World Journal of Advanced Engineering Technology and Sciences
eISSN: 2582-8266
Cross Ref DOI: 10.30574/wjaets
Journal homepage: https://wjaets.com/

(Review Article)

Decomposition of intra-household disparity sensitive fuzzy multi-dimensional poverty index: A study of vulnerability through Shapley machine learning

Sugata Sen [1,*] and Santosh Nandi [2]

[1] *Associate Professor of Economics, Panskura Banamali College (Autonomous), West Bengal, INDIA 721152.*
[2] *Research Scholar, Panskura Banamali College Research Center in Science affiliated to Vidyasagar University, India.*

World Journal of Advanced Engineering Technology and Sciences, 2024, 12(02), 057–063

Publication history: Received on 22 May 2024; revised on 02 July 2024; accepted on 05 July 2024

Article DOI: https://doi.org/10.30574/wjaets.2024.12.2.0276

Abstract

The well accepted multi-dimensional measures have failed to properly project the vulnerability of human-beings towards poverty. Some of the reasons behind this inability may be the failure of the existing measures to consider the graduality within the concept of poverty and the disparities within the household in wealth distribution. So, this work wants to develop a measure to estimate the vulnerability of households in becoming poor through incorporating the intra-household disparities through the factors which suffer from graduality. The decomposition of the grade of vulnerability on the causal factors is also under the purview of this work. To that respect the idea of fuzzy logic and decomposition through artificial intelligence has been used to develop a mathematical framework. So, the idea of Shapley Value Decomposition method has been used extensively. This decomposition is implemented here with the help of Shapley Machine Learning. This decomposition will help the planners to locate the role of different dimensions behind the vulnerability of human beings to become poor more efficiently.

Keywords: Multi-dimensional vulnerability; Graduality; Intra-household disparity; Shapley decomposition; Machine learning

1. Introduction

The well accepted multi-dimensional measures have failed to properly project the vulnerability of human-beings towards poverty. Some of the reasons behind this inability may be the failure of the existing measures to consider the graduality within the concept of poverty and the disparities within the household in wealth distribution. So this work wants to develop a measure to estimate the vulnerability of households in becoming poor through incorporating the intra-household disparities through the factors which suffer from graduality. The decomposition of the grade of vulnerability on the causal factors is also under the purview of this work. To that respect the idea of fuzzy logic and decomposition through artificial intelligence has been used to develop a mathematical framework.

2. Review of literature

One of the major impediment of the well accepted Multi-dimensional Poverty Indices(Alkire & Foster, Counting and Multidimensional Poverty Measurement, 2009)(Alkire & Foster, 2011)(Alkire, Kanagaratnam, & Suppa, The Global Multidimensional Poverty Index (MPI):2018 Revision, 2018) is that they have tried to distinguish the poor from the non-poor through a well defined threshold. Their argument is based upon the classical Boolean logic of either yes or no. In their idea the concept of poverty is rigorously defined as an ordinary proposition. But the idea of poverty suffers from vagueness and naturally cannot be defined through a well defined cut-off. So developing and discussing the multidimensional poverty on the light of Boolean logic is not correct (Qizilbash, 2006).

* Corresponding author: Sugata Sen; Email: sensugata@gmail.com

The graduality within a vague concept can well be represented by the idea of fuzzy logic (Zadeh, 1965). Naturally the logic of fuzzy sets started to reshape the discourses on poverty. Cerioli and Zani first attempted to use the fuzzy logic on the measurement of multidimensional poverty (MP)(Cerioli & Zani, 1989). They have tried to estimate the strength of poverty in each dimension through a relevant membership function. Then aggregated the strength of every dimension and normalized through the number of dimensions to get the overall strength of multidimensional poverty of each individual. Their idea has been improved further by Cheli and Lemmi through the idea of Total Fuzzy and Relative (TFR) (Chelli & Lemmi, 1995). After that a voluminous research appeared in this field to illustrate different forms of membership functions. Many of them have also depicted the aggregation and inference issues related with this type of fuzzy indices (Martinetti, 2006). On the basis of these works Betti *et. al.* have tried to develop an idea called "Integrated Fuzzy and Relative" (IFR) approach to the analysis of fuzzy multidimensional poverty. Under IFR the authors have tried to deliver a more acceptable membership function. They have also discussed different executable operations of fuzzy poverty sets (Betti, Cheli, Lemmi, & Verma, 2006). Chakraborty has provided an axiomatic interpretation of fuzzy multi-dimensional poverty index (Chakravarty, 2006).

Another important drawback of the current multidimensional poverty indices is that all of these measures have accepted that the resources are distributed equally or according to needs within the household and poverty status of the individuals within a household are equal of that of the household. Thus these indices have accepted the household as a homogenous unit and such that have failed to capture the intra-household differences. It is an established fact that the different members within a family enjoys varied endowment as well as bargaining power (Agarwal, 1997)(Duflo, 2003). So computing poverty measures taking households as the lowest unit leads to improper estimation of poverty. The earliest work to put importance on the individuals instead of the household was carried out by Haddad and Kanbur (Haddad & Kanbur, 1990). Vijaya *et. al.* (Vijaya, Lahoti, & Swaminathan, 2014) and Klasen *et. al.* (Klasen & Lahoti, 2016)have tried to discuss the importance of individual sensitive measures in the multi-dimensional framework.

Apart from measuring the composite effect of the multi-dimensional poverty, a large volume of research appeared on the decomposition of composite index. The sub-group decomposability of MP index became very important due to its special importance in policy formulation. Using the properties of sub-group decomposability Alkire *et. al.* have tried decompose Alkire-Foster Adjusted Headcount Ratio to find the importance of different sub-groups in the composite multidimensional poverty index (Alkire, Roche, & Vaz, 2017). Deutch and Silber have tried to decompose the fuzzy multidimensional poverty index through Shapley method to find the importance of the concerned determinants (Deutsch & Silber, 2006).

The Shapley Value Decomposition is a solution concept in the findings of influential causal factors. This type of decomposition takes into consideration the average of the marginal contributions of a factor under different combinations. To that respect, the concerned factor is first withdrawn from the model and the rest of the factors are permuted to form different distributions. Gradually, the withdrawn factor is added to each of the combination and the marginal contribution of the added factor in a specific distribution is counted. The average of marginal contributions of the stated factor from all the distributions is the influence of that very factor on the composite influence. In this way, the average contribution of all the factors are determined. The aggregation of all these factorial influences delivers the overall variation of the dependent factor. In this way, the Shapley Value Method decompose the overall variation of the composite dependent factor into the independent causal factors(Shorrocks, 2013).

To decompose the multi-dimensional poverty index machine learning can be used. Machine Learning(ML) is a technique of data analytics that instructs computer to learn from experience. Machine Learnings algorithms use computational methods to "learn" information directly from data without depending on a pre-set equation as a model. Thus the goal of Machine Learning is to understand the nature of (human and other forms of) learning, and to create those learning capabilities in computers (Kubat, 2017)(Theobald, 2017).

Supervised machine learning creates a model that makes predictions based on evidence in the appearance of ambiguity. Supervised learning applies classification and regression techniques to develop predictive models. (Chopra, 2018).Unsupervised learning reveals hidden patterns or inherent, structures in data. It is used to draw decisions from datasets consisting of input data without pre-set outputs. It is used for experimental data analysis to reveal hidden patterns or groupings in data (Srinivasaraghavan & Joseph, 2019).

An alternative to the traditional learning methods has been provided by Shapley Value Machine Learning. This method of machine learning can be applied to decompose the individual influence of causal factors on multi-dimensional poverty using Shapley Decomposition. Shapley Value Machine Learning entails the approximation of the factorial contributions through Shapley decomposition method. One of the framework called SHapley Additive exPlanation (SHAP) executes the Shapley Machine Learning in reality. SHAP puts an importance value to each causal factors. SHAP framework takes into consideration LIME, DeepLIFT and layer-wise relevance propagation. But in reality finding the exact value of SHAP is really challenging. But, those values can be approximated through combining the current Additive Feature Attribution Methods. Help of Max SHAP and Deep SHAP can also be taken for exact computation of SHAP values (Lundberg & Lee, 2017).

The preceding discussion substantiates that the traditional strict cut-off based poverty measures suffer from many short comings. So apart from developing fuzzy index with appropriate membership function it is needed to pull down the basic level of poverty measurement from household level to individual level. The extension of the basic level of poverty measurement requires the further justification of the individual dimensions on the individual sensitive composite measure of poverty. Dimensional decomposition of the dependent variable can examine the feasibility of the elements of causal set as influencing factors. To that respect Shapley Value Decomposition can play a potent role. In the presence of a large number of explanatory variables and very big amount of observation appropriate machine learning process with the help of artificial intelligence can be used to execute the decomposition. On the basis of this context the specific objectives of this study are the following.

Objectives

- Construction of fuzzy multi-dimensional poverty index taking individuals instead of the households as the basic units.
- Development of appropriate machine learning process with the help of artificial importance to execute Shapley Value Decomposition of the composite poverty index.

3. Results

Let, there are n individual, expressed as i=1,2, … n and k dimension expressed as j=1,2,…k. Then the performance level and each individual on each dimension can be expressed as a n×k real values non-negative matrix. Thus, each row vector y_i={yij} when j=1,2,…k.

Let, z is a vector of threshold values when, z={z_j}.

So, on the basis of y_i and z we can create g_i^0 for each individual, when g_i^0={1,0}.

Here g_{ij}^0 =1, when $y_{ij}<z_j$ and

$$g_{ij}^0 = 0, \text{ when } y_{ij} \geq z_j$$

Now it $c_i=|g_{ij}^0|$ i.e., the sum of the g_j^0 and any individuals. The vector c will show the numbers of dimensions where each individual is lying below the established threshold.

But, c_i is not sufficient to conclude about the multi-dimensional poverty of an individual.

Let us assume that to become multi-dimensionally poor it is necessary to be poor in d dimensions.

If, d=1 then, it will be union approach

And if d=k then, it will be intersection approach

Again, d may be 1 < d < k, so generalised statement can be

$$d = 1 \leq d \leq k$$

We can apply a value judgement to fix the value of d.

Then a household become multi-dimensionally poor if,

$$c_i \geq d$$

This identification of poor can be generalised through the introduction of fuzzy logic and intra-household disparities. It is assumed that poverty is a vague concept and cannot be determined through a strict threshold. At the same time it is also accepted that household poverty cannot capture the individual level poverty properly. To accommodate the individuals as the basic unit of poverty measurement we have tried to measure the achievement of a household in a particular dimension differently under the fuzzy perspective. Let us assume that each of the households consists of q individuals where q is a positive integer.

Let the grade of membership to the poor set of the qth member of the ith household in a specific dimension is expressed through the dimension specific individual membership function

$$\mu_p^q(\mathrm{i}) = 1 \text{ if } 0 \le y_j^q \le y'_j \text{and}$$

$$\mu_p^q(\mathrm{i}) = 0 \text{ if } y_j^q > y''_j$$

An individual is definitely poor if his achievement in a particular dimension j is from 0 uptoy'_j. On the other hand if individual achievement is above y''_j then the individual is not poor on dimension j. For individual achievement between y'_j and y''_j the membership function thakes on values in [0,1]. More clearly it can be interpreted that if

μ_p^q(i) = 0 if the ith individual is certainly not poor in the jth dimension.

μ_p^q(i) = 1 if the ith individual completely belongs to the poor set corresponding to jth dimension.

0 <μ_p^q(i) < 1 if the ith individual shows a partial membership in the poor set p of jth dimension.

The strength of membership of all the individuals of a particular household in a particular dimension can be added and deflated by the number of household members to get the collective strength of household membership in a particular dimension. Thus the individual sensitive grade of membership of ith household in jth dimension can be represented as

$$\mu_p(ij) = \frac{\sum \mu_p^q}{q}$$

The grade of membership of the ith individual to the multi-dimensional poor set M can be defined as

$$\mu_M(i) = \frac{\sum_{j=1}^{k} \mu_p(ij)}{k}$$

If the individual i is definitely poor in all the dimensions then

$$\mu_M(i)= 1 \text{ and}$$

If the ith individual is definitely not poor in any dimension then

$\mu_M(i)$= 0 otherwise

$$0 <\mu_M(i)< 1$$

Now if we assign weight w_j to the jth dimension then it can be written that

$$\mu_M(i) = \frac{\sum_{j=1}^{k} y_{ij.w_j}}{\sum_{j=1}^{k} w_j}$$

Appropriate value judgement can determine a system to term an individual as poor in the multidimensional setup on the basis of the above presented multi-dimensional membership function. From there the head count ratio of the multi-dimensionally poor can be determined. But this work is not interested to develop those criteria. Instead we are interested to decompose the influence of each contributing factor on the strength of membership to the composite multi-dimensionally poor set.

Let the strength of membership to M of the ith household is λ_i where λ_i can take any value from the interval [0,1]. So it is quite natural that, the desired value of λ_i is 0. Thus, the difference between desired and observed strength of membership is λ_i. To decompose this difference in the strength of membership to M Shapley value decomposition has been used. This method calculates the average of marginal contributions of each dimension to the strength of membership. In our model if we are interested to find the contribution of K^{th} dimension, then we would find different combination of K-1 dimensions. So, the total no of combinations without the Kth dimension is –

$$^{(K-1)}C_1 + {}^{(K-1)}C_2 + {}^{(K-1)}C_3 + {}^{(K-1)}C_4 + \ldots + {}^{(K-1)}C_{K-1}$$

Subsequently, the K^{th} dimension is added with each of the combination to find the marginal effect of K^{th} dimension, on that particular combination. Naturally, we would get,

$^{(K-1)}C_1 + {}^{(K-1)}C_2 + {}^{(K-1)}C_3 + {}^{(K-1)}C_4 + \ldots + {}^{(K-1)}C_{K-1}$ number of marginal contributions. According to Shapley decomposition, the contribution of K^{th} dimension is the average of all the marginal contributions of K.

So, the total no of combinations is –

$$\sum_{h=1}^{K-1}(k-1)_{C_h} = \theta$$

Let, the marginal contribution of K^{th} dimension from s^{th} combination is φ_s. So, the set of marginal contributions of the K^{th} dimension is-

$$\eta_k = (\varphi_1{}^k, \varphi_2{}^k, \ldots, \varphi_\theta^k)$$

So, the average of marginal contribution of the kth dimension is

$$CON_k = \frac{1}{\theta}\sum_{i=1}^{\theta} \varphi_i{}^k$$

We have θ combinations for each dimension that means in the whole model we have $K\theta$ combinations. The aim of the machine learning algorithm here is to find

$$CON_j \text{ where } j = 1, 2, \ldots, k$$

To find the contribution of jth dimension SHAP takes the help of Local Interpretable Machine-agnostic Explanation (LIME) algorithm. LIME deliberately perturbs the network by accepting input variables from the neighborhood and counts the effect that perturbation on the output. Finally the relevance of the particular input is determined through the average of deviation in the output due to the perturbations. Alternatively it can also be stated that in our decomposition model LIME will first count the effect on composite poverty due to jth factor through the perturbation through θ combinations and then find the average of deviations on multidimensional poverty due to θ perturbations. This activity is almost similar to the Shapley Value decomposition. The advantage of using LIME is that it takes the inputs from the locally available neighborhood and predict without considering the internal mechanism of the underlying model.

4. Conclusion

The traditional multi-dimensional poverty indices suffer from many shortcomings. This work has alternatively tried to estimate the vulnerability of households to become poor in the multi-dimensional perspective through developing a new mathematical framework. To that extent this work has applied fuzzy logic to accommodate intra-household disparities on the composite index. Further it has developed a mechanism to decompose the dimensional influence on the newly deduced index with the help of artificial intelligence. This decomposition will help the planners to locate the role of different dimensions behind the vulnerability of human beings to become poor.

Compliance with ethical standards

Disclosure of conflict of interest

The Authors proclaim no conflict of interest.

References

[1] Agarwal, B. (1997). Bargaining and Gender Relations: Within and Beyond the Household. *Feminist Economics, 3*(1), 1-51.

[2] Alkire, a., & Foster, J. (2011). Counting and multidimensional poverty measurement. *Journal of Public Economics*, 476-487.

[3] Alkire, S., & Foster, J. (2009). *Counting and Multidimensional Poverty Measurement.* Oxford: OPHI.

[4] Alkire, S., Kanagaratnam, U., & Suppa, N. (2018). *The Global Multidimensional Poverty Index (MPI):2018 Revision.* Oxford: OPHI.

[5] Alkire, S., Roche, J. M., & Vaz, A. (2017). Changes Over Tme in Multidimensional Poverty: Methodology and Results for 34 Countries. *World Development*, 232-249.

[6] Betti, G., Cheli, B., Lemmi, A., & Verma, V. (2006). Multidimentional and Longitudinal Poverty: an Integrated Fuzzy Approach. In A. L. Betti, *FUZZY SET APPROACH TO MULTIDIMENTIONAL POVERTY MEASUREMENT* (pp. 114-137). New York: Springer.

[7] Bourguignon, F., & Chakravarty, S. R. (2003). The Measurement of Multidimensional Poverty. *The Journal of Economic Inequality*, 25-49.

[8] Carroll, C. D. (1994, February 1). How does Future Income Affect Current Consumption? *The Quarterly Journal of Economics, 109*(1), 111-147.

[9] Cerioli, A., & Zani, S. (1989). A Fuzzy Approach To The Measurement of Poverty. *Income and Wealth Distribution, Inequality and Poverty* (pp. 272-284). Italy: Springer.

[10] Chakravarty, S. R. (2006). An Axiomatic Approach to Multidimentional Poverty Measurement via Fuzzy Sets. In A. Lemmi, & G. Betti, *FUZZY SET APPROACH TO MULTIDIMENTIONAL POVERTY MEASUREMENT* (pp. 49-72). New York: Springer.

[11] Chelli, B., & Lemmi, A. (1995). "Totally" Fuzzy and Relative Approach to the Multidimentional Analysis of Poverty. *Economic Notes*, 115-134.

[12] Chopra, R. (2018). *Machine Learning.* New Delhi: Khanna Book Publishing.

[13] Cowan, R. S. (1976, January). "Industrial Revolution" in the Home: Household Technology and Social Change in the 20th. *Technology and Culture, 17*, 1-23.

[14] Deutsch, J., & Silber, J. (2006). The "uzzy St" Approach to Multidimensional Poverty Analysis: Using the Shapley Decomposition to Analyze the Determinants of Poverty in Israel. In A. L. Betti, *FUZZY SET APPROACH TO MULTIDIMENSIONAL POVERTY MEASUREMENT* (pp. 155-174). New York: Springer.

[15] Duflo, E. (2003). Grandmothers and Granddaughters: Old Age Pension and Intrahousehold Allocation in South Africa. *World Bank Economic Review, 17*(1), 1-25.

[16] Haddad, L., & Kanbur, R. (1990). How Serious is the Neglect of Intra-Household Inequality ? *The Economic Journal*, 866-881.

[17] Klasen, S., & Lahoti, R. (2016, March 11). *How Serious is the Neglect of Intra-Household Inequality in Multi-dimensional Poverty Indices ?* Retrieved March 23, 2019, from SSRN: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2742083

[18] Kubat, M. (2017). *An Introduction to Machine Learning.* Cham: Springer.

[19] Lundberg, S. M., & Lee, S.-I. (2017). *A Unified Approach to Interpreting Model.* Retrieved March 19, 2019, from NIPS Proceedings: https://computer.org/csdl/proceedings-article/icpr/1994/00576879/12OmNx965uE

[20] Martinetti, E. C. (2006). Capability Approach and Fuzzy Set Theory: Description, Aggregation and Inference Issues. In A. L. Betti, *FUZZY SET APPROACH TO MULTIDIMENSIONAL POVERTY MEASUREMENT* (pp. 93-113). New York: Springer.

[21] Munshi, K., & Rosenzweig, M. (2016). Networks and Misallocation: Insurance, Migration, and the Rural-Urban Wage Gap. *American Economic Review, 106*(1), 46-98.

[22] Qizilbash, M. (2006). Philosophical Accounts of Vagueness, Fuzzy Poverty Measures and Multidimensionality. In A. Lemmi, & G. Betti, *FUZZY SET APPROACH TO MULTIDIMENSIONAL POVERTY MEASUREMENT* (pp. 9-28). New York: Springer.

[23] Ravallion, M. (2016). *THE ECONOMICS OF POVERTY - HISTORY, MEASUREMENT AND POLICY.* New Delhi: Oxford University Press.

[24] Sen, A. (1976). Poverty: An Ordinal Approach to Measurement . *Econometrica*, 219-231.

[25] Shorrocks, A. F. (2013). Decomposition Procedures for Distributional Analysis: A Unified Framework Based on the Shapley Value. *The Journal of Economic Ineuality*, 99-126.

[26] Srinivasaraghavan, A., & Joseph, V. (2019). *Machine Learning .* Hoboken: Wiley.

[27] Theobald, O. (2017). *Machine Learning For Absolute Beginners: A Plain English Introduction (Second Edition).* Seattle: Scatterplot Press.

[28] Vargha, L., Gál, R. I., & Crosby-Nagy, M. O. (2017, March 23). Household production and consumption over the life cycle: National Time Transfer Accounts in 14 European countries. *DEMOGRAPHIC RESEARCH, 36*(32), 905-944.

[29] Vijaya, R. M., Lahoti, R., & Swaminathan, H. (2014). Moving from the Household to the Individual: Multidimensional Poverty Analysis. *World Development, 59*, 70-81.

[30] Zadeh, L. A. (1965, June). Fuzzy Sets. *Information and Control, 8*(3), 338-353.