World Journal of Advanced Engineering Technology and Sciences
eISSN: 2582-8266
Cross Ref DOI: 10.30574/wjaets
Journal homepage: https://wjaets.com/

(Research Article)

Check for updates

Discrete time queue with interruptions and repeat/resumption of interrupted service

P.K. Pramod * and M. K. Sathian

Department of Mathematics, Panampilly Memorial Government College, Chalakudy, Kerala-680722, India.

World Journal of Advanced Engineering Technology and Sciences, 2025, 17(03), 464-472

Publication history: Received on 23 November 2025; revised on 28 December 2025; accepted on 31 December 2025

Article DOI: https://doi.org/10.30574/wjaets.2025.17.3.1581

Abstract

Here we consider an infinite-capacity, single-server, discrete-time queue in which customers arrive according to a PH-distributed interarrival process. If the system is idle, an arriving customer begins service immediately. During service, one or more interruptions may occur, and the interarrival times of the interruption process are geometrically distributed with parameter p. When an interruption occurs, a threshold random clock starts ticking. If the duration of the interruption exceeds the threshold random variable, the interrupted customer must restart the service from the beginning upon completion of the interruption; otherwise, the service is resumed. Several performance measures are evaluated, and numerical illustrations of the system behavior are also provided.

Keywords: Service interruptions; Phase type distribution; Geometric distribution; Threshold clock

1. Introduction

In this paper we consider interruption during the service of customers. Here we do not investigate queues with interruption in the form of vacation. White and Christie [18] is the first reported work on queues with interruption. Subsequently Heathcote [9], Keilson [8], Gaver [6], Aissani and Artalejo [1], among others, analyzed such queueing systems in continuous time. In a recent work Atentia and Moreno [3] discuss a discrete time queue with failure/interruption at the time when a new service starts. (See also Alfa [2]). A detailed review on queues with service interruption could be found in Krishnamoorthy et.al. [13]. For motivation in the investigation of such queues one may refer to Krishnamoorthy et.al. [12] which, we believe, is the first work to give concrete conditions for resumption/repetition of an interrupted service, on completion of the interruption. Almost simultaneously Fiems et.al. [5] considered an interruption queueing model with arbitrarily distributed service time and interruption duration, the arrival their constituting a Poisson process. They set a priori probability q for resumption of service; with complementary probability it is repeated. In this paper we assume that interruption occurs according to a geometric process of rate γ; an idle server does not get affected by interruptions. Only when service is proceeding does the interruption have effect. Further, we assume that at any time only one interruption can affect the service; that is, the service process behaves like a Type-I counter (see Karlin and Taylor [10]). Interruptions may occur due to events such as the server leaving the area to serve a priority customer or a server breakdown. We assume that the duration of each interruption is a random variable following a phase-type distribution.

At the instant the service is interrupted, a random clock (threshold clock) starts ticking. When the interruption ends, the customer whose service was interrupted either repeats or resumes service according to the following rule: if the duration of the interruption exceeds the threshold random variable, the customer must repeat the service from the beginning; otherwise, the service is resumed upon completion of the interruption. In the present work, the interruption process is assumed to be preemptive, that is, the customer in service is preempted immediately when an interruption occurs.

* Corresponding author: P.K. Pramod

The system capacity is assumed to be infinite. Customers arrive according to a Markovian arrival process with an appropriate representation, and the service discipline is first-come, first-served (FCFS), subject to server interruptions during service. This model exhibits the property that, for the customer in service, either a complete or a partial Markov property holds, depending on whether the threshold clock expires before the interruption ends or vice versa. Real life examples of the model discussed here are provided in Krishnamoorthy et.al.[12].

This paper is arranged as follows. Section 2 gives a model description and notations used in this article. In section 3, the stationary distribution of the model is provided. Description of the service process in discrete time queue is discussed in section 4 and stability Condition is investigated in section 5. The expected waiting time is provided in section 6. Then we provide numerous system characteristics in section 7. Numerical result corresponds to various performance charactersits are discussed in section 8.

2. Model Description

Here we consider a discrete-time queueing system where the time axis is divided into intervals of equal length, called slots. In continuous-time queues, the probability of more than one event such as an arrival and a departure or two departures or more event taking place during a very short interval of time is zero, whereas it is not so in discrete-time queues. Let the time axis be marked by 0,1,2,3,...,m,.......Suppose the departures and interruptions occur in (m-,m),and the arrival occur in (m,m+). In this queueing model arrival process follows phase type distribution represented by (η,L) of dimension r. Here the mean inter arrival time is $l = 1 - L1$. The service time, interruption time and threshold random variable are also phase type distributed. The phase type representation of service time, interruption time and threshold random variable are (α,S),(β,T),(δ,U) and mean service times are $s = 1-S1$,$t = 1-T1$,$u = 1-U1$ respectively. During service an interruption can occur where the interruption process is geometrically distributed with parameter γ. When the service is interrupted the server goes for interruption or vacation and threshold random clock starts ticking. Then a competition between interruption time and threshold random variable starts. On completion of interruption the interrupted customer may repeat /resume its service based on the following rule: If the interruption time exceeds threshold the interrupted customer gets its service repeated from the very beginning; else its is resumed.

The state of the system at time n be ρ_n and its value is 0 if it is in interrupted state and 1 otherwise. If $\rho_n = 0$, let the number of customers in the system including the one in service be H_n, the service phase be B_n, the phase of arrival be J_n, the phase of interruption time be D_n and the threshold random variable be E_n. If $\rho_n = 1$ there are only three r.vs; the number of customers, service phase and arrival process. The process $\Omega = (H_n, \mathrm{C}_n, \mathrm{B}_n, \mathrm{D}_n, E_n, \mathrm{J}_n)$ is a discrete time markov chain whose n^{th} level is given by $\mathrm{l(n)} = \bigcup_{n=0}^{\infty} \psi(n,l), n \geq 1, l = 0,1$. The subsets of $\psi(n,l)$ are defined as

$$\{(\mathrm{n},0,\mathrm{i}_1,\mathrm{i}_2,\mathrm{i}_3,\mathrm{i}_4) : 1 \leq \mathrm{i}_1 \leq a; 1 \leq \mathrm{i}_2 \leq b; 0 \leq \mathrm{i}_3 \leq c; 1 \leq \mathrm{i}_4 \leq r\}$$

for $1 \leq n \leq N$, $\{(n,1,i_1,i_4); 1 \leq i_1 \leq a, 1 \leq i_4 \leq r\}$. Consider the Markov chain described by

$\Delta = \{(0,J_n) \cup (H_n,0,B_n,D_n,E_n,J_n) \cup (H_n,1,B_n,J_n)\}, n \geq 0.$

The LIQBD has transition probability matrix $\psi = \begin{bmatrix} C_0 & C_1 & & & \\ C_2 & A_1 & A_0 & & \\ & A_2 & A_1 & A_0 & \\ & & A_2 & A_1 & A_0 \\ & & & . & . \end{bmatrix}$

Transition from level 0 to 0 is represented by the matrix $C_0 = L$. When the system is idle an arriving customer is immediately taken for service. This part of ψ is given by C_1. We have C_1 given by $C_1 = [[0],\beta \otimes l^1\alpha]$. The matrix C_2 corresponds to transitions on completion of the service of a customer when the server is in level 1 and the matrix is as follows: $C_2 = \begin{bmatrix} [0] \\ S^0 \otimes L \end{bmatrix}$.

Here A_0 records transitions to $\psi(n+1,l)$ from $\psi(n,l)$given by $A_0 = \begin{bmatrix} A_0^{(1)} & A_0^{(2)} \\ A_0^{(3)} & A_0^{(4)} \end{bmatrix}$. $A_0^{(1)}$corresponds to transition during interruption period. The matrix $A_0^{(1)} = I_{m_1} \otimes T \otimes \bar{U} \otimes l^1\alpha$ where $\bar{U} = \begin{bmatrix} 1 & 0 \\ U & U^0 \end{bmatrix}$.

Write the matrix$A_0^{(2)} = \begin{bmatrix} F_1 & F_2 & . & . & F_{m_1} \end{bmatrix}^T$ where $F_j = T^0 \otimes \begin{bmatrix} \beta e_j \; e_j \; e_j \ldots e_j \end{bmatrix}^T \otimes l^1\alpha$

corresponds to level change from n to n+1 when the interruption state changes to busy state. In a discrete time queueing system more than one event can takes place at an epoch. Here the interruption mode first changes to busy and then an arrival occur. Consider the matrix$A_0^{(3)} = \gamma S \otimes \eta \otimes \delta \otimes l^1\alpha$. This matrix is related to arrival of customers when the system state changes from busy to interruption. The matrix$A_0^{(4)} = S \otimes l^1\alpha$ corresponds to arrival of customers during busy period.

Now we describe A_2. The matrix A_2 can be represented as $\begin{bmatrix} [0] & [0] \\ A_2^{(1)} & A_2^{(2)} \end{bmatrix}$ where$A_2^{(1)} =$

$S^0\beta \otimes L$ and $A_2^{(2)} = \gamma S^0 e \otimes \eta \otimes \bar{\delta} \otimes L$ which designate level change from n to $n-1$. Here$A_2^{(1)}$

refers to departure of a customer along with the server changing from busy to interruption state and $A_2^{(2)} = \gamma S^0 e \otimes \eta \otimes \bar{\delta} \otimes L$ corresponds to departure and server continuing to be busy with the next customer in line.

The matrix A_1 which corresponds to transitions within level, and is as follows:

$A_1 = \begin{bmatrix} A_1^{(1)} & A_1^{(2)} \\ A_1^{(3)} & A_1^{(4)} \end{bmatrix}$. Here continuing in the interruption state is represented by the matrix $A_1^{(1)} = I_{m_1} \otimes T \otimes \bar{U} \otimes L$, where only transition due to interruption, threshold

and arrival phase change occur. Here $\bar{U} = \begin{bmatrix} 1 & \bar{0} \\ U^0 & U \end{bmatrix}$. The matrix $A_1^{(2)}$ in A_1 is $A_1^{(2)} = \begin{bmatrix} K_1 & K_2 & . & . & . & K_{m_1} \end{bmatrix}^T$ where $K_j = T^0 \otimes \begin{bmatrix} \beta & \bar{e}_j & \bar{e}_j & . & . & \bar{e}_j \end{bmatrix}^T \otimes l^1\alpha$ and e_j is

a row vector of appropriate order with 1 in the j^{th} place and 0 elsewhere. The matrix $A_1^{(2)}$correspond to transition from interruption to busy state. In this section when interruption time exceeds threshold, the interrupted customer gets its service repeated. Thus events in these transition corresponds to removal of interruption followed by repeat/resume of service.

Occurrence of interruption during service is indicated by the matrix $A_1^{(3)}$ and is given by

$A_1^{(3)} = \gamma S \otimes \eta \otimes \hat{\delta} \otimes L + \gamma S^0 e \otimes \eta \otimes \delta \otimes t^1\alpha$. The first term in $A_1^{(3)}$ stands for no service completion and arrival prior to interruption. The second term stands for a service completion and an arrival take place before it skips to interruption. The matrix $A_1^{(4)} = S \otimes L + S^0\beta \otimes l^1\alpha$where $\hat{\delta} = (0,\delta)$, shows that the system maintain its status when it is busy with no arrival and service completion in the first term and an arrival and departure in the second term.

3. Stationary Distribution

Let $A = A_0+A_1+A_2$ and $\pi = \pi A$, $\pi\mathbf{1} = 1$. The LIQBD is positive recurrent if $\pi A_0 1 < \pi A_2 1$. Let x be the invariant vector of P with $x = xP$, $x\mathbf{1} = 1$ where $x = [x_0, x_1, \ldots\ldots]$. By the matrix-geometric theorem (Neuts (1981)) we have $x_{i+1} = x_i R$, where R is the minimal nonnegative solution to $R = A_0 + RA_1 + R^2A_2$ and the vectors x_0, x_1 are obtained by solving the equations

$$x_0 D_0 + x_1 C_1 = 0,\ \ x_0 C_0 + x_1(A_1 + RA_2) = 0 \tag{1}$$

subject to the normalizing condition

$$x_0 e + x_1(I - R)^{-1} e = 1, \tag{2}$$

where $[x_0, x_1]$ is the invariant vector of the stochastic matrix $\begin{bmatrix} C_0 & C_1 \\ C_2 & A_1 + RA_2 \end{bmatrix}$. Also $x_{i+1} = x_i R_{,i} \geq 1$.

4. Description of the service process in Discrete time queue

In this section we describe the time it takes to process a job once it enters into the service facility. We assume that service time, interruption time, threshold random variables are all independent phase type distributed random variables, with representations (α,S), (β,T) and (δ,U) respectively. Let X denote the duration of the effective service for a job. ie X is the time between the arrival of a job to the service facility until it leaves the facility. There is no restriction on number of interruptions during the service of a customer. The interruption occurs to the service of a customer with rate γ which is distributed geometrically. We define $J_1(t)$,$J_2(t)$,$J_3(t)$ respectively, to the phase of service, phase of interruption and phase of threshold random variable.

The states and their description are given in the following table:

$\{J_1\}, 1 \leq J_1 \leq a$	The service is in the Phase J_1
$\{J_1, J_2\}$	The threshold clock is expired, the interruption clock is in state J_2 the service phase is frozen in state J_1.
$\{J_1, J_2, J_3\}$	The interruption clock is in state J_2, the threshold clock is in state J_3 and the service phase is frozen in state J_1.

The Markov process $\{J_1(t), J_2(t), J_3(t)\} : t \geq 0$ with absorbing state $*$ is defined on the state space

$\Omega = \{(0,J_2);1 \leq J_2 \leq b\} \cup \{(J_1,J_2);1 \leq J_1 \leq a, 1 \leq J_2 \leq b\} \cup \{(J_1,J_2,J_3);1 \leq J_1 \leq a, 1 \leq J_2 \leq b, 1 \leq J_3 \leq c\}$

and its infinitesimal generator matrix is given by

$$Q = \begin{bmatrix} \Delta & \Delta^0 \\ 0 & 0 \end{bmatrix} \tag{3}$$

where

$$\Delta = \begin{bmatrix} \Delta_{01} & 0 & \Delta_{03} \\ \Delta_{11} & \Delta_{12} & 0 \\ \Delta_{21} & \Delta_{22} & \Delta_{23} \end{bmatrix} \tag{4}$$

Where

$\Delta_{01} = S - \gamma I,\ \Delta_{03} = \gamma\beta \otimes \delta \otimes I$

$\Delta_{11} = T^0 \otimes \alpha,\ \Delta_{12} = I \otimes T$

$\Delta_{21} = I \otimes T^0 \otimes e,\ \Delta_{22} = I \otimes T \otimes U^0$

$\Delta_{23} = I \otimes (T \oplus U)$

and

$\Delta^0 = (S^0, 0, 0)'$

Theorem 1 *The effective service time, X, has phase type distribution with representation* (ζ, Δ) *of order a+bc+abc, where* $\zeta = (\alpha, 0, 0)$ *is given in (4).*

Proof 1 *First note that a new service will begin in level* 0 *in state* J_1 *with probability* α_{j1}*. Once the service begins it can end with or without interruptions and looking through all possible transitions, one will see that the transition matrix is given in* Δ*. Thus, the service time is nothing but the time until absorption into state * starting from level* 0*.*

Mean $\mu'_\Delta = \zeta(-\Delta)^{-1}e$ *and standard deviation of X is* $\sigma\Delta = \sqrt{2\zeta(-T)^{-2}c - \mu\Delta^1}$

Due to the special structure of the matrix given in (4), we can compute the mean' as well

as the standard deviation of X explicitly and recursively. First, we define

$\beta(-\Delta)^{-1} = (u, v, w)$,

Using the above equation and exploiting the special structure of Δ*, we can easily calculate u,v and w. Also we get the expression for* $\mu_\Delta{}' = ue + ve + we.$

5. Stability Condition.

Theorem 2 *The markov chain* $\{X_t, t \geq 0\}$ *is stable if only if* $\frac{1}{\eta(L)^{-1}e} < \frac{1}{\mu_\Delta{}'}$.

6. Expected waiting time

We compute the expected waiting time of a tagged customer and is positioned r in the queue at the arrival epoch. We consider the Markov process $\{N(t), S(t), J(t) / t = 0, 1, 2, \ldots\}$ where $N(t)$ is the rank of the customer, $S(t) = \{0,1\}$ as the state of the server, $J(t)$ as the phase of the service process at time t. The r^{th} rank of the customer may decrease to $r - 1$ when the present customer in service leave the system after completing his service. We arrange the state space of $X(t)$ as:$\{r, r-1, \ldots\ldots, 3, 2, 1\} \times \{0,1\} \times \{1, 2, \ldots\ldots, a\} \times \{*\}$ where $*$ is an absorbing state which denote the tagged customer is selected for service.

The infinitesimal generator $\bar{Q} = \begin{bmatrix} \tilde{\Delta} & \tilde{\Delta}^0 \\ \bar{0} & 0 \end{bmatrix}$ where $\Delta = \begin{bmatrix} A_1 & A_2 & & & & \\ & A_1 & A_2 & & & \\ & & A_1 & A_2 & & \\ & & & \cdot & \cdot & \\ & & & & \cdot & \cdot \\ & & & & & A_1 \end{bmatrix}$ and

$\Delta^0 = \begin{bmatrix} 0 & 0 & 0 & . & . & \beta \end{bmatrix}'$ where $\beta = \begin{bmatrix} 0, S^0 \end{bmatrix}'$. The matrix $\tilde{A}_1 = \begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} \\ \tilde{A}_{13} & \tilde{A}_{14} \end{bmatrix}$ and $A_{11} = I_\alpha \otimes T \otimes \overline{U}$,

$A_{12} = \begin{bmatrix} B_{11} \; B_{12} \ldots B_{1\alpha} \end{bmatrix}'$ where $B_{1j} = T^0 \otimes \begin{bmatrix} \beta \; \overline{e_j} \ldots \overline{e_j} \end{bmatrix}'$

$A_{13} = \gamma S \otimes \eta \otimes \bar{\delta}$ and $A_{14} = S$. The matrix $\tilde{A}_2 = \begin{bmatrix} 0 & 0 \\ \tilde{A}_{21} & \tilde{A}_{22} \end{bmatrix}$ where $A_{21} = \gamma S^0 \beta \otimes \delta$

$A_{22} = (S^0 \beta \otimes L)(1-\gamma)$

7. Performance Characteristics

Some useful descriptors of the model are listed below.

- Mean number of customers in the system $= \sum_{n=1}^{\infty} n x_n e = x_1 (I-R)^{-2} e$
- Fraction of time the server is busy $= \sum_{n=1}^{\infty} x_{n1} e$
- Fraction of time the server remains interrupted $= \sum_{n=1}^{\infty} x_{n0} e$
- Thus the fraction of time the server is idle $= x_0 e$
- Fraction of time service is in interrupted state

  +Fraction of time service is going on $= \sum_{n=1}^{\infty} x_{n0} e + \sum_{n=1}^{\infty} x_{n1} e$

- The rate at which server break down occurs $= \gamma \sum_{n=1}^{\infty} x_{n1} e$
- The rate at which repair completion (removal of interruption) takes place before the
- Threshold is reached

  $R^c{}_{NT} = \sum_{n=1}^{\infty} \sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{k=1}^{c} \sum_{l=1}^{r} x_{n,0,i,j,k,l}\, S^0{}_{2j}$ where

  S_{2j}^0 is the j^{th} component of S_2^0

- Rate at which repair completion takes place after the threshold is reached

  $R_T^c = \sum_{n=1}^{\infty} \sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{l=1}^{r} x_{n,0,i,j,0,l}\, S_{2j}^0$

- Effective service rate $R^c{}_T = \sum_{n=1}^{\infty} \sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{l=1}^{t} x_{n,1,i,1}\, S^0{}_{2i}$
- The probability of a customer completing service without any interruption=P(service time< an exponentially distributed random variable with parameter γ) and is given by $\int_0^{\infty} (\alpha e^{Tu} T^0) e^{-\gamma u} du = \alpha(\gamma I - S_1)^{-1} S_1^0$

8. Numerical Results

In order to illustrate the performance of the system, we fix the following values:

$$L = \begin{bmatrix} 0.3 & 0.2 \\ 0.2 & 0.3 \end{bmatrix}, S = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}, T = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}$$

$$U = \begin{bmatrix} 0.5 & 0.2 \\ 0.2 & 0.5 \end{bmatrix} \; L^0 = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \; S^0 = \begin{bmatrix} 0.7 \\ 0.7 \end{bmatrix}, T^0 = \begin{bmatrix} 0.7 \\ 0.7 \end{bmatrix}, U^0 = \begin{bmatrix} 0.3 \\ 0.3 \end{bmatrix}$$

γ	0.0	0.05	0.1	0.15	0.2	0.25
Mean	1.1	1.5	2.0	3.2	5.5	16

Table above indicates that mean number of customers in the system increases with increasing interruption rate γ.

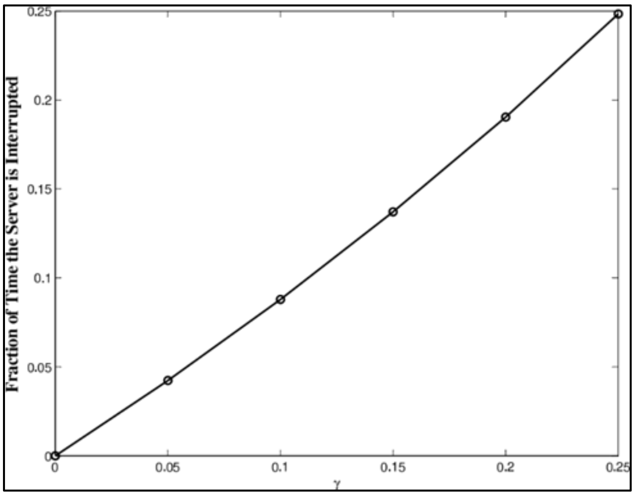

Figure 1 Gamma versus Fraction of time the server is interrupted

In this model, fraction of time the server is interrupted increases with increasing interruption rate. It is seen from Fig. 1 that the graph is almost linear in shape. Fraction of time the server is idle decreases with increasing interruption rate γ in figure 2.

With increasing interruption rate from 0 to 0.25, the effective service rate decreases gradually, which is seen in Fig. 3.

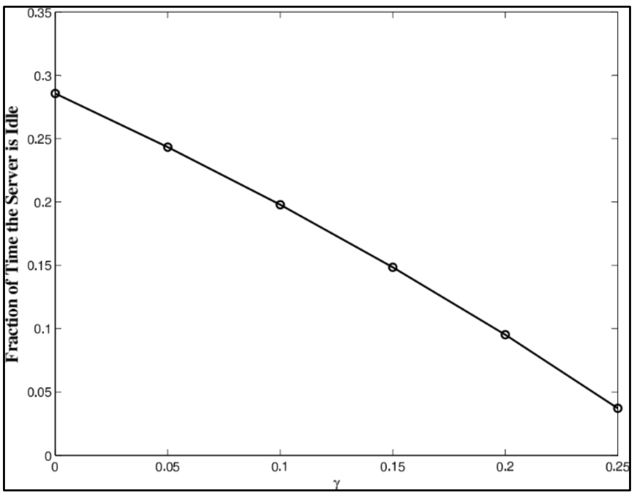

Figure 2 Gamma versus Fraction of time the server is Idle

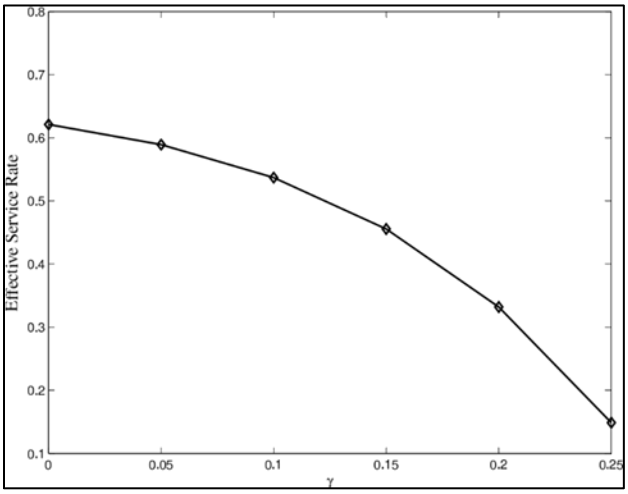

Figure 3 Gamma versus Effective service Rate

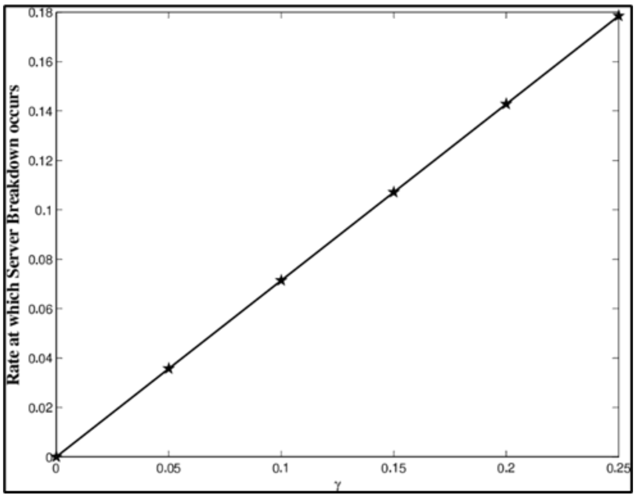

Figure 4 Gamma versus Rate at which server breakdown occurs

The rate at which sever break down also increases with increasing interruption rate and the curve is linear (see Fig. 4).

9. Conclusion

In this paper we investigated a discrete time queueing system where the server is subject to interruption. The purpose of introducing threshold clock is to check the interruption duration of a customer's service and to decide whether the service to be repeated or resumed. We analyzed such a queueing system in stable regime to obtain several characteristics. Numerical illustrations are provided for the measures that are investigated.

Compliance with ethical standards

Disclosure of conflict of interest

No conflict of interest to be disclosed.

References

[1] Aissani A and Artalejo J R 1998 On the single server retrial queue subject to breakdowns, *Queueing Sysitesm: Theory and Applications*, v30 n, 3-4, 309-321.

[2] Alfa A S 2007 Discrete time queues and matrix-analytic methods, *TOP, Springer publication*, 147-185.

[3] Atencia I and Moreno P 2006 A Discrete-Time Geo/G/1 retrial queue with the server subject to starting failures, *Annals of OR*, 141:85-107.

[4] Bellman R E 1960 Introduction to matrix analysis, *McGraw Hill*, New York, NY. .

[5] Fiems D, Maertens T and Brunee H 2008 Queueing systems with different types of interruptions, *European Journal of Operations Research*, 188 (3), 838-845.

[6] Gaver D 1962 A waiting line with interrupted service including priority, *Journal of Royal Statistical Society*, B24, 73-90 .

[7] Ibe O and Trivedi K S 1990 Two Queues with alternating service and server breakdown, *Queueing Systems*, Vol.7(3-4), 253-268.

[8] Julian Keilson 1962 Queues Subject to Service Interruptions, *The Annals of Mathematical Statistics*.

[9] Heathcote, C R 1959 The time-dependent problem for a queue with preemptive priorities, *Operations Research*, 670-680.

[10] Karlin S and Taylor H E 1975 A first course in Stochastic Processes, 2nd edition, *Elsevier*.

[11] Kao E P C and Smith M S 1992 On excess, Current and Total life distributions of Phase-type renewal processes, *Naval Research Logistics*, Vol.32, pp.789-799.

[12] Krishnamoorthy A, Pramod P K and Deepak T G 2009 On a queue with interruptions and repeat or resumption of service. *Nonlinear Analysis, Theory, Methods and Applications*, 71 (12), 1673-1683.

[13] Krishnamoorthy A, Pramod P K and Chakravarthy S R 2014 A survey on queues with interruptions, *TOP* , 22,290-320.

[14] Krishnamoorthy A, Pramod P K and Chakravarthy S R 2013 A Note on Characterizing Service Interruptions with Phase-Type Distribution, *Stochastic Analysis and Applications*,Taylor and Francis, 31 (4), 671-683.

[15] Marcus M and Minc H 1964 A Survey of matrix theory and matrix inequalities, *Allyn and Bacon*, Boston, MA.

[16] Neuts M F 1981 Matrix-Geometric solutions in stochastic models: An algorithmic approach, *The Johns Hopkins University Press*, Baltimore, MD.

[17] Tewfik Kernane 2009 A Single Server Retrial Queue with Different Types of Server Interruptions,*HAL Open Science*.

[18] White H and Christie L S 1958 Queuing with Preemptive Priorities or with Breakdown, *Operational Research*. Vol. 6, pp. 79-95. .