World Journal of Advanced Engineering Technology and Sciences

eISSN: 2582-8266

Cross Ref DOI: 10.30574/wjaets

Journal homepage: https://wjaets.com/

(Research Article)

Check for updates

Mathematical Modelling of Poor Democratic Governance in Nigeria and Its Implications for National Development

MutahWadai [1,*], Ibekwe Jacob John [1] and Adem Kilicman [2]

[1] *Department of Mathematics, Federal University of Health Sciences, Otukpo, Benue State, Nigeria.*

[2] *Department of Computer and Mathematical Sciences, Universiti of Teknologi MARA, 40450 Shah Alam, Selangor, Malaysia.*

World Journal of Advanced Engineering Technology and Sciences, 2026, 18(02), 104-121

Publication history: Received on 01January 2026; revised on 07 February 2026; accepted on 10 February 2026

Article DOI: https://doi.org/10.30574/wjaets.2026.18.2.0084

Abstract

Poor democratic governance continues to constrain sustainable national development in Nigeria; however, most existing studies rely on qualitative or econometric approaches that do not adequately capture the dynamic evolution of governance practices. This study develops a nonlinear compartmental dynamical model to examine the persistence of poor democratic governance and its implications for national development. The population is divided into politically apathetic citizens, participants in poor democratic practices, and advocates of good governance, while a national development index is introduced to explicitly link governance dynamics with development outcomes. Fundamental qualitative properties of the model, including positivity and boundedness of solutions, are established. A threshold parameter, the basic reproduction number R_0, is derived and shown to determine whether poor governance dies out or persists. Stability analysis reveals that the poor governance-free equilibrium is globally asymptotically stable when $R_0 < 1$, whereas an endemic equilibrium exists and is globally stable when $R_0 > 1$. Numerical simulations based on Nigeria-informed baseline parameters drawn from published sources support the analytical results. Normalized sensitivity analysis identifies the transmission of poor governance behaviours and political recruitment as the strongest drivers of persistence, while reform, sanctions, and turnover exert stabilizing effects. The findings highlight the importance of preventive institutional reforms and sustained civic engagement for long-term national development.

Keywords: Basic Reproduction Number; Democratic Governance; National Development; Nonlinear Dynamical Model; Sensitivity Analysis; Stability Analysis

1. Introduction

Several studies have applied mathematical and dynamical systems approaches to the study of corruption and governance-related behaviours. Using compartmental or epidemiological-style models, these studies examine how corrupt practices spread within a population, identify corruption-free and endemic equilibria, and establish threshold conditions analogous to a basic reproduction number [1, 2, 8]. While such models provide valuable insights into corruption persistence and control, they are generally limited to corruption dynamics alone and do not explicitly incorporate broader democratic governance structures or development outcomes.

In the Nigerian context, most governance-related studies are qualitative or econometric in nature. The existing literature extensively documents how poor democratic governance, corruption, weak institutions, and lack of accountability have negatively affected Nigeria's economic growth, social welfare, and human development [3, 7, 12]. Although these studies establish strong empirical relationships between governance quality and development indicators, they do not employ mathematical models capable of capturing the dynamic evolution of governance practices over time.

*Corresponding author: Mutah Wadai

Beyond Nigeria, system dynamics and institutional modelling approaches have been used to explore the relationship between governance quality, institutional effectiveness, and socioeconomic outcomes [6, 10]. While these studies acknowledge feedback mechanisms between governance and development, they typically rely on simulation-based frameworks rather than analytical differential-equation models that permit equilibrium and stability analysis.

Overall, the literature reveals a clear gap. Although mathematical models of corruption exist and empirical studies linking governance to development are abundant, there remains a lack of models that formally integrate democratic governance dynamics with national development outcomes within a unified dynamical systems framework, particularly in the Nigerian context.

This study addresses this gap by developing a mathematical model of poor democratic governance that captures transitions among political apathy, poor governance, and good governance, while explicitly linking these dynamics to a national development variable. Through equilibrium analysis, stability conditions, and policy-relevant thresholds, the model contributes a novel analytical framework to the governance–development literature.

2. Model Formulation and Assumptions

2.1. Model formulation

This study develops a deterministic, nonlinear compartmental model to examine the dynamics of poor democratic governance in Nigeria and its implications for national development. The model is inspired by epidemic-type transmission frameworks, in which governance behaviours spread through social, political, and institutional interactions within the population.

At time t, the total population $N(t)$ is partitioned into three governance-related compartments: politically apathetic citizens $A(t)$, participants in poor democratic practices $P(t)$, and advocates of good governance $G(t)$. A national development index $D(t)$ is introduced to explicitly link governance dynamics to development outcomes.

Individuals enter the political system at a constant recruitment rate Λ and exit naturally at rate μ. Poor governance spreads through social influence and political interaction at rate β. Reform, civic education, and institutional correction convert poor governance participants into good governance advocates at rate γ, while reform fatigue or political discouragement causes some advocates to revert to apathy at rate σ. Poor governance also attracts institutional sanctions or additional removal at rate δ. National development is directly influenced by governance quality.

2.2. Model Assumptions

To reflect the Nigerian context, the following assumptions are made:

- The population is divided into mutually exclusive classes based on governance participation.
- Poor democratic practices can spread through political patronage, electoral malpractice, and the normalization of corruption.
- Effective institutions and civic education reduce poor governance participation.
- National development deteriorates as poor governance intensifies.
- All parameters are non-negative, and governance dynamics evolve continuously over time

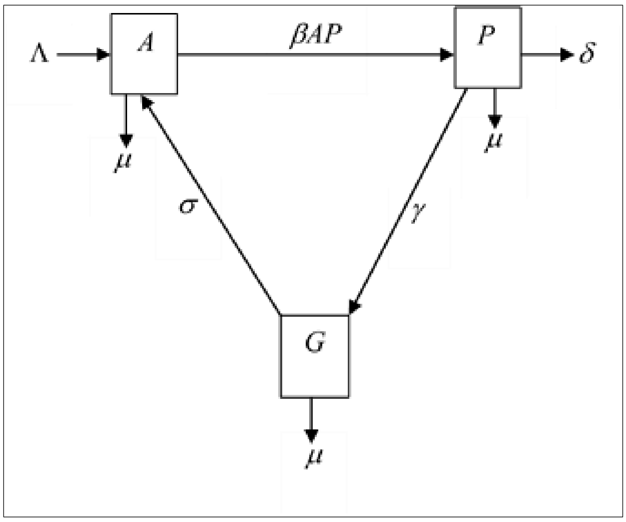

Figure 1 The model flow diagram

3. Mathematical Analysis of the Model

3.1. Model equations for Poor Democratic Governance.

The governing equations of the model are given as follows:

$$\frac{dA}{dt} = \Lambda - \beta AP - \mu A + \sigma G, \tag{1}$$

$$\frac{dP}{dt} = \beta AP - (\gamma + \delta + \mu)P, \tag{2}$$

$$\frac{dG}{dt} = \gamma P - (\sigma + \mu)G, \tag{3}$$

The national development index satisfies:

$$\frac{dD}{dt} = \rho G - \kappa P - \eta D \tag{4}$$

Equation (4) captures the net evolution of national development as the balance between positive contributions from good governance, deterioration due to poor governance, and natural decay effects.

Table 1 Variables for Poor Democratic Governance

Variable	Description
$A(t)$	Politically apathetic citizens.
$P(t)$	Population actively engaging in poor democratic practices (vote-buying, corruption, political thuggery).
$G(t)$	Citizens promoting good democratic governance.
$D(t)$	National development index.

Table 2 Parameters for Poor Democratic Governance

Parameter	Description
Λ	Recruitment rate into political system.
β	Rate of influence into poor governance.
γ	Rate of reform, civic education, and institutional correction.
σ	Disillusionment rate from good governance.
δ	Sanctions against poor governance.
μ	Natural exit rate.
ρ	Contribution of good governance to development.
κ	Negative impact of poor governance on development.
η	Development decay rate.

3.2. Positivity of Solutions

3.2.1. Theorem 1

For non-negative initial conditions $A(0) \geq 0,\ P(0) \geq 0,\ G(0) \geq 0,$ all solutions of the system remain non-negative for all t>0.

3.2.2. Proof.

We examine each equation on the boundary planes

From equation (1), on the boundary A=0:

$$\frac{dA}{dt} = \Lambda + \sigma G \geq 0,$$

since $\Lambda > 0$ and $G \geq 0$.

Hence, trajectories cannot cross into the negative A-region.

From equation (2), on the boundary P = 0:

$$\frac{dP}{dt} = 0.$$

Thus, P(t) cannot become negative once it reaches zero.

From equation (3), on the boundary G = 0:

$$\frac{dG}{dt} = \gamma P \geq 0,$$

since $\gamma > 0$ and $P \geq 0$.

By evaluating the vector field on the boundary planes A = 0, P = 0, and G = 0, it can be shown that trajectories cannot cross into the negative region. Hence, the non-negative orthant in the region $\Omega = \{(A, P, G) \in \Re^3 : A, P, G \geq 0\}$ is positively invariant, and all solutions remain non-negative for all t > 0.

Overall, since the vector field points inward or is tangential to the non-negative orthant on all boundary planes, the region Ω is positively invariant.

3.3. Invariant (Feasible) Region

3.3.1. Theorem 2

Let $\Omega = \left\{ (A, P, G) \in \Re_{+}^{3} : A + P + G \leq \frac{\Lambda}{\mu} \right\}$ denote the feasible region of the model. For any initial condition in Ω, the solution remains in Ω for all t > 0.

3.3.2. Proof

To establish boundedness, we study the total population dynamics. Let the total population be

$$N(t) = A(t) + P(t) + G(t) \tag{5}$$

Differentiating equation (5) with respect to t, we have;

$$\frac{dN(t)}{dt} = \frac{dA(t)}{dt} + \frac{dP(t)}{dt} + \frac{dG(t)}{dt} \tag{6}$$

Substituting equation (1) to (3) into (6) and simplifying, we have;

$$\frac{dN}{dt} = \Lambda - \mu(A + P + G) - \delta P,$$

$$\frac{dN}{dt} \leq \Lambda - \mu N. \tag{7}$$

Solving equation (7) by method of comparison, we have;

$$N(t) \leq \frac{\Lambda}{\mu} + \left(N(0) - \frac{\Lambda}{\mu} \right) e^{-\mu t}.$$

Thus,

$$\lim_{t \to \infty} \sup N(t) \leq \frac{\Lambda}{\mu}.$$

Hence, all solutions eventually enter and remain in the region:

$$\Omega = \left\{ (A, P, G) \in \Re_{+}^{3} : A + P + G \leq \frac{\Lambda}{\mu} \right\}.$$

3.4. Poor Governance-Free Equilibrium (PGFE)

By definition, at the poor governance-free equilibrium: $P^{*} = 0$. At Equilibrium;

$$\frac{dA}{dt} = \frac{dP}{dt} = \frac{dG}{dt} = 0.$$

From equation (3): $0 = \gamma P^* - (\sigma + \mu)G.$

Substituting, $P^* = 0$ into (3), we have; $G^* = 0.$

From equation (1): $0 = \Lambda - \mu A^* + \sigma G^*.$

Substituting, we have; $A^* = \frac{\Lambda}{\mu}.$

Hence, the PGFE is given by: $E_0 = \left(\frac{\Lambda}{\mu}, 0, 0\right)$ and represents a political system devoid of poor democratic practices.

3.5. Basic Reproduction Number

Considering equation (1) to (3), the infected compartment is P(t), since it represents active engagement in poor democratic practices using next-generation matrix approach [15].

Considering the P(t) equation: $\frac{dP}{dt} = \beta AP - (\gamma + \delta + \mu)P.$

The new infection term is: $f = \beta AP.$

The transition (removal) term: $v = (\gamma + \delta + \mu)P.$

Constructing the next-generation matrices and evaluating at E_0, we have;

New-infection matrix F: $F = \left[\frac{\partial f}{\partial}\right]_{E_0} = \beta A^* = \frac{\beta \Lambda}{\mu}.$

Transition matrix V: $V = \left[\frac{\partial v}{\partial}\right]_{E_0} = \gamma + \delta + \mu..$

Since there is only one infected compartment, the matrices reduce to scalars.

Therefore, the basic reproduction number is computed by $R_0 = \rho(FV^{-1})$.

$$R_0 = \frac{F}{V} = \frac{\beta \Lambda}{\mu(\gamma + \delta + \mu)}. \quad (8)$$

3.6. Local Stability Analysis of the Poor Governance-Free Equilibrium (PGFE)

3.6.1. Theorem 3

The poor governance-free equilibrium is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.

3.6.2. Proof

We analyze the local stability of the poor governance-free equilibrium $E_0 = \left(\frac{\Lambda}{\mu}, 0, 0\right)$ using the Jacobian matrix method.

The Jacobian J (A, P, G) is given by:

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial A} & \frac{\partial f_1}{\partial P} & \frac{\partial f_1}{\partial G} \\ \frac{\partial f_2}{\partial A} & \frac{\partial f_2}{\partial P} & \frac{\partial f_2}{\partial G} \\ \frac{\partial f_3}{\partial A} & \frac{\partial f_3}{\partial P} & \frac{\partial f_3}{\partial G} \end{bmatrix} \qquad (9)$$

Taking the partial derivative of equation (1) to (3), we have;

$$J = \begin{bmatrix} -\beta P - \mu & -\beta A & \sigma \\ \beta P & \beta A - (\gamma + \mu + \delta) & 0 \\ 0 & \gamma & -(\sigma + \mu) \end{bmatrix} \qquad (10)$$

Evaluating (10) at E_0, we have;

$$J(E_0) = \begin{bmatrix} -\mu & \frac{-\beta A}{\mu} & \sigma \\ 0 & \frac{\beta A}{\mu} - (\gamma + \mu + \delta) & 0 \\ 0 & \gamma & -(\sigma + \mu) \end{bmatrix}$$

Since $J(E_0)$ is upper triangular, the eigenvalues are the diagonal entries:

$$\lambda_2 = \frac{\beta\Lambda}{\mu} - (\gamma + \delta + \mu),\ \lambda_3 = -(\sigma + \mu) < 0.$$

From the eigenvalues, $\lambda_1 < 0, \lambda_3 < 0$. Thus stability depend entirely on λ_2

$\frac{\beta\Lambda}{\mu} - (\gamma + \delta + \mu) < 0.$ Rearranging, we have; $\frac{\beta\Lambda}{\mu(\gamma + \delta + \mu)} < 1.$

Recall $R_0 = \frac{\beta\Lambda}{\mu(\gamma + \delta + \mu)}$. This implies that $R_0 < 1$. Hence, PGFE is locally asymptotically stable and unstable if $R_0 > 1$.

3.7. Global stability of the Poor Governance-Free Equilibrium (PGFE)

3.7.1. Theorem 4

Let E_0 be the poor governance-free equilibrium of the system. If $R_0<1$, then E_0 is globally asymptotically stable in the feasible region Ω.

3.7.2. Proof

Let the Lyapunov function be

$$V(P) = P \geq 0 \qquad (11)$$

This means V(P)=0 if and only if P=0.

Taking the derivative of (11), we have;

$$\frac{dV}{dt} = \frac{dP}{dt} = \beta AP - (\gamma + \delta + \mu)P = P[\beta A - (\gamma + \delta + \mu)]$$

From the positivity and invariant region analysis, the bound of A(t) is given by:

$$0 \leq A(t) \leq N(t) \leq \frac{\Lambda}{\mu}.$$

Hence,

$$\beta A - (\gamma + \delta + \mu) \leq \beta \frac{\Lambda}{\mu} - (\gamma + \mu + \delta)(R_0 - 1).$$

Using the boundedness of A(t) and the definition of R_0, it follows that:.

$$\frac{dV}{dt} \leq \beta \frac{\Lambda}{\mu} - (\gamma + \mu + \delta)(R_0 - 1)$$

If $R_0 < 1$, then $(R_0 - 1) < 0$. Therefore, $\frac{dV}{dt} \leq 0$, with equality only at P = 0. By LaSalle's Invariance Principle, the largest invariant set where $\frac{dV}{dt} = 0$ is P = 0, $\frac{dG}{dt} = -(\sigma + \mu)G$ as $G \to 0$ and $\frac{dA}{dt} = \Lambda - \mu A$ as $A \to \frac{\Lambda}{\mu}$.

Hence, the PGFE is globally asymptotically stable whenever $R_0 < 1$.

4. Endemic equilibrium analysis

4.1. Endemic (persistent poor governance) equilibrium

At the endemic equilibrium, all derivatives of system (1) to (3) are set to zero. That is, $\frac{dA}{dt} = \frac{dP}{dt} = \frac{dG}{dt} = 0$ and $P^* > 0$ (persistent poor governance). Let $E_1 = (A^*, P^*, G^*)$.

From equation (3),

$$0 = \gamma P^* - (\sigma + \mu)G^* \Rightarrow G^* = \frac{\gamma}{\sigma + \mu} P^* \quad (12)$$

From equation (1), $0 = \Lambda - \beta A^* P^* - \mu A^* + \sigma G^*$

Substituting the expression for G^*, and simplifying further gives;

$$A^* = \frac{\Lambda + \frac{\sigma\gamma}{\sigma + \mu} P^*}{\mu + \beta P^*} \quad (13)$$

From equation (2), $0 = \beta A^* P^* - (\gamma + \delta + \mu)P^*$,

$$0 = (\beta A^* - (\gamma + \delta + \mu))P^*.$$

Divide both sides by P^*, we have

$$\beta A^* - (\gamma + \delta + \mu) = 0 \Rightarrow A^* = \frac{\gamma + \delta + \mu}{\beta} \tag{14}$$

Substitute equation (14) into (13), we have;

$$\frac{\gamma + \delta + \mu}{\beta} = \frac{\Lambda + \frac{\sigma\gamma}{\sigma + \mu} P^*}{\mu + \beta P^*}$$

Cross multiply, we have;

$$\beta\left(\Lambda + \frac{\sigma\gamma}{\sigma + \mu} P^*\right) = (\mu + \beta P^*)(\gamma + \delta + \mu),$$

$$\beta\Lambda + \frac{\beta\sigma\gamma}{\sigma + \mu} P^* = \mu(\gamma + \delta + \mu) + \beta P^*(\gamma + \delta + \mu)$$

Collecting like terms and simplifying further gives;

$$P^* = \frac{\mu(\gamma + \delta + \mu) - \beta\Lambda}{\beta\left(\frac{\sigma\gamma}{\sigma + \mu} - (\gamma + \delta + \mu)\right)} \tag{14a}$$

Recall that $R_0 = \dfrac{\beta\Lambda}{\mu(\gamma + \delta + \mu)}$, this implies that $R_0 \mu(\gamma + \delta + \mu) = \beta\Lambda$ (14b)

Put (14b) into (14a), and simplifying further gives;

$$P^* = \frac{\mu(\gamma + \delta + \mu)(R_0 - 1)}{\beta\left((\gamma + \delta + \mu) - \frac{\sigma\gamma}{\sigma + \mu}\right)} > 0 \text{ if } R_0 > 1.$$

Hence, the endemic equilibrium is: $E_1 = \left(\dfrac{\gamma + \delta + \mu}{\beta}, \dfrac{\mu(\gamma + \delta + \mu)(R_0 - 1)}{\beta\left((\gamma + \delta + \mu) - \frac{\sigma\gamma}{\sigma + \mu}\right)}, \dfrac{\gamma}{\sigma + \mu} P^*\right)$ when $P^* > 0$.

4.2. Local stability of the endemic (persistent poor governance) equilibrium

The Jacobian for (A, P, G) from (10) is

$$J(A,P,G)=\begin{bmatrix}-\beta P^{*}-\mu & -\beta A^{*} & \sigma\\ \beta P^{*} & \beta A^{*}-(\gamma+\mu+\delta) & 0\\ 0 & \gamma & -(\sigma+\mu)\end{bmatrix}$$

$$J(E_1)=\begin{bmatrix}-\beta P^{*}-\mu & -(\gamma+\delta+\mu) & \sigma\\ \beta P^{*} & 0 & 0\\ 0 & \gamma & -(\sigma+\mu)\end{bmatrix} \quad (15)$$

Let x = $\beta P^{*} > 0$. The characteristic polynomial of (15) is a cubic: $\chi(\lambda)=\lambda^3+a_1\lambda^2+a_2\lambda+a_3$,

with coefficient: $a_1 = x+2\mu+\sigma>0,$ (15a)

$$a_2 = x(\gamma+2\mu+\delta+\sigma)+\mu(\mu+\sigma)>0, \quad (15b)$$

$$a_3 = x(\gamma+\mu+\delta(\mu+\sigma)-\gamma\sigma)>0. \quad (15c)$$

Applying Routh–Hurwitz conditions on the cubic: $\lambda^3+a_1\lambda^2+a_2\lambda+a_3=0$,

The local asymptotic stability holds if and only if: $a_1>0,\ a_2>0,\ a_3>0,$ and $a_1,a_2>a_3$.

Since equation (15a)-(15c) are all greater than zero, then we will prove that $a_1,a_2-a_3>0$ as shown below;

$$a_1,a_2-a_3 = x^2(\gamma+\delta+2\mu+\sigma)+x(\gamma\mu+\gamma\sigma+\delta\mu+4\mu^2+4\mu\sigma+\sigma^2)+(2\mu^3+3\mu^2\sigma+\mu\sigma^2) \quad (16)$$

Every term on the right-hand side of (16) is strictly positive for $\mu > 0$, $x > 0$ and nonnegative parameters. Therefore,

$$a_1,a_2>a_3.$$

All Routh–Hurwitz conditions hold, so all eigenvalues of $J(E_1)$ have negative real parts**.** Hence, the endemic equilibrium E_1 is locally asymptotically stable whenever $P^{*} > 0$.

4.3. Global stability of the endemic equilibrium

The endemic equilibrium exists if R >1. Recall that,

$$E_1=(A^{*},P^{*},G^{*})=\left(\frac{\gamma+\delta+\mu}{\beta},P^{*},\frac{\gamma}{\sigma+\mu}P^{*}\right)$$

We will be using logarithmic Lyapunov function and it is given by

$$V(A,P,G)=A-A^{*}-A^{*}In\frac{A}{A^{*}}+P-P^{*}-P^{*}In\frac{P}{P^{*}}+G-G^{*}-G^{*}In\frac{G}{G^{*}}. \quad (17)$$

Taking the partial derivative of (17), we have;

$$\frac{dV}{dt}=\left(1-\frac{A}{A^{*}}\right)\frac{dA}{dt}+\left(1-\frac{P}{P^{*}}\right)\frac{dP}{dt}+\left(1-\frac{G}{G^{*}}\right)\frac{dG}{dt}. \quad (18)$$

Substitute equation (1) to (3) into (18), we have;

$$\frac{dV}{dt} = \left(1 - \frac{A}{A^*}\right)\left[-\beta\left(A - A^*\right)\left(P - P^*\right) - \mu\left(A - A^*\right) + \sigma\left(G - G^*\right)\right]$$
$$+\left(1 - \frac{P}{P^*}\right)\left[\beta\left(A - A^*\right)\left(P - P^*\right) - (\gamma + \delta + \mu)\left(P - P^*\right)\right]$$
$$+\left(1 - \frac{G}{G^*}\right)\left[\gamma\left(P - P^*\right) - (\sigma + \mu)\left(G - G^*\right)\right]$$

Next we will prove that $\frac{dV}{dt} \leq 0$ for all A > 0, P > 0, G > 0 and $\frac{dV}{dt} = 0 \Leftrightarrow (A, P, G) = E_1$.

Using y > 0**:** $1 - \frac{y^*}{y} \leq In \frac{y}{y^*}$ allows bounding cross-terms like $\left(A - A^*\right)\left(P - P^*\right)$. Thus,

$$\frac{dV}{dt} = -\mu \frac{\left(A - A^*\right)^2}{A} - (\gamma + \delta + \mu)\frac{\left(P - P^*\right)^2}{P} - (\sigma + \mu)\frac{\left(G - G^*\right)^2}{G} \leq 0$$

The largest invariant set contained in {V˙=0} consists only of the equilibrium point, satisfying the conditions of LaSalle's Invariance Principle. Therefore, the endemic equilibrium E_1 is globally asymptotically stable in the positive orthant.

5. Sensitivity Analysis and Numerical Simulations

5.1. Computational Implementation

All analytical sensitivity indices were derived symbolically from the closed-form expression of the basic reproduction number R_0. Numerical simulations of the full dynamical system were carried out using MATLAB R2023a. The system of nonlinear ordinary differential equations was integrated using the adaptive Runge-Kutta solver ODE45, with simulations performed over a sufficiently long-time horizon to ensure convergence to equilibrium states. Multiple initial conditions were considered to verify global stability properties. Graphical outputs and sensitivity plots were generated directly from the numerical solutions.

5.2. Sensitivity Analysis of the Effective Reproduction Number

Sensitivity analysis is used to determine how variations in model parameters influence key outcomes of the system. In this study, sensitivity analysis assesses the extent to which changes in governance-related parameters affect the basic reproduction number R_0 , the trajectory of poor governance participation P(t), and the national development index D(t). By quantifying parameter influence, the analysis identifies the most critical drivers of governance persistence and development outcomes, and helps evaluate the robustness of model predictions to uncertainty in parameter values. Recall from (8), the basic reproduction number is given by

$$R_0 = \frac{\beta\Lambda}{\mu(\gamma + \delta + \mu)}$$

The normalized forward sensitivity index of 'R_0' with respect to 'p' is defined as

$$S_p^{R_0} = \frac{\partial R_0}{\partial p} \cdot \frac{p}{R_0}$$

This index quantifies the relative change in 'R_0' produced by a one-percent variation in 'p' (Chitnis, *et. al.* 2008). Carrying out the sensitivity analysis on (8) analytically using the parameter values on table 3 gives the sensitive indices of the respective parameter on table 4 with R_0 = 4.31 as shown below.

$$R_0 = \frac{0.9 \times 0.045}{0.15(0.25 + 0.02 + 0.15)} = 4.31$$

Using Nigeria-informed baseline parameters drawn from published sources in table 4, the normalized sensitivity analysis reveals that the basic reproduction number R_0 is most sensitive to the influence rate into poor governance and political recruitment, each with sensitivity index +1. Reform and civic correction reduce R_0 with index −0.532, while sanctions exhibit a weaker stabilizing effect (−0.426). The exit rate displays the strongest negative sensitivity (−1.043), indicating its compounded role in governance turnover dynamics as shown in figure 2. These results suggest that policies aimed at reducing the transmission of poor governance behaviours and strengthening reform mechanisms are critical for governance stabilization.

Table 3 Baseline parameter values for governance dynamics model (Nigeria-informed sources)

Parameter	Value	Data source
Λ	0.045	[14, 16].
β	0.9	[4, 11].
γ	0.25	[4, 18].
δ	0.20	[9].
μ	0.02	[17].
σ	0.15	[13].
α	0.40	[13,18].
κ	0.50	[11,13].
ρ	0.075	[13].

Table 4 Normalized forward sensitivity indices of R_0

Parameter	Sensitivity index
Λ	+1.000
β	+1.000
γ	-0.532
δ	-0.426
μ	-1.043

5.3. Numerical simulation and convergence to the endemic equilibrium

Using Table 3 baseline parameters, the basic reproduction number satisfies $R_0 \approx 4.31>1$, confirming the existence of an endemic equilibrium characterized by persistent poor governance participation. The endemic equilibrium (A^*, P^*, G^*, D^*) was computed numerically by solving the steady-state equations. Numerical integration of the full model from multiple initial conditions over a long horizon (0-200 years) showed convergence of P(t) toward P^* and convergence of D(t) toward D^*, indicating that the endemic equilibrium acts as the long-run attractor under the Table 3 parameter regime. The persistence of $P^*>0$ implies that development improvements remain constrained by the negative governance impact term $-\kappa P(t)$, unless policy interventions reduce the effective transmission of poor governance or strengthen reform and sanction mechanisms. The graphical representations are shown in figure 3 to figure 7.

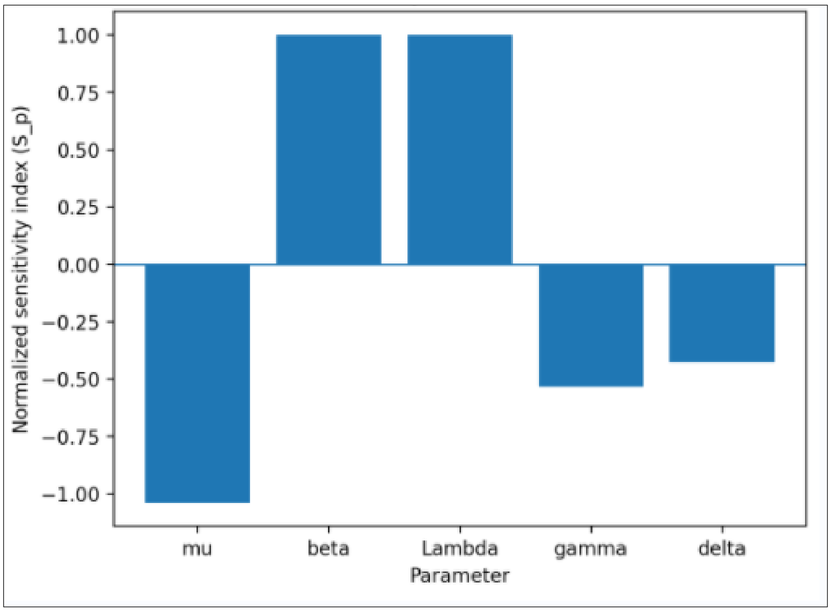

Figure 2 Sensitivity analysis of R_0 showing each parameter

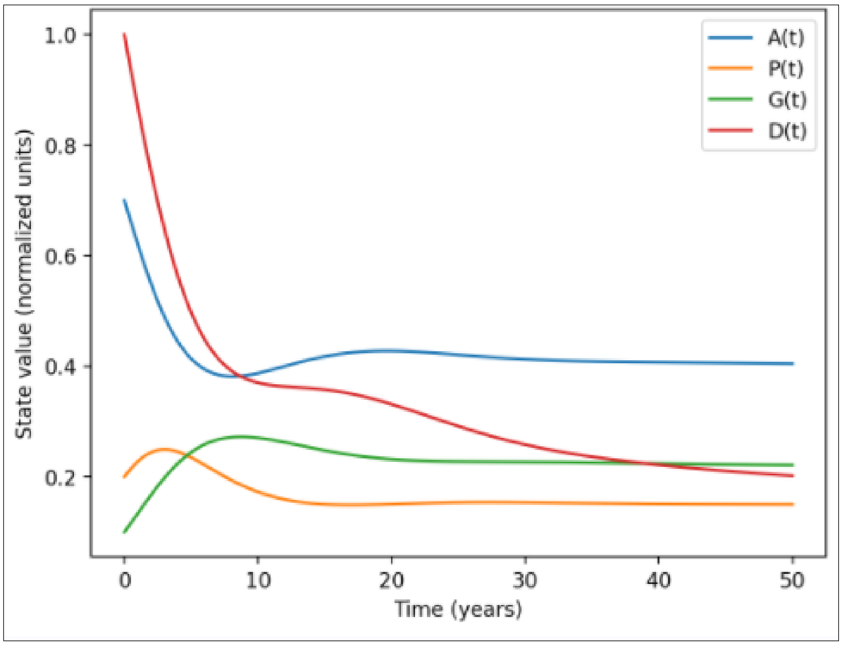

Figure 3 Graph of Governance-Development Model Dynamics

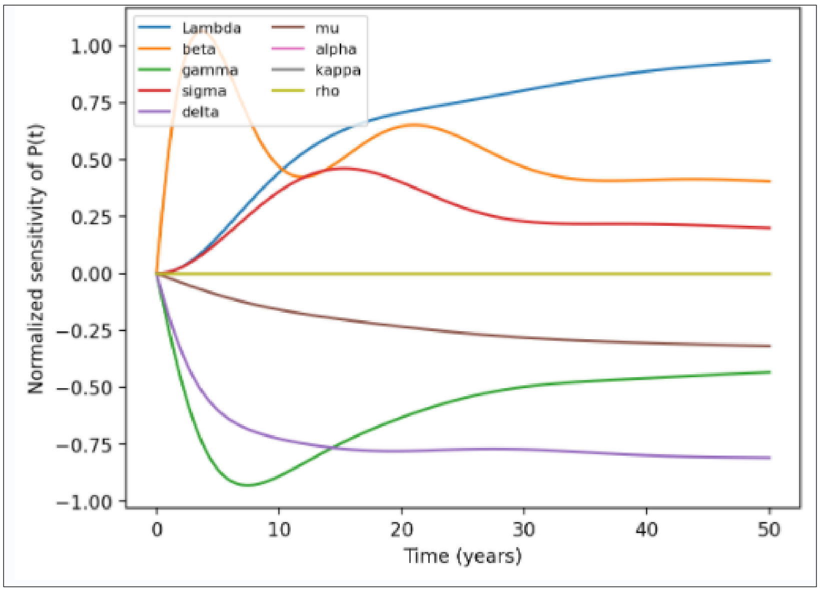

Figure 4 Graph of normalized forward sensitivity of poor governance P(t)

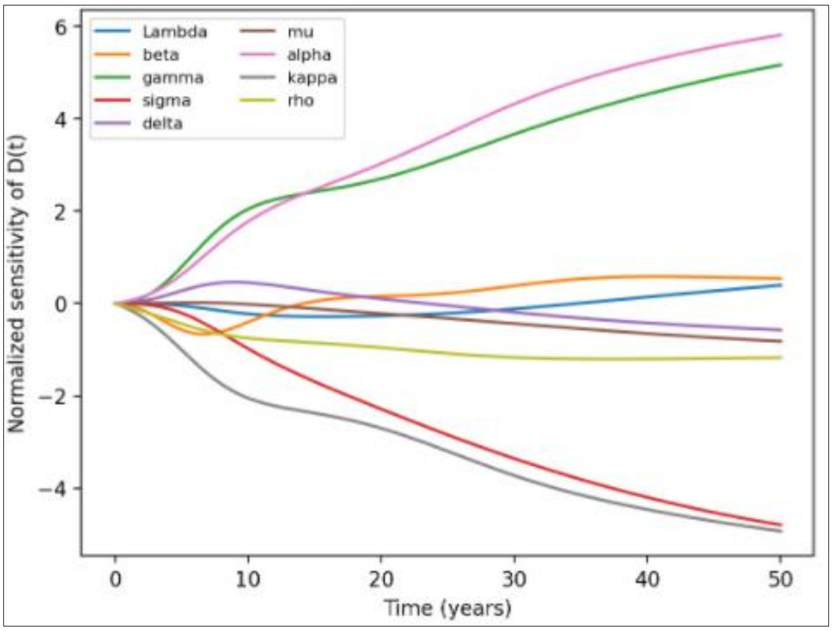

Figure 5 Normalized forward sensitivities of development D(t)

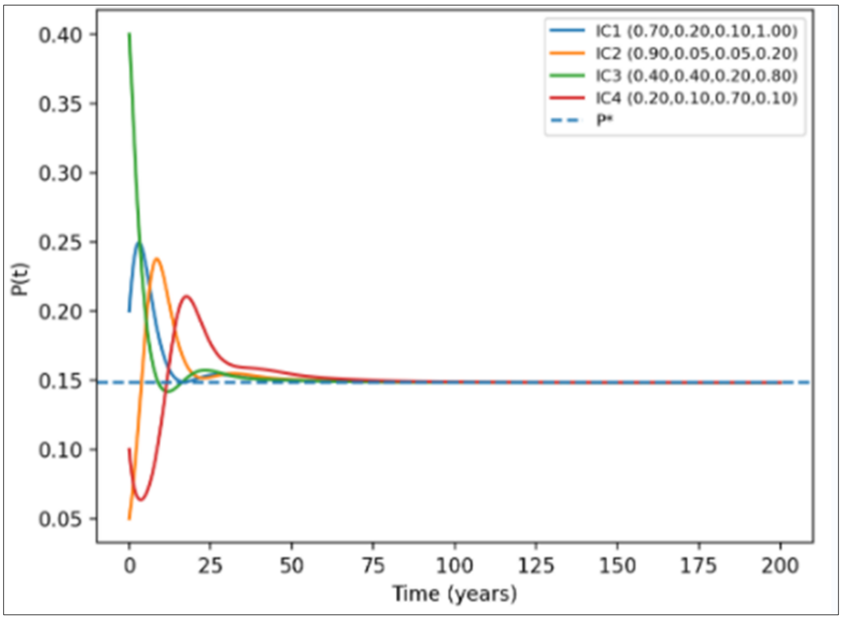

Figure 6 Graph of Convergence of poor governance participation P(t)

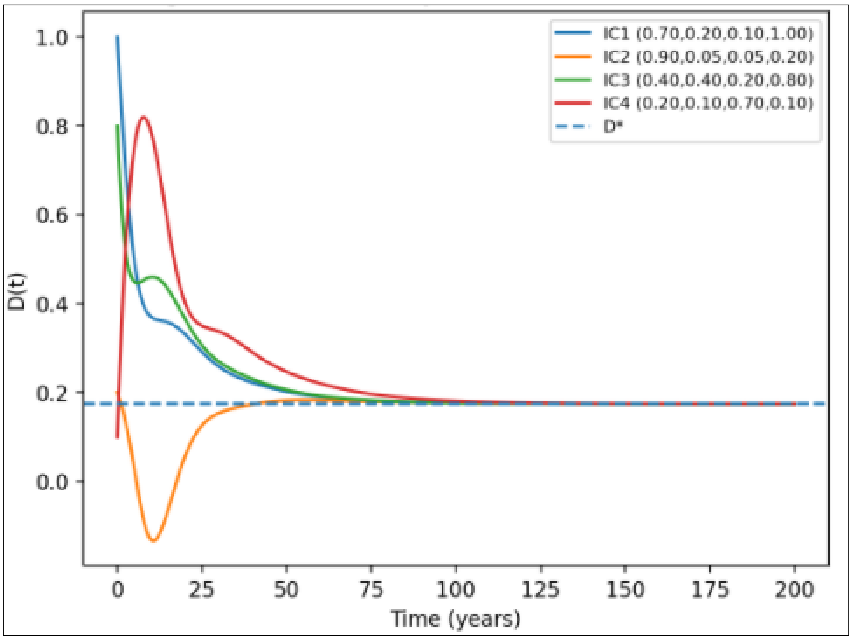

Figure 7 Graph of Convergence of development index D(t)

6. Discussions

6.1. Normalized Sensitivity Analysis Bar Chart of R_0(figure 2)

Figure 2 shows the parameters that most strongly influence the basic reproduction number R_0, which governs the persistence of poor democratic governance in the model. The influence rate into poor governance (β) and the recruitment rate into the political system (Λ) exhibit the largest positive sensitivity indices, indicating that proportional increases in these parameters lead to equivalent increases in R_0. This confirms that the spread of poor governance practices and unchecked political participation are the dominant drivers of governance persistence.

In contrast, reform and civic correction (γ) and sanctions against poor governance (δ) display negative sensitivity indices, demonstrating their stabilizing effects. However, their magnitudes are smaller than those of β and Λ, implying that reforms and sanctions must be significantly strengthened to counteract the strong transmission of poor governance. The exit or turnover rate (μ) has the largest negative sensitivity, reflecting its compounded role in reducing participation in poor governance.

Overall, the sensitivity analysis shows that preventing the transmission of poor governance behaviors is more effective than relying solely on corrective or punitive measures, highlighting the importance of institutional strengthening and preventive governance reforms.

6.2. Convergence of poor governance participation P(t)(figure 3)

Each curve is P(t) from a different initial condition. All curves converge to the same horizontal dashed line P^*.This means the long-run level of poor governance is persistent and predictable, and is largely determined by the balance between: spread/contagion effects (β, contact with A), versus removal/correction (γ + δ + μ).

6.3. Convergence of development index D(t) (figure 4)

D(t) converges to a steady state D^*. The long-run level depends on the steady governance mixture: $\dot{D} = \alpha G - \kappa P - \rho D$. At equilibrium: $D^* = \frac{\alpha G^* - \kappa P^*}{\rho}$. Because $P^*>0$, the term $-\kappa P^*$ continues to reduce development in the long run, and D(t) settles at a level that reflects persistent governance drag. This shows that, even if reforms raise G(t), development will not substantially improve unless the model moves toward lower P^* (that is, lowering β or raising γ,δ).

6.4. Model State Trajectories (figure 5)

The model state trajectories show that the governance-development system converges to a stable endemic equilibrium under the baseline parameter values. Poor governance participation initially declines but stabilizes at a persistent non-zero level, consistent with the condition $R_0>1$. Good governance advocacy increases temporarily but settles at a moderate level, while political apathy remains relatively high due to reform fatigue and disengagement effects.

The national development index declines initially and stabilizes at a low equilibrium value, reflecting the sustained negative impact of persistent poor governance on development outcomes. Overall, the trajectories indicate that without stronger reform and preventive governance measures, poor democratic practices persist and continue to constrain long-term national development.

6.5. Normalized Forward Sensitivities of P(t)(figure 6)

The normalized forward sensitivity analysis of poor governance participation P(t) reveals that the influence rate into poor governance is the dominant driver of persistence over time, with consistently positive and large sensitivity values. Political recruitment also contributes positively, particularly in the early stages of the dynamics. In contrast, reform, sanctions, and exit dynamics exert stabilizing effects, as indicated by their negative sensitivities, though their magnitudes are smaller than those of the influence parameter.

Overall, the results show that preventing the spread of poor governance behaviors is more effective than reactive correction, and that sustained institutional reform and governance renewal are required to significantly reduce long-term poor governance participation.

6.6. Normalized Forward Sensitivities of D(t)(figure 7)

The normalized forward sensitivity analysis of the national development index D(t) shows that development outcomes are positively driven by good governance dynamics and negatively affected by persistent poor governance. Parameters associated with governance quality exhibit positive sensitivities, while those linked to poor governance participation display sustained negative sensitivities, indicating a long-term drag on development. The decay parameter consistently contributes to development erosion, highlighting the fragility of development gains in the absence of sustained institutional improvement.

Overall, the results indicate that development responds more strongly to long-term structural governance improvements than to short-term corrective actions, emphasizing the need for durable and preventive governance reforms to achieve sustained national development.

7. Conclusion

This study presents a novel dynamical systems framework for understanding the persistence of poor democratic governance and its consequences for national development in Nigeria. By modeling governance behaviors as interacting compartments and explicitly linking them to a development index, the analysis provides a unified quantitative perspective on governance-development dynamics. The results show that poor governance persists when its effective transmission outweighs reform and sanction mechanisms, as captured by the threshold parameter R_0. Numerical simulations and sensitivity analyses further reveal that the spread of poor governance behaviors and unchecked political recruitment are the most influential drivers of persistence, while reform, sanctions, and demographic turnover exert stabilizing but comparatively weaker effects. Importantly, the findings demonstrate that persistent poor governance imposes a lasting constraint on development outcomes, even in the presence of moderate reform efforts. Overall, the model underscores the need for preventive governance strategies that reduce the transmission of poor democratic practices, alongside sustained institutional reform. The proposed framework offers a foundation for future extensions incorporating data calibration, heterogeneity, or stochastic effects, and provides a quantitative tool for informing long-term governance and development policy in Nigeria.

Recommendation

Based on the model results, the model suggests that policy efforts may benefit from prioritizing the limit of transmission of poor democratic practices rather than relying primarily on corrective measures after such practices become entrenched. This can be achieved by embedding transparency and accountability mechanisms directly into political recruitment, electoral processes, and public administration, thereby reducing exposure to poor governance norms at entry points. In parallel, reform and sanction mechanisms should be strengthened and sustained over time to prevent reform fatigue and institutional backsliding. Targeted civic education that emphasizes long-term institutional values, combined with regular leadership renewal and performance-based political incentives, is recommended to gradually shift the system toward a governance regime where poor practices cannot persist.

Compliance with ethical standards

Acknowledgements

We the authors give all glory and gratitude to Almighty God for His guidance, wisdom, and strength throughout the course of this research. We also sincerely appreciate the constructive comments and suggestions provided by anonymous reviewers, which greatly improved the quality and clarity of this manuscript. Appreciation is further extended to our institution for providing a supportive academic environment for this work.

Disclosure of conflict of interest

We the authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Statement of ethical approval

This study is based on mathematical modeling and numerical simulations using parameter values obtained from published literature and publicly available reports. It does not involve human participants, animal subjects, or identifiable personal data. Therefore, ethical approval and informed consent were not required.

Funding

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Data Availability

No new data were generated or analyzed in this study. All parameter values and supporting data were obtained from previously published studies and publicly accessible sources cited in the manuscript.

References

[1] Adeniran, A. O., Yusuf, A., and Adebayo, T. S. (2022). Mathematical modeling of corruption dynamics with control strategies. Journal of Applied Mathematics and Computing, 68(2), 901–920. https://doi.org/10.1007/s12190-01567-3

[2] Ahmed, M., Khan, M. A., and Ullah, S. (2023). Dynamical analysis of corruption transmission models with stability and sensitivity analysis. Mathematical Methods in the Applied Sciences, 46(5), 5678–5695. https://doi.org/10.1002/mma.8891

[3] Akindele, S. T. (2019). Governance, corruption and development in Nigeria. African Journal of Political Science and International Relations, 13(3), 45–56. https://doi.org/10.5897/AJPSIR2019.1152

[4] African Governance Institute. (2025). African governance indicators report 2025.

[5] Chitnis, N., Hyman, J. M., and Cushing, J. M. (2008). Determining important parameters in the spread of malaria through the sensitivity analysis of a mathematical model. Bulletin of Mathematical Biology, 70(5), 1272–1296. https://doi.org/10.1007/s11538-008-9299-0

[6] Mensah, J., Adams, S., and Boateng, E. (2022). Governance quality, institutional effectiveness and sustainable development in sub-Saharan Africa. Journal of Institutional Economics, 18(4), 617–635. https://doi.org/10.1017/S174413742100060X

[7] Ojo, E. O., and Adebayo, P. F. (2021). Democratic governance and development challenges in Nigeria. Journal of African Studies and Development, 13(2), 23–34. https://doi.org/10.5897/JASD2021.0623

[8] Owolabi, K. M., and Yusuf, A. (2024). Stability and bifurcation analysis of corruption dynamics models. Chaos, Solitons and Fractals, 176, 113956. https://doi.org/10.1016/j.chaos.2023.113956

[9] Reuters. (2025). Nigeria governance reforms and political accountability report. https://www.reuters.com

[10] Sterman, J. D. (2018). Business dynamics: Systems thinking and modeling for a complex world. McGraw-Hill Education.

[11] Transparency International. (2024). Corruption perceptions index 2024. https://www.transparency.org

[12] Umar, A., Sadiq, M., and Bello, S. (2020). Institutional quality and economic development in Nigeria. Journal of Public Administration and Policy Research, 12(1), 1–11. https://doi.org/10.5897/JPAPR2019.0591

[13] United Nations Development Programme (UNDP). (2023). Human development report 2023. https://hdr.undp.org

[14] United Nations Department of Economic and Social Affairs (UN DESA). (2023). World population prospects 2023. www.un.org/development/desa

[15] van den Driessche, P., and Watmough, J. (2002). Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission. Mathematical Biosciences, 180(1–2), 29–48. https://doi.org/10.1016/S0025-5564(02)00108-6

[16] World Bank. (2024a). World development indicators 2024. https://data.worldbank.org

[17] World Bank. (2024b). Worldwide Governance Indicators. https://info.worldbank.org/governance/wgi

[18] World Bank. (2024c). Nigeria public sector governance diagnostic. https://info.worldbank.org