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Torque and Power Computation for a Vertical Wind Turbine with Controlled Blade Angles

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Abstract

In this paper, the available torque and power are computed for a vertical wind turbine that has a mechanism of controlling the approaching angle of each blade. The blades are controlled such that they are kept horizontal in the upwind zone and vertical in the downwind zone. Thus, the torque and power are zero in the upwind zone and hence are computed only in the downwind zone. The positive torque and power due to the wind energy and the negative torque and power due to the rotation of the blades are both included in the formulation of the torque and power equations. Based on the results of the torque and power computation, the design of the vertical wind turbine is updated.

Keywords: Vertical wind turbine; VAWT; Savonius turbine; Torque; Power; Energy

1. Introduction

As an environmentally friendly way of generating energy, various types of wind turbines have been designed and built. The vertical wind turbines produce less noises [1-4] than their horizontal counterparts. However, the blades of the vertical wind turbines, by nature, move upwind (i.e., against the direction of the winds) for half of the turning cycle, which reduces the torque and power of the turbines. As a result, the design objective of the vertical wind turbines is to minimize the negative torque and power due to the rotation of the blades against the winds.

There are two types of vertical wind turbines: Savonius and Darrieus types of wind turbines [1-8]. The drag-type Savonius wind turbines have high torque at low speed, but their efficiency is relatively low. In contrast, Darrieus turbines have much higher efficiency than Savonius counterparts. However, Darrieus turbines have very low torque at low wind speed, causing a serious self-starting problem at wind speed. They also have storm protection problems and vibration problems at high speed.

In this paper, the available torque and power are computed for a Savonius type of vertical wind turbine designed in the preliminary research [9], in which the blades are controlled such that they are kept horizontal in the upwind zone and vertical in the downwind zone. Thus, the wind component vertical to the wind blades and the relative velocity component due to the blade rotation are considered only in the downwind zone. The equations of the torques and power acting on the turbine shaft are formulated for a single blade. Then, the torques and power are computed by using the derived equations. Finally, the computation of torque and power by all the 4 blades are computed based on those of s single blade.

This paper is organized as follows. The equations of torque and power acting on the turbine shaft are formulated for a single blade in Section 2. The torque and power are computed in Section 3. The results of this study and future work are discussed in the Conclusion section.

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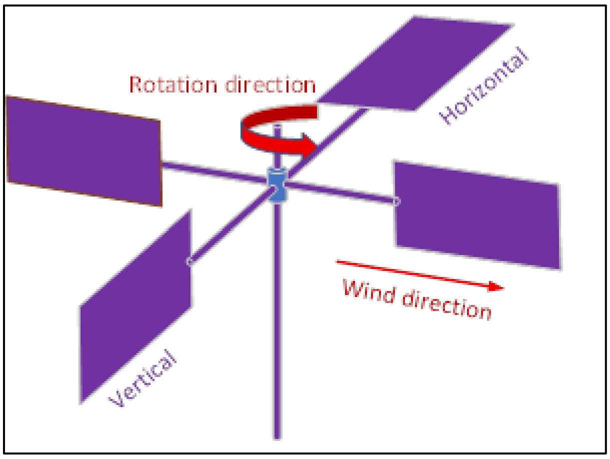

Figure 1 Schematic diagram of a vertical wind turbine having a mechanism of blade angle control

2. Torque and Power Formulation

In this section, a vertical wind turbine [9] having a mechanism that controls the blade angle shown in Fig. 1 is considered. The approaching angles (i.e., the angles of attack) of all the blades are controlled such that the blades are kept horizontal, thus resulting in no wind resistance, when travelling upwind. Then the blade angles are changed to vertical to extract maximum wind energy while travelling downwind.

2.1. Relative velocity components acting on a blade

Figure 2 shows the top view of a single blade in the downwind direction ($0 \leq \theta \leq \pi$), where r_i and r_o denote the inner and outer radii of the blade, respectively; θ and ω denote the rotation angle and angular velocity of the blade, respectively; v_w, v_{wp}, and v_{wv} denote the wind velocity, its component parallel to the blade, and its component vertical to the blade, respectively.

Figure 3 shows relative velocity components vertical to the blade. The wind velocity component v_{wv} ($= v_w \sin\theta$) is vertical to the blade, thus generating positive torque and power for the wind turbine, and the relative velocity component ($r\omega$) is due to the rotation of the blade, thus causing negative torque and power for the wind turbine.

2.2. Torque and power acting on the turbine shaft

The force acting on a vertical surface due to steady fluid flow, shown in Fig. 4, is given [10] as

$$f = \rho Q v = \rho A v^2, \qquad \text{.........} \qquad (1)$$

where ρ is the density of fluid; v denotes the velocity of fluid; Q (=Av) denotes the volume flow rate; A is the area of the surface vertical to fluid direction.

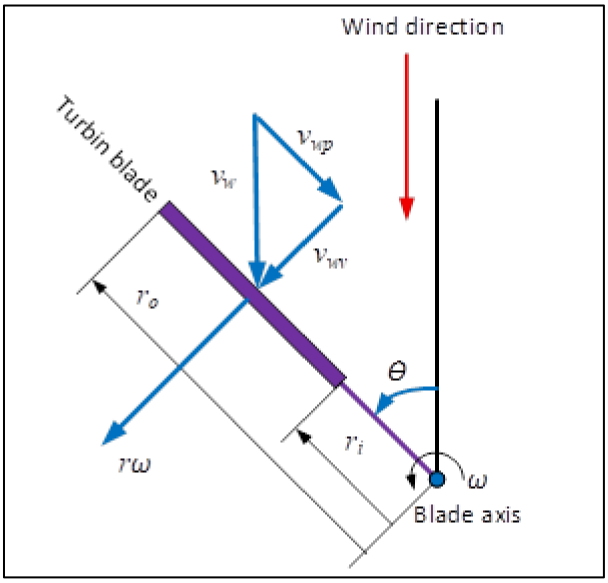

Figure 2 Top view of a single blade in the downwind zone

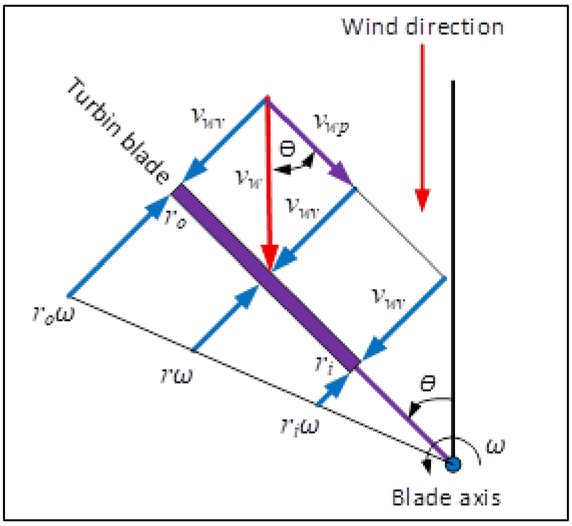

Figure 3 Relative velocity components acting on both sides of the blade

As shown in Figs. 2 and 3, the vertical component of the wind acting on the blade is v_{wv}. The air is neither compressed nor pressurized until it hits the blades. Thus, it is assumed that the density of air ρ_a is constant in this study. Then, according to Eq. (1), the force f_w acting on the blade due to v_{wv}, is computed as

$$f_w = \rho_a A v^2 = \rho_a (r_o - r_i) w v_{wv}^2, \qquad \text{..........} \qquad (2)$$

where w is the width of the blade and r_o and r_i are the outer and inner end of the blade, respectively. Thus, $(r_o - r_i)w$ denotes the area of the blade.

Since the force f_w passes through the centroid $(r_o + r_i)/2$, the positive torque τ_w (= $f_w(r_o + r_i)/2$) acting on the turbine shaft due to the wind can be computed as

$$\tau_w = \rho_a \frac{(r_o^2 - r_i^2)}{2} w v_{wv}^2 \qquad \text{..........} \qquad (3)$$

The power p_w generated by the wind is $\tau_w \omega$ and thus the positive power is

$$p_w = \tau_w \omega = \rho_a \omega \frac{(r_o^2 - r_i^2)}{2} w v_{wv}^2 \quad \ldots\ldots\ldots\ldots \quad (4)$$

The relative velocity component ($r\omega$) against the blade due to the rotation of the blade is proportional to the radius r. Thus, the force f_r and torque τ_r due to the rotation of the blade can be computed as

$$f_r = \int_A \rho_a v^2 \, dA = \int_{r_i}^{r_o} \rho_a w \omega^2 r^2 \, dr = \frac{\rho_a w \omega^2}{3}(r_o^3 - r_i^3) \quad (5)$$

$$\tau_r = \int_A \rho_a r v^2 \, dA = \int_{r_i}^{r_o} \rho_a w \omega^2 r^3 \, dr = \frac{\rho_a w \omega^2}{4}(r_o^4 - r_i^4) \quad (6)$$

The negative power due to the blade rotation $p_r = \tau_r \omega$ is computed as

$$p_r = \frac{\rho_a w \omega^3}{4}(r_o^4 - r_i^4) \quad \ldots\ldots\ldots\ldots \quad (7)$$

Finally, the net torque and power are computed as

$$\tau = \tau_w - \tau_r = \rho_a \frac{(r_o^2 - r_i^2)}{2} w v_{wv}^2 - \frac{\rho_a w \omega^2}{4}(r_o^4 - r_i^4) \quad \ldots\ldots\ldots \quad (8)$$

$$p = p_w - p_r = \rho_a \omega \frac{(r_o^2 - r_i^2)}{2} w v_{wv}^2 - \frac{\rho_a w \omega^3}{4}(r_o^4 - r_i^4) \quad \ldots\ldots\ldots \quad (9)$$

where $v_{wv} = v_w \sin\theta$.

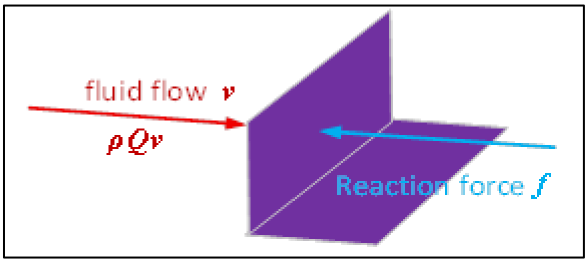

Figure 4 Reaction force acting on the vertical surface due to steady fluid flow

2.3. Computation of Torque and Power

The blades do not generate or consume any power while rotating in the upwind direction, i.e., for $\pi < \theta < 2\pi$, since the blades are kept horizontal in this zone. Thus, the torque and power generation are computed only in the downwind zone ($0 < \theta < \pi$) with the blades kept vertical.

The torque and power acting on the turbine shaft due to the wind v_w (= 5.0 m/s) are computed based on Eqs. (8) and (9) for $0 < \theta < \pi$, where ρ_a = 1.225 kg/m^3 (the density of air at 15° C [11]), r_i = 0.5 m, r_o= 1.0 m, and w = 0.2 m. Figures 5 and 6 show the torque and power for five different angular velocities of the shaft ω = 0, 1.5, 3.0, 4.5, and 6.3 rad/s.

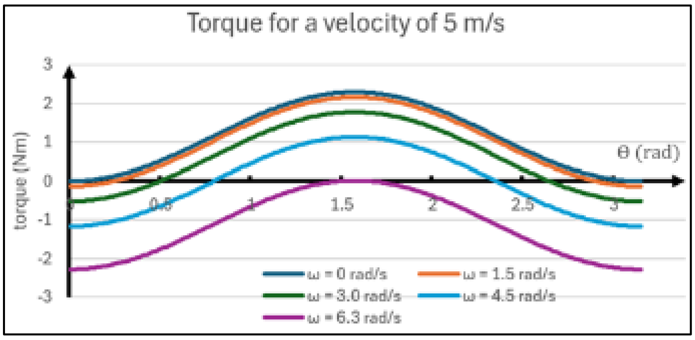

Figure 5 Torque generation for a velocity of 5 m/s as functions of θ

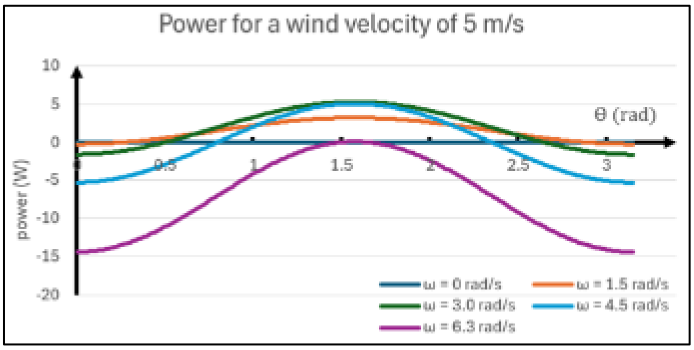

Figure 6 Power generation for a velocity of 5 m/s as functions of θ

Figures 5 and 6 show that the torque and power are functions of θ for given wind velocity v_w and given rotational speed ω of the blades. The torque and power are symmetric with respect to $\theta = \pi/2$, where the torque and power are maximum. Figure 5 shows that the torque is maximum with ω = 0, but Fig. 6 shows that the power is zero.

As the blade rotational speed ω increases, the net torque for $0 < \theta < \pi$ decreases. The net torque is almost zero at ω = 4.5 rad/s, as shown in Fig. 5. This means that the turbine blade can be rotated no more than ω = 4.5 rad/s for the wind speed of 5.0 m/s. In this case, the tangential speed of the center of each blade is 3.375 m/s (= 4.5 * 0.75). The max power is generated at ω = 3.0 rad/s, as shown in Fig. 6.

According to Figs. 5 and 6, when the horizontal blade control in the upwind zone is extended to $0 \leq \theta \leq 0.5$ and $2.64 \leq \theta \leq \pi$ in the downwind zone, the negative torque and power due to the rotation of the blades can be removed, thus resulting in more torque and power generation at ω = 3.0 rad/s.

Since the wind turbine has 4 blades spaced $\pi/2$ radians apart, the total torque and power due to all 4 blades are computed by adding the torque and power of a single blade to those of the preceding blade with a lag of $\pi/2$, like in 3-phase AC voltage.

3. Conclusion and Future Work

In this paper, the torque and power have been computed for a vertical wind turbine having a mechanism of controlling the approaching angles of the blades. As expected, the torque is maximum with ω = 0 rad/s and the maximum velocity of the center of the blades can be no more than 67.5% (=3.375/5.0) of the wind speed.

The negative torque and power due to the rotation of the blades can be removed, thus resulting in more torque and power generation for ω = 3.0 rad/s, when the horizontal blade control in the upwind zone is extended to $0 \leq \theta \leq 0.5$ and $2.64 \leq \theta \leq \pi$ in the downwind zone.

Based on this result, the future work of this research will be addressed to the design of a more efficient mechanism for vertical wind turbines. In addition, a mechanism to limit the rotational speed of the blades will be researched to protect the vertical turbine from hurricane-strength winds.

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