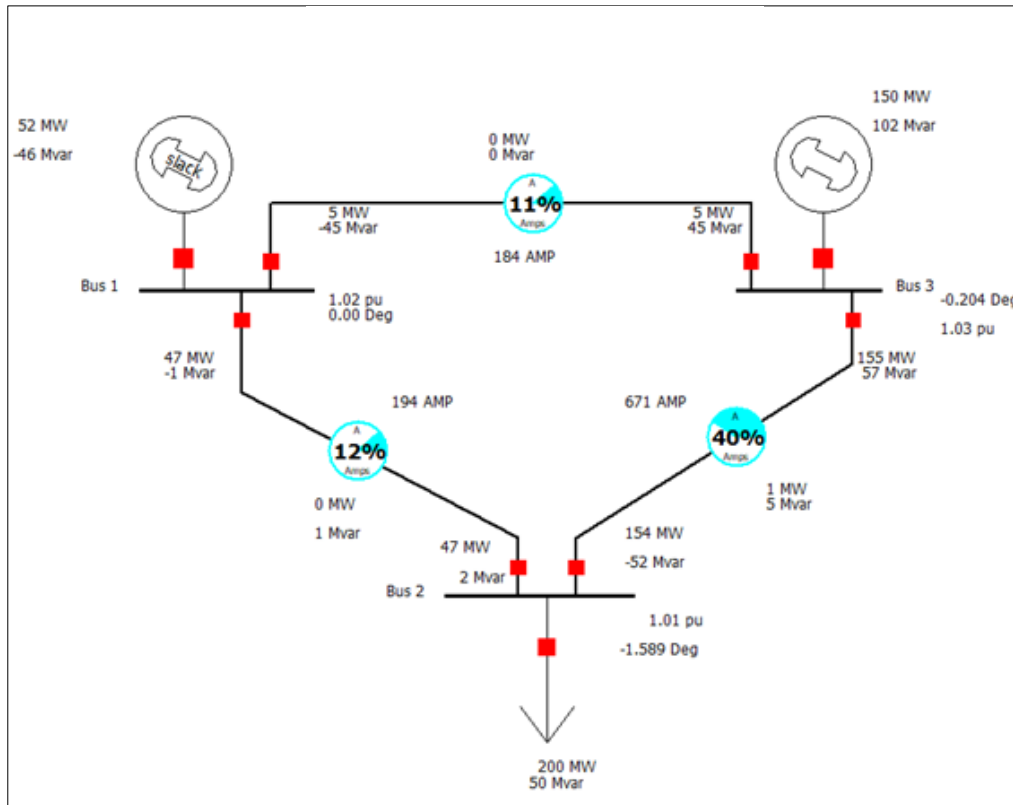


Figure 6 Fast Decoupled Method – Simulation

5.4. IEEE 9-Bus System - Simulations using Newton Raphson Method

Figure 9 is the simulation diagram for an IEEE 9-bus system. Table 5.0 is the corresponding power flow and voltage at the buses for one iteration. The results are actual values instead of per unit values .

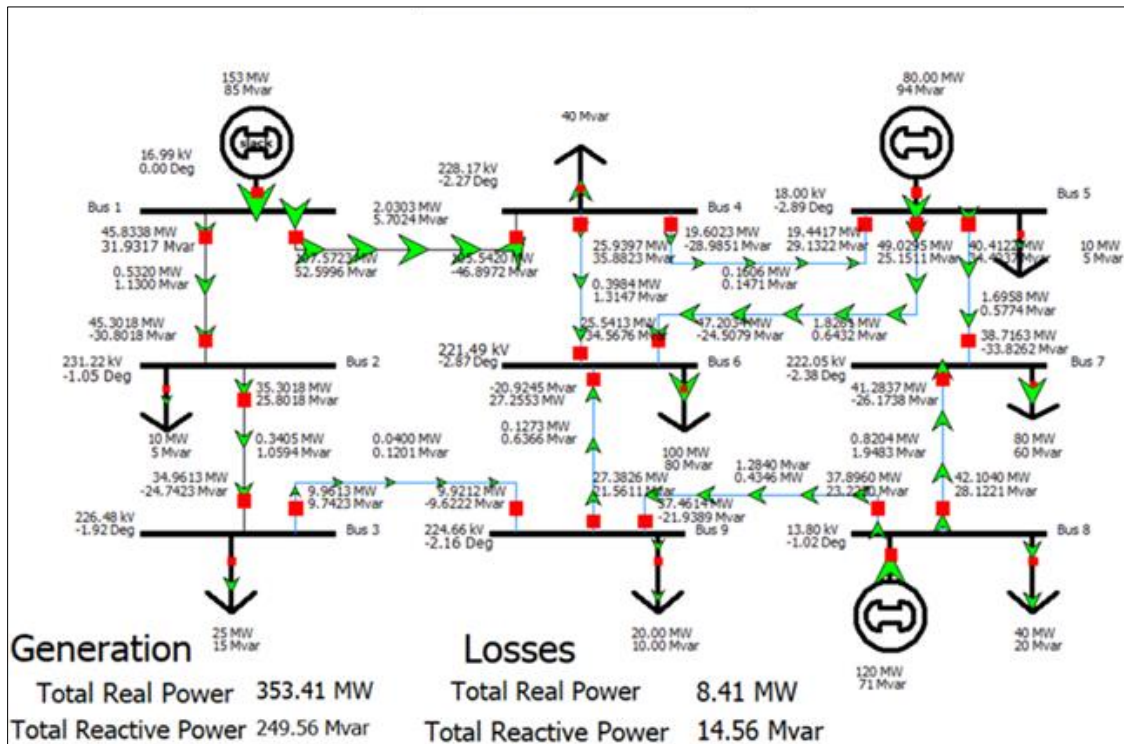


Figure 7 IEEE 9-bus System [16]

Table 5 IEEE 9-Bus System [16]

Bus No	Angle Degree	Voltage (kV)	Load P MW	Q Mvar	Generation P MW	Q Mvar	Injected Power P	
							MW	Q Mvar
1	16.99	0.000	0.000	0.000	153.000	85.000	0.000	0.000
2	231.22	-1.050	10.000	5.000	0.000	0.000	0.000	0.000
3	226.480	-1.920	25.000	15.000	0.000	0.000	0.000	0.000
4	228.170	-2.270	60.000	40.000	0.000	0.000	0.000	0.000
5	18.000	-2.890	0.000	0.000	80.000	94.000	0.000	0.000
6	221.490	-2.870	100.000	80.000	0.000	0.000	0.000	0.000
7	222.050	-2.380	80.000	60.000	0.000	0.000	0.000	0.000
8	13.800	-1.020	40.000	20.000	120.000	71.000	0.000	0.000
9	224.660	-2.160	20.000	10.000	0.000	0.000	0.000	0.000

6. Results and Discussions

Power flow analysis, or load flow analysis, has a wide range of applications in power systems operation and planning. The illustrative examples of the IEEE 3-bus system agree with the simulation results. The results of the IEEE 9-bus system are satisfactory. The student version of Power World software is user friendly and can handle up to 22-bus system. The professional version can handle up to 250,000 nodes or buses.

7. Conclusion

Power flow analysis is performed by solving power network equations. Since these equations are nonlinear, iterative techniques such as the Gauss-Seidel, the Newton-Raphson, and the Fast-Decoupled power flow methods are commonly used for the analysis. In general, the Gauss-Seidel method is simple but converges slower than the Newton-Raphson method and the Fast-Decoupled method. However, the Newton Raphson method requires the Jacobian matrix formation at every iteration. The fast-decoupled power flow method is a simplified version of the Newton-Raphson method. This simplification is achieved in two steps: 1) decoupling real and reactive power calculations; 2) obtaining of the Jacobian matrix elements directly from the Y-bus matrix. The illustrative examples serve to guide the novel reader to understand in a tutorial fashion the techniques and procedures for the solution of load flow problem. The Power World software student version is freely available for download and Engineering students can take advantage of its free availability.

Compliance with ethical standards

Acknowledgments

I acknowledge the contributions made by my co-authors and those who supported us during our research.

Disclosure of conflict of interest

No conflict of interest to be disclosed.

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