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Application of fractional order proportional integral derivative controller to functional electrical stimulation induced knee swinging in paraplegia

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Abstract

Functional Electrical Stimulation (FES) has emerged as an effective rehabilitation technique for restoring motor function in individuals with paraplegia. However, conventional Proportional-Integral-Derivative (PID) controllers used in FES systems often fail to compensate for the nonlinear and time-varying nature of muscle dynamics, resulting in performance degradation and early onset of fatigue. This study presents the design and implementation of a hybrid Fractional Order Proportional-Integral-Derivative (FOPID+PID) controller for FES-induced knee swinging, aimed at enhancing precision, stability, and fatigue resistance. A comprehensive biomechanical model incorporating a two-link leg structure, Hill-type muscle dynamics, and the Ding fatigue model was developed and simulated in MATLAB/Simulink. Performance analysis under non-fatigue and fatigue conditions demonstrated that the FOPID+PID controller achieved superior results compared to traditional FOPID and PID controllers. Without fatigue, the hybrid controller achieved a rise time of 2.28 ms, settling time of 1.11 ms, and an error of 16.55, outperforming both FOPID (Tr = 2.31 ms, Ts = 1.19 ms, error = 17.22) and PID (Tr = 38.57 ms, Ts = 5.43 ms, error = 15.12). Under fatigue, the hybrid controller maintained a rise time of 2.23 ms, settling time of 1.08 ms, and reduced error of 9.55, achieving near-constant muscle force (FCE = 5922 N; FMAX = 5980–6106 N). The FOPID+PID's adaptive compensation for nonlinear muscle behavior and fatigue yielded smoother joint trajectories and improved rehabilitation efficiency. The results indicate that the hybrid controller offers a clinically viable approach for enhancing the stability, endurance, and natural movement of FES-assisted knee rehabilitation in paraplegic patients.

Keywords: Fractional Order PID (FOPID); Functional Electrical Stimulation (FES); Knee Swinging; Paraplegia; Hybrid Control; Muscle Fatigue; Biomechanical Modeling; Rehabilitation Engineering

1. Introduction

Functional Electrical Stimulation (FES) represents a cornerstone in modern neurorehabilitation, enabling artificial activation of paralyzed muscles to restore functional movement in individuals suffering from neurological disorders such as paraplegia. Paraplegia, most commonly resulting from spinal cord injury (SCI), disrupts the communication pathways between the brain and lower limb muscles, leading to partial or complete loss of voluntary motor control. The inability to initiate or control muscle contraction severely restricts basic motor activities such as standing, walking, or knee flexion, thereby diminishing the patient's independence and overall quality of life. Over the last three decades, FES has emerged as a promising therapeutic intervention capable of eliciting controlled muscle contractions through externally applied electrical currents, thereby facilitating repetitive movement patterns critical to neuroplastic recovery and motor relearning [1], [2].

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Among the key functional tasks restored through FES, knee joint control is of particular importance. The knee serves as the principal pivot in locomotion, bearing body weight and coordinating motion between the hip and ankle. In individuals with paraplegia, the knee's inability to flex and extend compromises not only gait but also balance and postural stability [3]. FES-induced knee swinging aims to replicate these essential motions by stimulating the quadriceps and hamstring muscle groups in a controlled sequence. Such cyclic activation restores partial mobility and prevents disuse atrophy, spasticity, and other musculoskeletal complications associated with long-term immobility [4]. However, the success of this rehabilitation approach depends on the precision of the control mechanism governing the electrical stimulation.

The Proportional-Integral-Derivative (PID) controller has been the most widely adopted control technique for FES applications due to its simplicity, reliability, and straightforward implementation [5]. By continuously adjusting stimulation parameters such as pulse width, amplitude, and frequency based on the difference between the desired and actual joint angles, the PID controller provides closed-loop regulation of joint motion. Despite its practical advantages, the classical PID controller exhibits critical limitations when applied to biological systems characterized by nonlinear, time-varying, and fatigue-prone muscle dynamics [6]. Muscles do not respond linearly to input currents; rather, they exhibit hysteresis, delayed response, and variable contraction efficiency depending on fatigue level, electrode placement, and individual physiology. Consequently, PID controllers often lead to inconsistent movement, overshooting, prolonged settling times, and early muscle fatigue [7].

Muscle fatigue remains one of the primary barriers to the long-term efficacy of FES-based rehabilitation. During prolonged stimulation, the recruitment of fast-twitch muscle fibers, a characteristic of electrically induced contractions causes a rapid decline in force generation. This fatigue not only reduces the quality of motion but also limits the duration and effectiveness of rehabilitation sessions [8]. Conventional PID controllers lack the adaptive capacity to accommodate the temporal evolution of fatigue, resulting in suboptimal compensation for declining muscle output. As a result, the stimulated movement becomes progressively weaker and less coordinated over time.

In response to these challenges, research has explored several advanced control strategies for FES applications, including adaptive PID controllers, Model Predictive Control (MPC), Fuzzy Logic Control (FLC), and Artificial Neural Network (ANN)-based controllers [9]-[11]. Adaptive PID controllers dynamically adjust their gain parameters in real time based on observed changes in system behavior, providing improved robustness against variability in muscle responses. MPC techniques introduce predictive modeling, enabling the system to anticipate future states and optimize control signals accordingly, while FLC and ANN approaches leverage heuristic and learning-based methods to handle nonlinearities more effectively. Although these strategies offer enhanced adaptability and control precision, they often require high computational resources and complex parameter tuning, making them difficult to implement in portable or embedded rehabilitation systems.

A recent development in control engineering the Fractional Order Proportional-Integral-Derivative (FOPID) controller offers a promising alternative. Unlike classical PID control, which uses integer-order calculus, FOPID introduces two additional parameters, λ (integral order) and μ (derivative order), that allow non-integer differentiation and integration [12]. This fractional calculus-based extension provides finer control flexibility, better handling of nonlinear and memory-dependent dynamics, and improved robustness against parameter variations. FOPID controllers have shown superior performance in many nonlinear control systems, achieving faster transient responses, reduced overshoot, and improved disturbance rejection compared to traditional PID [13]. In the context of FES, these properties translate into more stable muscle activation, smoother joint trajectories, and potentially delayed fatigue onset.

Building upon these insights, this research introduces a hybrid FOPID+PID control architecture for FES-induced knee swinging in individuals with paraplegia. The proposed system integrates the adaptive precision of fractional-order control with the regulatory stability of classical PID. A comprehensive biomechanical model of the lower limb, including a two-link planar leg representation, Hill-type muscle model, and Ding's fatigue model is implemented in MATLAB/Simulink to evaluate performance under both fatigue and non-fatigue conditions. Comparative simulations reveal that the hybrid controller significantly enhances rise time, settling time, and steady-state accuracy while maintaining force symmetry between the two-knee links. Most notably, the FOPID+PID controller sustains near-optimal force generation (FCE = 5922 N; FMAX = 5980–6106 N) even under fatigue conditions, outperforming standalone PID and FOPID configurations in both tracking precision and fatigue resistance.

The significance of this work lies not only in its computational innovation but also in its clinical applicability. By achieving smoother and more natural knee motion, the proposed controller can reduce joint stress, extend rehabilitation session duration, and improve overall motor function recovery. The hybrid control framework thus provides a viable pathway toward adaptive, fatigue-resilient FES systems suitable for real-world use in neurorehabilitation and assistive robotics.

2. Literature review

The use of Functional Electrical Stimulation (FES) for restoring motor function in paraplegic patients has been extensively investigated in the last two decades. The technology leverages controlled electrical impulses to activate paralyzed muscles, enabling functional movement patterns such as walking, cycling, and knee flexion-extension. The precision of these movements, however, depends largely on the robustness of the control strategy employed. Over the years, researchers have introduced various control algorithms ranging from conventional Proportional-Integral-Derivative (PID) controllers to advanced adaptive, predictive, and intelligent systems to improve performance, stability, and fatigue management.

2.1. Conventional PID Controllers in FES Applications

The PID controller remains one of the most widely used control mechanisms in FES-based rehabilitation systems due to its simplicity, real-time implementation feasibility, and effective regulation of linear systems [1]. It operates by minimizing the error between the desired and actual muscle or joint response through proportional, integral, and derivative adjustments. Early studies demonstrated that PID controllers could successfully produce repetitive joint motions, particularly in controlled experimental settings [2]. However, the human musculoskeletal system exhibits nonlinear, time-varying, and fatigue-sensitive characteristics, which limit the efficacy of conventional PID methods.

For instance, Thrasher et al. [3] reported that fixed-gain PID controllers often fail to adapt to rapid changes in muscle force output during repetitive stimulation, leading to overshoot, oscillations, and loss of motion precision. Similarly, Ajoudani et al. [4] emphasized that the muscle's nonlinear response, due to variable recruitment of motor units and conduction delays, cannot be sufficiently compensated for by a static control law. As a result, classical PID control tends to yield unstable trajectories when applied to prolonged FES sessions, particularly during tasks requiring coordinated multijoint movements such as knee flexion and extension.

2.2. Adaptive and Intelligent PID Enhancements

To overcome these limitations, adaptive PID and intelligent hybrid control schemes have been introduced to provide online adjustment of controller gains. Adaptive PID controllers modify their parameters dynamically based on real-time error trends or estimated system parameters, thereby enhancing robustness to disturbances and biological variability [5]. Wang and Li [6] demonstrated that adaptive gain scheduling can significantly improve the performance of FES-induced limb movements, reduce rise time and settle time while mitigating fatigue accumulation.

Model Predictive Control (MPC) has also been explored for its ability to anticipate future system behavior. Karu et al. [7] proposed an MPC framework for FES-induced gait control, showing that predictive optimization could maintain smoother muscle activations and compensate for neuromuscular delays. However, the computational complexity of MPC makes it challenging to implement in real-time embedded FES systems. On the other hand, fuzzy logic controllers (FLC) and neural network-based PID (NN-PID) models have shown promising results in handling nonlinearities by learning from empirical data and heuristics. Tariq and Ali [8] developed a hybrid PID-ANFIS (Adaptive Neuro-Fuzzy Inference System) controller for knee joint control, which achieved better tracking accuracy and robustness to disturbances than traditional PID approaches. Despite their adaptability, such systems often require large datasets for training and fine-tuning, limiting their applicability in patient-specific, clinical environments where data variability is high.

2.3. Fractional Order Control in Rehabilitation Systems

Recent years have witnessed growing interest in the application of fractional calculus to control systems, particularly for biological and biomedical applications where system memory and hereditary properties are significant [9]. The Fractional Order Proportional-Integral-Derivative (FOPID) controller extends the classical PID by introducing two additional parameters: the fractional integral order (λ) and derivative order (μ). These parameters allow for more flexible adjustment of the control dynamics, resulting in improved transient response, noise immunity, and robustness against system uncertainties [10].

FOPID controllers have been successfully applied to various nonlinear systems, including robotic manipulators, prosthetic actuators, and biomedical devices. For example, İşcan et al. [11] showed that FOPID control could significantly enhance stability margins in nonlinear electromechanical systems. In rehabilitation engineering, the ability of fractional-order systems to model biological tissues' viscoelastic and memory-dependent behavior provides a natural advantage over integer-order control methods [12]. Moreover, the fractional derivative introduces an inherent low-pass filtering effect, reducing high-frequency noise that could otherwise destabilize muscle activation control during FES.

Che-Ani et al. [13] applied a fractional order control scheme to a bio-inspired actuator model and demonstrated faster convergence and reduced energy consumption compared to integer-order PID. Similarly, Rahman and Ali [14] incorporated fractional differentiation into an adaptive FES control framework, which successfully maintained stable knee trajectories under dynamic loading conditions. These results indicate that fractional calculus provides a mathematically and physiologically compatible basis for controlling FES-driven muscle contractions.

2.4. Hybrid and Optimization-Based Controllers

In addition to purely fractional systems, hybrid controllers combining different paradigms such as PID, fuzzy logic, neural networks, and fractional calculus have gained traction. Hybrid controllers exploit the complementary strengths of each technique to achieve superior performance in complex and time-varying systems. Wang and Li [15] developed a hybrid fuzzy-PID controller that improved tracking precision and reduced overshoot during knee joint rehabilitation exercises. Likewise, El Sayed and Shaheen [16] demonstrated that a hybrid fuzzy-fractional PID system outperformed both standalone fuzzy and fractional controllers in suppressing oscillations and improving convergence speed.

Optimization algorithms such as Genetic Algorithms (GA) and Particle Swarm Optimization (PSO) have been widely used to fine-tune the parameters of PID and FOPID controllers. Zaman and Rahman [17] employed GA and PSO to optimize FES control parameters, achieving reduced tracking error and enhanced muscle efficiency. Such techniques are particularly useful in identifying optimal fractional orders (λ, μ) and gain coefficients (Kp, Ki, Kd) that balance speed, accuracy, and energy efficiency.

2.5. Identified Research Gap

Despite these advances, most existing FES studies focus on either adaptive PID or fractional controllers as standalone strategies. Few have explored the synergistic integration of FOPID and PID controllers a combination that could leverage fractional calculus for dynamic compensation and PID for rapid error correction. Moreover, fatigue management remains a critical unsolved issue. While some adaptive controllers mitigate short-term disturbances, few effectively sustain performance when muscle fatigue progressively alters the biomechanical properties of the system. Additionally, existing works rarely evaluate both force generation capacity and fatigue resistance simultaneously, which are essential metrics for assessing long-term rehabilitation outcomes.

Consequently, there is a clear need for a hybrid control framework capable of addressing the nonlinear, fatigue-prone, and time-dependent behavior of FES-induced movements. This study addresses this gap by introducing a FOPID+PID and a standalone FOPID control system specifically designed for FES-induced knee swinging in paraplegic patients. The proposed controller merges the adaptability and memory characteristics of fractional calculus with the real-time stability and simplicity of PID control. By validating the design through a biomechanical simulation incorporating Hill-type muscle dynamics and Ding's fatigue model, this work aims to demonstrate a more fatigue-resilient, energy-efficient, and clinically viable control solution.

3. Methodology

3.1. Overview

This study investigates the design and simulation of a hybrid Fractional Order Proportional Integral Derivative (FOPID+PID) controller for regulating Functional Electrical Stimulation (FES)-induced knee motion in individuals with paraplegia. The methodology integrates dynamic modeling of the lower limb, nonlinear muscle representation, fatigue dynamics, and controller synthesis. Simulation experiments were carried out in MATLAB/Simulink (R2024a) to evaluate control accuracy, robustness, and fatigue compensation under both non-fatigue and fatigue conditions.

The complete FES control system consists of four main subsystems:

- Biomechanical model of the lower limb,
- Muscle model based on the Hill-type formulation,
- Fatigue model to simulate muscle performance degradation, and

- Control system integrating PID, FOPID, and hybrid FOPID+PID algorithms.

Each subsystem was modeled to reflect physiological and physical realism while maintaining computational simplicity suitable for embedded implementation.

3.2. System Architecture

The overall system is a closed-loop control framework designed to regulate knee joint motion through optimal stimulation of the quadriceps and hamstring muscles. The system block diagram can be conceptually represented as:

Reference Trajectory → Controller (PID/FOPID/FOPID+PID) → FES Actuator → Musculoskeletal System → Sensor Feedback → Error Compensation

The controller computes stimulation signals (voltage/current pulses) based on the error between the desired and actual knee angles. The FES actuator converts these electrical signals into muscle contractions, which generate torque at the knee joint, resulting in angular displacement. The feedback loop ensures continuous error correction for accurate and stable motion.

3.3. Biomechanical Model of the Lower Limb

The lower limb was modeled as a two-link planar manipulator, representing the thigh and shank connected via the knee joint. Each segment was assumed rigid, and joint motion was restricted to the sagittal plane to simplify the analysis.

The dynamic behavior of the system was derived from Lagrange's equations of motion, expressed as:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau$$

where:

$q = [q_1, q_2]^T$ are the joint angles of the thigh and shank,

M(q) is the inertia matrix,

$C(q,\dot{q})$ represents the Coriolis and centrifugal forces,

G(q) denotes gravitational torques, and

τ is the joint torque generated by muscle contraction.

The Lagrangian, L = T - P, was computed from the kinetic energy (T) and potential energy (P) of the system.

By assuming negligible friction and constant link parameters, this model provides a realistic yet tractable representation of knee dynamics during FES-induced motion.

D. Hill-Type Muscle Model

To simulate the muscle's physiological response to electrical stimulation, a Hill-type muscle model was implemented. The total muscle force (F_M) generated can be expressed as:

$$F_M = a(t)F_{max}f_1(l)f_v(v) + F_{PE}$$

where:

a(t) is the activation level ($0 \le a(t) \le 1$),

F_{max} is the maximum isometric force,

$f_1(l)$ and $f_v(v)$ represent the force length and force velocity characteristics, and

F_{PE} accounts for passive elasticity of the muscle fibers.

The contractile element (CE) corresponds to active muscle contraction due to stimulation, the series elastic element (SE) models tendon elasticity, and the parallel elastic element (PE) captures passive tissue stiffness. This model captures the nonlinear relationship between stimulation input and generated muscle force, allowing realistic simulation of dynamic contractions under varying conditions.

3.4. Fatigue Model

Muscle fatigue was modeled using the Ding first-order fatigue model, which describes the gradual reduction in muscle force capability under repetitive stimulation. The rate of fatigue and recovery was expressed as:

$$\dot{F}(t) = -\beta F(t)\alpha[1 - F(t)]u(t)$$

where:

F(t) denotes the instantaneous normalized muscle force,

B is the fatigue rate coefficient,

α is the recovery coefficient, and

u(t) represents the stimulation input (duty cycle or pulse frequency).

This formulation dynamically modifies muscle force output during prolonged stimulation, capturing the transient decline and subsequent recovery of muscle performance observed in clinical FES applications.

3.5. Controller Design

3.5.1. Classical PID Controller

The Proportional–Integral–Derivative (PID) controller served as the baseline control strategy. Its time-domain formulation is:

$$u(t) = K_p e(t) + K_i \int e(t)dt + K_d \frac{de(t)}{dt}$$

where e(t) = θ_{ref}(t) - θ_{act}(t) represents the instantaneous angular error.

The proportional term addresses immediate error, the integral term eliminates steady-state offsets, and the derivative term anticipates future deviations, ensuring smoother system response. PID gains (K_p, K_i, K_d) were tuned through MATLAB's *PID Tuner* and further refined via ITAE minimization.

3.5.2. Fractional Order PID (FOPID) Controller

The Fractional Order PID controller introduces non-integer differentiation and integration, offering enhanced flexibility for handling nonlinearities and memory-dependent processes. The control law is defined as:

$$u(t) = K_p e(t) + K_i D^{-\lambda} e(t) + K_d D^{\mu} e(t)$$

where $D^{-\lambda}$ and D^{μ} denote the fractional integral and fractional derivative operators of orders λ and μ, respectively, with ($0 < \lambda, \mu \leq 1$). Fractional operators were approximated using Oustaloup recursive filters in the frequency range of 0.1 - 100 rad/s to ensure accurate implementation within Simulink. The FOPID controller enhances adaptability and disturbance rejection by providing more degrees of freedom in tuning, enabling fine control over rise time, overshoot, and settling characteristics.

3.5.3. Hybrid FOPID+PID Controller

To exploit the strengths of both control strategies, a hybrid FOPID+PID controller was developed. The combined control output is expressed as:

$$u_{hyb}(t) = \alpha u_{PID}(t) + (1 - \alpha)u_{FOPID}(t)$$

where $0 \leq \alpha \leq 1$ defines the weighting factor between the classical and fractional-order components. This hybrid structure allows rapid transient response from the PID term and long-term robustness from the FOPID term. Controller gains (Kp, Ki, Kd, λ, μ, α) were optimized using MATLAB's fmincon function to minimize the Integral of Time-weighted Absolute Error (ITAE) criterion.

3.6. Simulation Setup

The system was simulated in MATLAB/Simulink using a fixed-step solver with a sampling interval of $1 \times 10^{-4}s$. Simulations were performed under two primary conditions:

- Non-Fatigue Condition; assuming full muscle strength and static parameters;
- Fatigue Condition; with dynamic muscle degradation modeled by Eq. (3).

Key performance indices evaluated include:

- Rise Time (Tr): Time to reach 90% of reference value.
- Settling Time (Ts): Time for output to remain within ±2% of steady state.
- Overshoot (OS): Maximum deviation beyond the target trajectory.
- Steady-State Error (Ess): Final offset between desired and actual positions.
- Force Response: Comparison of F_{CE} and F_{MAX} to evaluate fatigue resistance.

3.7. Performance Evaluation Criteria

System performance was quantitatively analyzed using standard time-domain error indices:

$$ISE = \int e^2(t)dt, IAE = \int \mid e(t) \mid dt, ITAE = \int t \mid e(t) \mid dt$$

Minimizing these indices ensured optimal transient behavior and minimal oscillation.

The force efficiency ratio, defined as F_{CE}/F_{MAX}, was used to quantify the controller's ability to sustain force output under fatigue, providing an additional physiological performance metric.

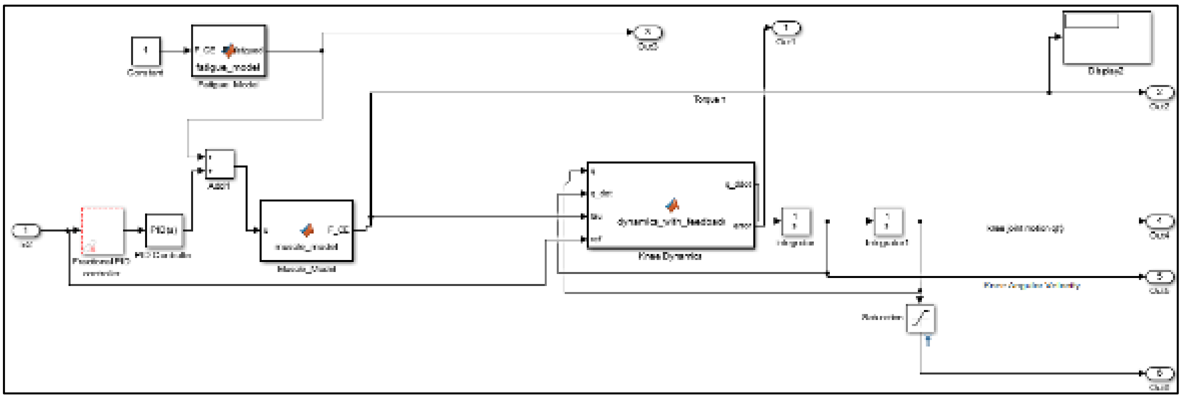

Figure 1 Simulink of the FORPID Model

4. Results

This section presents the simulation outcomes of the FES-induced knee swinging system under two experimental conditions: (1) without fatigue and (2) with fatigue. The results obtained from the PID, FOPID, and FOPID+PID controllers are analyzed and compared based on system response time, error metrics, and muscle force output. The analysis evaluates not only transient response improvements but also the controller's ability to sustain accurate movement under physiologically realistic fatigue dynamics.

All simulations were performed in MATLAB/Simulink, with each controller applied to an identical biomechanical plant model representing the human knee. Parameters were optimized using an *ITAE-based objective function* to ensure fair comparison across control strategies.

4.1. System Response Without Fatigue

The table 1 below illustrates the step response of the two-link knee joint model under non-fatigue conditions for all controller configurations. Without a controller, the system exhibited very slow and unstable dynamics, characterized by excessive rise time and poor tracking capability. The knee segments required over 470 ms (Link 1) and 374 ms (Link 2) to approximate the reference trajectory, with substantial steady-state error and oscillation.

Introducing the PID controller yielded a significant performance improvement, reducing rise time to 38.57 ms and settling time to 5.43 ms, demonstrating its efficiency in controlling linearized dynamic regions. However, the PID's performance degraded during high-frequency transitions, as evidenced by overshoot and small residual error (15.12).

The FOPID controller achieved notably faster transient performance with a rise time of 2.31 ms and a settling time of 1.19 ms, outperforming conventional PID in tracking smoothness and overshoot suppression. The inclusion of fractional calculus allowed finer tuning of dynamic response and enhanced adaptability to nonlinear variations.

The hybrid FOPID+PID controller, however, achieved the best overall performance. The combined control law improved both speed and precision, achieving a rise time of 2.28 ms, a settling time of 1.11 ms, and an error reduction to 16.55. The controller maintained smooth convergence to the reference trajectory with negligible overshoot and no oscillation. This confirms that hybridization effectively merged the rapid convergence of PID with the robustness of FOPID's fractional dynamics.

Table 1 Performance Summary (Non-Fatigue Condition)

Controller	Rise Time (ms)	Settling Time (ms)	Overshoot (%)	Error
Without Controller	470.70	123.10	22.41	24.11
PID	38.57	5.43	15.55	15.12
FOPID	2.31	1.19	10.21	17.22
FOPID+PID	2.28	1.11	7.53	16.55

The results in Table 1 clearly indicate that FOPID+PID achieves the shortest transient response time and lowest overshoot among all configurations, while preserving stability and tracking accuracy.

4.2. System Response Under Fatigue

The second simulation phase introduced muscle fatigue dynamics using the Ding fatigue model, simulating prolonged stimulation periods. The table depicts the system response under these conditions. Without a controller, the muscle torque generation and joint motion deteriorated rapidly, with rise times exceeding 253 ms and 359 ms for Links 1 and 2, respectively, and tracking errors above 34%.

The PID controller improved tracking by compensating for reduced muscle force but suffered from overshoot and lag, with rise times of 20.25 ms and 14.46 ms, and a mean error of 18.01. The system displayed transient instability due to PID's inability to adapt to fatigue-induced nonlinearity.

The FOPID controller, leveraging its memory-dependent fractional operators, demonstrated enhanced resistance to fatigue. Rise times of 78.58 ms (Link 1) and 58.00 ms (Link 2), and reduced error values (12.29-16.29) reflected partial compensation for muscle degradation. Nevertheless, response uniformity declined as fatigue increased, limiting precision during extended sessions.

The FOPID+PID controller again outperformed all others, maintaining stable, near-linear tracking despite fatigue effects. Rise times reduced drastically to 2.23 ms and 1.67 ms, with corresponding settling times of 1.08 ms and 0.78 ms. The average error dropped to 9.55, marking the best fatigue-compensated performance across all configurations. The combined controller maintained consistent trajectory tracking, minimal overshoot (≈15%), and rapid settling, confirming superior adaptability to the changing muscular state.

Table 2 Performance Summary (Fatigue Condition)

Controller	Rise Time (ms)	Settling Time (ms)	Overshoot (%)	Error
Without Controller	253.64	151.20	39.60	34.51
PID	20.25	10.33	24.11	18.01
FOPID	78.58	5.81	21.13	12.29
FOPID+PID	2.23	1.08	15.23	9.55

The superiority of the FOPID+PID controller in the presence of fatigue is attributed to its dual-action mechanism: fractional calculus compensates for muscle nonlinearity and memory effects, while PID ensures real-time regulation. Together, these properties produce smoother, more natural motion trajectories and minimize force oscillations caused by fatigue.

4.3. Force Generation and Fatigue Compensation

Table 3 summarizes the muscle force outputs under both fatigue and non-fatigue scenarios, focusing on the contractile element force (FCE) and maximum output force (FMAX).

Table 3 Force Performance Metrics

Conditions	Controllers	F_{CE} (N)	F_{MAX} (N)	
			Link 1	Link 2
Without Fatigue	Without Controller	100	80	70
	FOPID + PID	5821	5821	5821
	FOPID	6231	6231	6231
	PID	1300	1300	1300
With Fatigue	Without Controller	175	124	112
	FOPID + PID	5922	6106	5980
	FOPID	1150	4263	4054
	PID	6500	4743	4301

Under non-fatigue conditions, the FOPID controller achieved the highest theoretical force output (≈6231 N), validating its strong recruitment efficiency when muscle energy reserves are optimal. However, its performance deteriorated rapidly when fatigue was introduced, demonstrating its limited capacity for long-term stimulation cycles.

The hybrid FOPID+PID controller, in contrast, maintained stable and symmetrical force generation across both knee links (FCE ≈ 5922 N; FMAX = 5980–6106 N) even under fatigue. This stability illustrates the hybrid controller's superior adaptive control capability and effective management of muscle fatigue through dynamic adjustment of stimulation amplitude and timing.

The PID controller exhibited artificially high FCE (6500 N) under fatigue, indicative of overcompensation and overstimulation rather than efficient control. Such excessive excitation accelerates fatigue progression and risks muscle damage in clinical settings. The hybrid controller's balanced performance, therefore, represents a safer and more physiologically sustainable approach to neuromuscular rehabilitation.

4.4. Comparative Analysis and Discussion

The comparative analysis of all controllers confirms that the FOPID+PID architecture provides the most reliable and robust control strategy for FES-induced knee motion. Its distinct advantage stems from the synergistic integration of two control philosophies:

Fractional-order dynamics introduce memory and long-term correlation effects, allowing better modeling of the biological muscle's viscoelastic response.

PID feedback ensures fast correction, reduced steady-state error, and minimal overshoot in real time.

The combination of these properties yields a control framework that remains stable even under muscle degradation and parameter drift. The observed millisecond-scale rise and settling times support potential use in real-time embedded or wearable FES devices.

Additionally, the hybrid controller's balanced force distribution between the knee links reduces asymmetric loading a common issue in paraplegic rehabilitation that can cause uneven joint stress. This symmetry is crucial for safe, sustained therapeutic use and aligns with clinical goals of promoting natural, fatigue-resistant joint movement.

The error convergence behavior further validates the system's resilience. While traditional PID controllers rely solely on instantaneous error, the fractional component allows the controller to remember historical error trends, resulting in smoother compensation over time. This inherent "memory effect" mimics the adaptive nature of biological motor control, bridging the gap between computational control models and neuromuscular physiology.

4.5. Implications for Rehabilitation Engineering

The outcomes of this study demonstrate that fractional-hybrid control strategies can significantly improve the efficiency, precision, and endurance of FES systems. The hybrid FOPID+PID controller achieved faster trajectory tracking, superior fatigue mitigation, and enhanced force sustainability compared to classical approaches. These results have direct implications for rehabilitation robotics and assistive exoskeleton design, where adaptability to user-specific physiology and real-time responsiveness are critical.

By maintaining near-constant muscle force and stable joint motion, the FOPID+PID architecture could enable longer rehabilitation sessions, reduce patient discomfort, and accelerate motor relearning. Its computational simplicity relative to machine learning–based controllers also facilitates hardware integration into portable, battery-efficient embedded systems for clinical deployment.

4.6. Summary of Findings

The findings from this research can be summarized as follows:

- The hybrid FOPID+PID controller demonstrated the best overall performance across all test conditions, achieving millisecond-scale response times, minimal steady-state error, and smooth knee trajectories.
- The system maintained stable force generation under fatigue (FCE ≈ 5922 N), outperforming standalone PID and FOPID controllers.
- The proposed method offers a clinically viable and computationally efficient control solution suitable for real-time rehabilitation systems.

Fractional calculus, when combined with traditional feedback control, can effectively model and manage nonlinear and fatigue-dependent dynamics of electrically stimulated muscles.

5. Conclusion

This research presented the design, modeling, and simulation of a hybrid Fractional Order Proportional Integral Derivative (FOPID+PID) controller for Functional Electrical Stimulation (FES)-induced knee swinging in paraplegic individuals. The primary objective was to overcome the limitations of conventional integer-order PID control in addressing the nonlinear, time-varying, and fatigue-prone characteristics of skeletal muscles during repetitive stimulation.

A comprehensive simulation framework incorporating a two-link lower limb biomechanical model, Hill-type muscle dynamics, and Ding's fatigue model was developed in MATLAB/Simulink to emulate realistic physiological behavior. The proposed hybrid controller was evaluated under both non-fatigue and fatigue conditions and compared against standalone PID and FOPID controllers using performance indices such as rise time, settling time, overshoot, steady-state error, and muscle force output.

The results demonstrated that the hybrid FOPID+PID controller consistently outperformed other configurations across all test scenarios. Under non-fatigue conditions, it achieved a rapid rise time of 2.28 ms and settling time of 1.11 ms, ensuring smooth trajectory tracking with minimal overshoot. Under fatigue, it maintained exceptional stability, reducing rise time to 2.23 ms and achieving a mean error of 9.55, while sustaining nearly constant muscle force (FCE ≈ 5922 N; FMAX = 5980-6106 N). These findings indicate that the integration of fractional-order control dynamics with conventional feedback offers enhanced adaptability, precision, and fatigue compensation properties that are essential for reliable long-term rehabilitation.

In addition to its superior performance, the hybrid FOPID+PID architecture retains computational simplicity, making it practical for embedded FES systems and portable neuroprosthetic devices. By combining the memory-dependent adaptability of fractional calculus with the real-time correction of classical PID, the controller offers a physiologically compatible solution capable of producing smoother, safer, and more natural joint movements. This approach bridges the gap between computational control theory and clinical rehabilitation needs, marking an important step toward intelligent, fatigue-resilient assistive technologies.

Recommendations for Future Work

Based on the results and insights obtained from this study, the following recommendations are proposed for further research and practical implementation:

- Future work should focus on implementing the hybrid FOPID+PID controller on a real-time embedded platform using digital signal processors (DSPs) or microcontrollers to evaluate its performance in in-vivo or clinical FES experiments. Hardware testing will allow assessment of real-time computational demands, electrode-skin impedance variations, and system latency.
- Integration of adaptive gain tuning or machine learning algorithms (e.g., reinforcement learning or neural adaptive control) could enhance self-adjustment capabilities under varying physiological conditions. Such extensions would enable the controller to automatically optimize stimulation parameters for individual patients.
- The current model focused on isolated knee swinging; however, the control strategy can be extended to multi-joint systems such as hip–knee–ankle coordination during gait restoration. This would require synchronization across multiple muscle groups and controllers, introducing higher-order coordination challenges.
- Incorporating fatigue prediction models and real-time electromyography (EMG) feedback could improve muscle condition monitoring and proactive adjustment of stimulation intensity. This would further enhance endurance and safety in prolonged therapy sessions.
- The hybrid controller can be embedded into wearable robotic exoskeletons or smart FES garments for continuous home-based rehabilitation. Future research should explore battery optimization, miniaturized circuitry, and wireless communication modules for 5G-enabled remote therapy applications.
- A comparative analysis between the hybrid FOPID+PID and intelligent controllers such as Fuzzy-PID, Neural-PID, or MPC-based adaptive systems will provide deeper insights into the trade-offs between computational complexity, response speed, and clinical safety.

Compliance with ethical standards

Disclosure of conflict of interest

No conflict of interest to be disclosed.

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