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(Research Article)

Combination of dispatching rules to minimize makespan and total weighted tardiness for identical parallel machines scheduling problem: A case study

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Abstract

This study would present a combination of dispatching rules in scheduling jobs on two identical parallel production lines to minimize the values of makespan and total weighted tardiness. Although, individual dispatching rule could achieve good results for a specific objective function value, combination of the LPT-SPT or LPT-ATC methods would bring better results for considered objectives. The results of the study indicate that combining the LPT-SPT methods would yields the best outcomes in comparison with individual dispatching rules. It could help enhance the efficiency and effective of limited resources.

Keywords: Identical Parallel Machines; Dispatching Rules; Longest Processing Time Rule; Shortest Processing Time Rule; Apparent Tardiness Cost Rule; Scheduling Problem

1. Introduction

In this paper, a problem of scheduling eight jobs on two identical parallel machines would be considered. There are assumptions that the machine setup time is negligible and it has already been included in the processing time of each job. The machine processes only one job at a time and without interruption.

Makespan, defined as the longest completion time of the job or the latest time the job releases the parallel workstation, is a critical metric for evaluating the efficiency of machine utilization. A shorter makespan implies a higher throughput and better utilization of resources, which is essential in competitive manufacturing environments where maximizing productivity is paramount. Total weighted tardiness, on the other hand, measures the delay of each job beyond its due date, weighted by the job's priority. This metric is particularly important in scenarios where different jobs have varying levels of urgency and importance. Minimizing total weighted tardiness ensures that jobs with higher priority are completed in a timely manner, thereby enhancing customer satisfaction and reducing potential penalties associated with late deliveries.

The scheduling of jobs on parallel machines presents a complex challenge due to the need to balance these dual objectives. Achieving an optimal or near-optimal schedule requires sophisticated algorithms and strategic decision-making, as the problem is combinatorial in nature. Each possible job sequence can significantly impact the makespan and total weighted tardiness, making it essential to explore various scheduling strategies and heuristics.

In this case study, advanced scheduling techniques to address this problem would be employed. By analyzing different scheduling algorithms, including heuristic and metaheuristic approaches and dispatching rules, methods that effectively

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minimize both makespan and total weighted tardiness would be identified. The results of this study are intended to provide practical insights and guidelines for industrial applications where similar scheduling issues arise in applicable ways.

Furthermore, this research contributes to the broader field of operations research by exploring the interplay between two crucial scheduling objectives. Through detailed simulations and comparative analysis, the trade-offs involved in optimizing for makespan versus total weighted tardiness would be highlighted. The findings could serve as a valuable reference for practitioners and researchers seeking to enhance scheduling efficiency and effectiveness in various operational contexts.

2. Literature review

The problem of scheduling on identical parallel machines, particularly minimizing makespan and total weighted tardiness, is well-documented in operations research and industrial engineering.

2.1. Minimizing Makespan

Minimizing makespan, the total time to complete all jobs, is critical for efficient machine utilization. Graham et al. (1979) reviewed scheduling problems and proposed heuristics for minimizing makespan. The Shortest Processing Time (SPT) rule, introduced by Johnson (1954), prioritizes jobs with the shortest processing times and is widely used for this purpose. Genetic algorithms (GA) and simulated annealing (SA) have also been effective in exploring large solution spaces to achieve near-optimal makespan solutions, as demonstrated by Nawaz, Enscore, and Ham (1983) with their NEH heuristic.

2.2. Minimizing Total Weighted Tardiness

Total weighted tardiness considers delays beyond due dates weighted by job priorities. Lawler (1977) developed dynamic programming algorithms for minimizing total weighted tardiness, which have been extended to parallel machines. The Apparent Tardiness Cost (ATC) heuristic effectively balances job priorities and processing times. Koulamas (1994) introduced a branch and bound algorithm for parallel machines, significantly improving total weighted tardiness minimization. Heuristic methods like Tabu Search (TS) and Particle Swarm Optimization (PSO) have also shown promise in this domain.

2.3. Dual Objective Optimization

Simultaneous optimization of makespan and total weighted tardiness presents a complex multi-objective problem. McNaughton (1959) laid the groundwork for considering multiple objectives in scheduling. Deb et al. (2002) introduced the NSGA-II algorithm for finding Pareto-optimal solutions that balance these dual objectives. The Longest Processing Time (LPT) heuristic, which prioritizes jobs with the longest processing times, helps distribute workloads evenly, potentially reducing both makespan and total weighted tardiness. Ruiz and Maroto (2005) combined genetic algorithms with local search techniques to enhance performance in parallel machines scheduling.

3. Methodology

Dispatching rules are heuristics used in production scheduling to determine the sequence in which jobs or tasks should be processed. They play a critical role in optimizing the performance of manufacturing systems, service operations, and other environments where resources need to be allocated efficiently [9]. Some common dispatching rules that are usually used such as First-Come, First-Served (FCFS), Shortest processing time (SPT), Longest processing time (LPT), Earliest Due Date (EDD), etc... Each dispatching rule has its own strengths and weaknesses and is chosen based on the specific goals and constraints of the scheduling environment. Often, hybrid approaches combining multiple rules are used to achieve better overall performance.

3.1. General assumptions and objective function of problem

This study presents a case study of a wet wipes manufacturing company. A list of eight different jobs to be assigned to two identical parallel machines would be considered. Detail data showed as below table:

Some general assumptions are described as follows:

The capacity of the two identical parallel machines is the same.

No.	Product code	Product name	Order Quantity (packs)	PPM (packs/min)	Processing time	Due date	Weighted
1	A	NRBW30(U)	112,000	48	8	15	2
2	B	PDFC10	34,000	48	3	12	3
3	C	CDHK15	235,900	72	11	23	1
4	D	MSAB15	120,000	72	6	26	3
5	E	GDMS40	89,000	60	5	8	4
6	F	PGBW70(X3)	156,000	30	17	29	4
7	G	PKTW100	165,720	60	9	22	5
8	H	MNFC25(MS)	112,140	60	7	16	2

Figure 1 List of jobs need to be assigned

- Materials for all jobs are always available.
- Machine setup time does not affect and is included in the processing time of the job.
- The jobs are non-preemptive.
- The machines run reliably throughout the production process, without stops or breakdowns.
- All jobs can be started at any time without being constrained by release time.

Notation:

- n - number jobs need to be scheduled
- m - number of machines
- S_{jk} - Setup time of job j on machine k
- d_j - due date of job j
- p_j - processing time of job j
- w_j - weighted of job j
- ATC_i - Apparent Tardiness Cost index of job i
- R - Due date range factor
- K - scaling parameter
- C_{max} - Latest completion time of the job or Makespan
- P_2 – two identical parallel machines
- d_{max} - the longest due date of a job in the list of jobs.
- d_{min} - the shortest due date of a job in the list of jobs.

In this study, the objectives are to minimize makespan (C_{max}) and the total weighted tardiness ($\sum w_j T_j$). The study is conducted in two steps. First, individual rules are used to find the mininize C_{max} value. Then, methods are combined to determine the scheduling sequence to improve the total weighted tardiness index ($\sum w_j T_j$).

The scheduling model and objective function are described as follows:

$$P_2 \, || C_{max} \, , \sum W_j T_j$$

3.2. Shortest processing time (SPT) rule

The Shotest Processing Time (SPT) rule is a widely used dispatching heuristic in production scheduling. It is particularly applicable to situations where multiple jobs or tasks need to be processed on a single machine or resource, and the primary objective is to minimize the overall completion time of these tasks. The SPT rule states that jobs should be scheduled in ascending order of their processing times (p_j). This means that the job with the shortest processing time should be scheduled first, followed by the job with the next shortest processing time, and so on. The underlying principle of the SPT rule is to minimize the average flow time or completion time of jobs, thereby enhancing system efficiency and throughput.

3.3. Longest processing time (LPT) rule

The Longest Processing Time (LPT) rule is a dispatching heuristic used in production scheduling to prioritize jobs based on their processing times. In contrast to the Shortest Processing Time (SPT) rule, the LPT rule schedules jobs with the longest processing times first. This approach aims to maximize resource utilization and minimize the makespan (C_{max}) of all jobs in the system.

3.4. Weighted shortest processing time (WSPT) rule

A rule sequences jobs to be processed on a resource in descending order of the ratio of their weight to their processing time (w_j/d_j). Jobs with high weights and low processing times tend to be sequenced early. If all jobs have equal weights, then WSPT yields the same results as SPT. Otherwise, if there are different types of jobs and their weights are vastly different, then WSPT becomes a simple priority rule in which the resource works on the highest-priority job before switching to lower-priority jobs.

3.5. Apparent tardiness cost (ATC) rule

The apparent tardiness cost (ATC) is a dispatching rule that demonstrates excellent performance in minimizing the total weighted tardiness (TWT) in single-machine scheduling. The ATC rule's performance is dependent on the lookahead parameter of an equation that calculates the job priority index. The ATC rule aims to balance the urgency of job due dates with their processing times. It prioritizes jobs based on an apparent tardiness cost index (ATC_i), which is calculated using a combination of the job's processing time, due date, and a parameter that adjusts the balance between processing time and due date urgency. The ATC_i index for the job i is typical calculated as follows:

$$ATC_i(t) = \frac{w_j}{p_j} \exp\left(-\frac{\max(d_j - p_j - t, 0)}{K\bar{p}}\right) \quad (1)$$

Where:

$$K = \begin{cases} 4.5 + R & if\ R \leq 0.5 \\ 6 - 2R & if\ R > 0.5 \end{cases}$$

With $R = (d_{max} - d_{min})/C_{max}$

In this study, the ATC method do not use alone but combine it with the LPT method to improve the total weighted tardiness objective function. First, using the LPT method, the scheduling sequence on the machines to minimize C_{max} is determined. Then, these machines would be separated into individual units and the ATC method would be applied to improve the total weighted tardiness without altering the C_{max} value of the entire system.

4. Results

4.1. Result follow SPT rule

	Machine 1				Machine 2			
Job	B	D	A	C	E	H	G	F
p_j	3	6	8	11	5	7	9	17
d_j	12	26	15	23	8	16	22	29
W_j	3	3	2	1	4	2	5	4
C_j	3	9	17	28	5	12	21	38
T_j	0	0	2	5	0	0	0	9
WjTj	0	0	4	5	0	0	0	36
Cmax =	38							
Total WjTj =	45							

Figure 2 SPT rule scheduling results

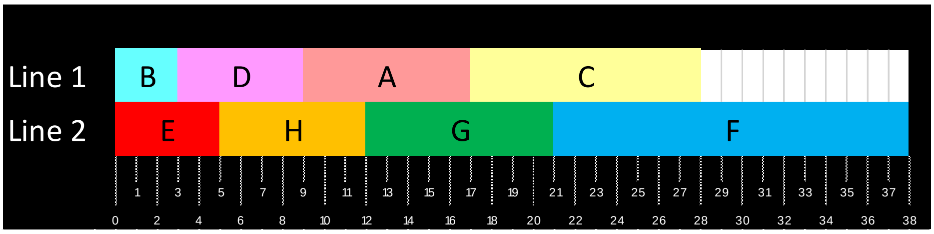

Figure 3 Gantt chart for SPT rule scheduling results

The sequences of the jobs on two machines were shown on the Figure 2 and Figure 3. The C_{max} value is 38 time units and total weighted tardiness values is 45. The valuation of weighted tardiness is under control but C_{max} needs to be improved.

4.2. Results follow WSPT rule

After applying WSPT rule, similarly the sequences of the jobs on two parallel machines is shown in the Figure 4, and Figure 5. C_{max} value and total weighted tardiness in turn of 35 and 46. C_{max} value is improved shorter, but total weighted tardiness get higher in comparison with SPT rule results.

	Machine 1				Machine 2			
Job	B	G	A	C	E	D	H	F
p_j	3	9	8	11	5	6	7	17
d_j	12	22	15	23	8	26	16	29
w_j	3	5	2	1	4	3	2	4
Pj/Wj	1.0	1.8	4.0	11.0	1.3	2.0	3.5	4.3
C_j	3	12	20	31	5	11	18	35
Tj	0	0	5	8	0	0	2	6
WjTj	0	0	10	8	0	0	4	24
Cmax =	35							
Total WjTj =	46							

Figure 4 WSPT rule scheduling results

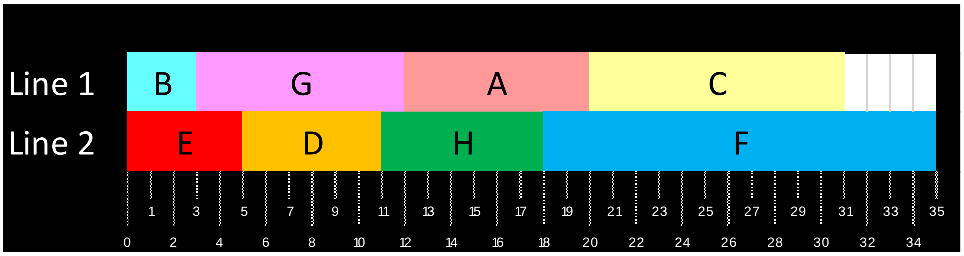

Figure 5 Gannt chart for WSPT rule scheduling results

4.3. Results follow LPT rule

The better result on C_{max} value is achieved after using LPT rule, but total weighted tardiness is out of expectation.

	Machine 1				Machine 2			
Job	F	A	D	B	C	G	H	E
p_j	17	8	6	3	11	9	7	5
d_j	29	15	26	12	23	22	16	8
w_j	4	2	3	3	1	5	2	4
C_j	17	25	31	34	11	20	27	32
Tj	0	10	5	22	0	0	11	24
WjTj	0	20	15	66	0	0	22	96
Cmax =	34							
Total WjTj =	219							

Figure 6 LPT rule scheduling results

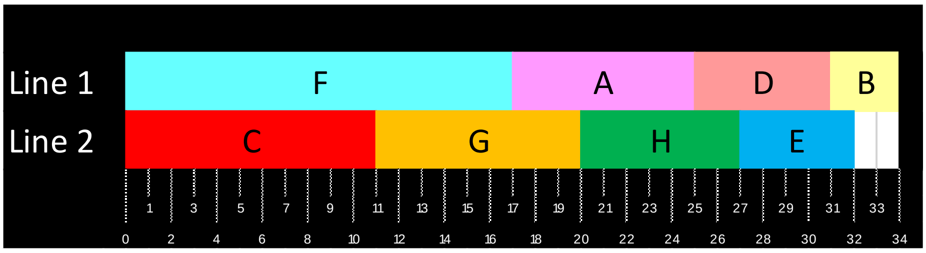

Figure 7 Gantt chart for LPT rule scheduling results

4.4. Results follow LPT-SPT combination

	Machine 1				Machine 2			
Job	B	D	A	F	E	H	G	C
p_j	3	6	8	17	5	7	9	11
d_j	12	26	15	29	8	16	22	23
w_j	3	3	2	4	4	2	5	1
c_j	3	9	17	34	5	12	21	32
Tj	0	0	2	5	0	0	0	9
WjTj	0	0	4	20	0	0	0	9
Cmax =	34							
Total WjTj =	33							

Figure 8 LPT-SPT combine scheduling results

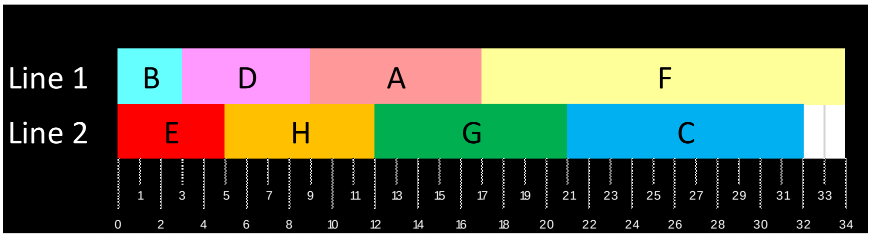

Figure 9 Gantt chart for LPT-SPT combine scheduling results

After combining LPT for parallel machines and then SPT for each single machine, C_{max} value and total weighted tardiness get better results. C_{max} is now 34 time units and weighted tardiness is 33.

4.5. Results follow LPT-ATC combining

	Machine 1				Machine 2			
Job	B	D	A	F	E	G	H	C
p_j	3	6	8	17	5	9	7	11
d_j	12	26	15	29	8	22	16	23
w_j	3	3	2	4	4	5	2	1
C_j	3	9	17	34	5	14	21	32
T_j	0	0	2	5	0	0	5	9
CjTj	0	0	4	20	0	0	10	9
Cmax =	34							
Total WjTj =	43							

Figure 10 LPT-ATC combine scheduling results

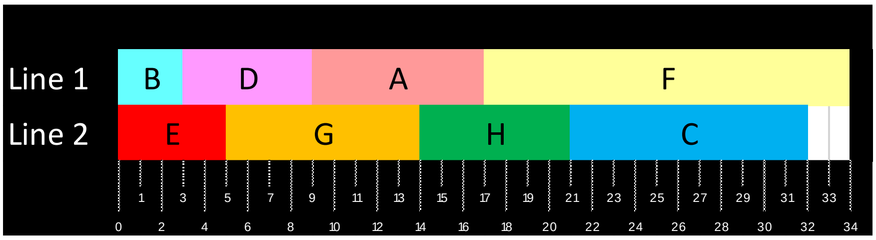

Figure 11 Gantt chart for LPT-ATC combine scheduling results

Comparation between two the combination methods, LPT-ACT and LPT-SPT, LPT-SPT brings better results in terms of C_{max} value and Total weighted tardiness value.

Actually, by combining individual dispatching rules, better results in comparison with applying each dispatching rule separately. Although, each dispatching rule can only optimize a specific objective function value on some special cases, with more than one objective functions, combine methods would bring better solutions as shown on Figure 12.

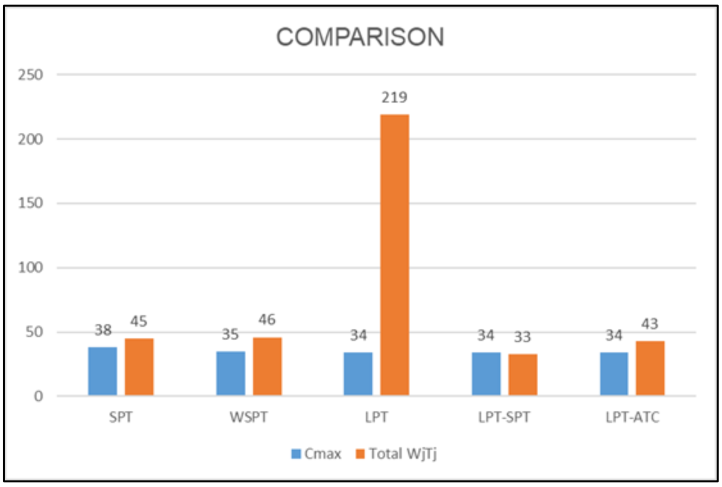

Figure 12 Comparison C_{max} value and Total weighted tardiness value (Total W_jT_j)

5. Conclusions

In this paper, the problem of scheduling eight jobs on two identical parallel machines with the goals of minimizing makespan and total weighted tardiness was addressed. The study demonstrates that a combined use of dispatching rules — Longest Processing Time (LPT) - Apparent Tardiness Cost (ATC), Longest Processing Time (LPT) - Shortest Processing Time (SPT), and Weighted Shortest Processing Time (WSPT)— yields superior results compared to applying any single rule in isolation.

By integrating these dispatching rules, the unique strengths of each approach to create a more balanced and efficient scheduling strategy were shown. ATC excels in prioritizing jobs based on their due dates and processing times, ensuring that high-priority tasks are completed on time. SPT reduces overall makespan by focusing on jobs with the shortest processing times. LPT helps to distribute the workload more evenly across machines, mitigating bottlenecks. WSPT further refines prioritization by considering both processing times and job weights, optimizing for job importance.

The flexibility to combine and switch between these rules dynamically allows for tailored adjustments based on specific job characteristics and production requirements. This combined approach leverages the advantages of each rule, resulting in a more robust and adaptable scheduling framework.

In production environments, the ability to flexibly apply these combined dispatching rules significantly enhances scheduling performance. This methodology not only improves outcomes in terms of makespan and total weighted tardiness but also provides a versatile tool for addressing complex and varying scheduling challenges.

In conclusion, the strategic combination of ATC, SPT, LPT, and WSPT dispatching rules, rather than relying on any single rule, offers a powerful solution for optimizing job schedules on identical parallel machines. This integrated approach leads to better overall performance, providing a practical and effective framework for improving production efficiency and effectiveness.

Despite the promising results achieved by combining dispatching rules such as Apparent Tardiness Cost (ATC), Shortest Processing Time (SPT), Longest Processing Time (LPT), and Weighted Shortest Processing Time (WSPT), several limitations exist in this study. Firstly, the research was conducted on a fixed set of eight jobs and two identical parallel machines, which may not be directly applicable to scenarios with larger numbers of jobs or different machine configurations. Additionally, the study relies on simplified assumptions such as negligible setup times and uninterrupted job processing, which do not reflect the complexities of real-world production environments where setup times, machine breakdowns, and interruptions are common. Moreover, the scheduling environment considered is static, unlike the dynamic nature of real production settings where job arrivals and priorities can change over time, necessitating more adaptive and real-time scheduling approaches. Furthermore, while ATC, SPT, LPT, and WSPT are effective, there are other dispatching rules and hybrid approaches that were not explored, leaving the potential of other rules and their combinations unexamined.

To address these limitations and further enhance the effectiveness of scheduling on identical parallel machines, several avenues for future research are recommended. Future studies should consider larger job sets and more complex machine configurations to understand the scalability of the combined dispatching rule approach and its applicability to more extensive production systems. Incorporating real-world factors such as varying setup times, machine breakdowns, maintenance schedules, and job interruptions can provide a more realistic assessment of the scheduling strategies.

Additionally, developing adaptive scheduling algorithms that can respond to dynamic changes in the production environment, such as new job arrivals, shifting priorities, and unexpected delays, is crucial. This could involve real-time data integration and machine learning techniques. Exploring other dispatching rules and their combinations, including newer heuristic and metaheuristic approaches, could uncover additional strategies for optimizing scheduling performance.

Hybrid algorithms that blend multiple rules and adapt based on the specific context could be particularly promising [10]. Conducting case studies in various industrial settings would validate the effectiveness of the combined dispatching rule approach in real-world applications, providing valuable insights and practical feedback. Finally, integrating advanced optimization techniques such as genetic algorithms, simulated annealing, and machine learning-based approaches can further refine the combined dispatching rules strategy, helping to find more optimal solutions and handle more complex scheduling problems.

While the combined use of ATC, SPT, LPT, and WSPT dispatching rules offers a significant improvement in scheduling performance, addressing the outlined limitations and exploring the suggested future work will enhance the robustness, scalability, and applicability of scheduling strategies on identical parallel machines.

Compliance with ethical standards

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Disclosure of conflict of interest

No conflict of interest to be disclosed.

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