



(RESEARCH ARTICLE)



The 'Farthest Equals Nearest' Paradox in Cosmology: Antipodal Points, Cosmic Horizons, and Universe Topology

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Abstract

This study examines how the concepts of "farthest" and "nearest" points change depending on the geometric structure of the universe. The research comparatively analyzes three fundamental cosmological models: (1) the static closed universe model ($\Omega > 1$), (2) the flat or semi-open universe model ($\Omega \leq 1$), and (3) the dynamic expanding universe model (Λ CDM). The philosophical proposition expressed as "The point infinitely approached is the farthest point, and at the same time the nearest point" has been evaluated within each model framework. Analysis results reveal that this paradoxical statement is mathematically valid only in the closed universe model, remains conceptually meaningless in the flat universe model, and becomes physically impossible in the dynamic expanding universe due to the cosmological event horizon. The study utilizes Planck Collaboration (2020) observational data, Einstein (1915, 1917) field equations, and Friedmann (1922) cosmology equations as the theoretical framework.

Keywords: Universe Topology; Antipodal Point; Cosmological Horizon; Riemannian Geometry; Λ CDM Model; Event Horizon; Cosmic Expansion; Spacetime Curvature

1. Introduction

The geometric structure of the universe contains conceptual paradoxes that challenge human intuition. Our everyday experience, accustomed to Euclidean geometry, is based on an understanding of infinite and flat space. However, Einstein's (1915) general theory of relativity fundamentally shook this intuition by revealing that spacetime can be curved by matter and energy. As J. B. S. Haldane (1927) famously stated: "The universe is not only queerer than we suppose, but queerer than we can suppose."

The focal point of this study is the cosmological evaluation of two propositions expressed as "The point infinitely approached is the farthest point, and at the same time the nearest point" and "When traveling 46 billion years away, half of the circle is completed." While these propositions contain deep insights regarding the topological structure of the universe, they have different levels of validity in different cosmological models.

The research systematically examines three fundamental universe models: the static closed universe model (Einstein, 1917), the flat or semi-open universe model (Bennett et al., 2013), and the dynamic expanding universe model Λ CDM (Planck Collaboration, 2020). For each model, the concepts of "farthest" and "nearest" points are analyzed in light of mathematical frameworks and observational data.

Understanding these concepts is not merely an academic exercise; it fundamentally affects our comprehension of cosmic distances, the fate of light signals sent into space, and the ultimate boundaries of human exploration. The observable universe, with its radius of approximately 46.5 billion light-years, represents only a fraction of what might exist beyond

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our cosmological horizon, making these topological considerations increasingly relevant to modern astrophysics and cosmology.

2. Theoretical Framework

2.1. Einstein Field Equations and Spacetime Geometry

Einstein's (1915) field equations, which form the foundation of general relativity, define the relationship between matter-energy distribution and spacetime geometry:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = (8\pi G/c^4)T_{\mu\nu}$$

In this equation, $G_{\mu\nu}$ represents the Einstein tensor, Λ the cosmological constant, $g_{\mu\nu}$ the metric tensor, and $T_{\mu\nu}$ the energy-momentum tensor. The cosmological constant Λ was added by Einstein (1917) to maintain a static universe model but was later characterized as his "biggest blunder" following Hubble's (1929) discovery of cosmic expansion. Ironically, the discovery of dark energy by Riess et al. (1998) and Perlmutter et al. (1999) re-established the central role of the cosmological constant in modern cosmology.

The geometric interpretation of these equations reveals that massive objects create curvature in the fabric of spacetime, and this curvature dictates how objects move through space. On cosmic scales, the total matter-energy content determines whether the universe curves back on itself (closed), extends infinitely in all directions (flat), or curves away from itself (open).

2.2. Friedmann Equations and Cosmic Curvature

Friedmann (1922) laid the foundation for modern cosmology by deriving solutions to Einstein's field equations for a homogeneous and isotropic universe. The Friedmann equation is expressed as follows:

$$H^2 = (8\pi G/3)\rho - kc^2/a^2 + \Lambda c^2/3$$

In this equation, H represents the Hubble parameter, ρ the matter density, k the curvature parameter (-1, 0, +1), $a(t)$ the scale factor, and c the speed of light. The curvature parameter k determines the global geometry of the universe: $k = +1$ indicates closed (spherical), $k = 0$ flat (Euclidean), and $k = -1$ open (hyperbolic) topology.

The scale factor $a(t)$ is particularly crucial as it describes how distances in the universe change over time. In an expanding universe, $a(t)$ increases, meaning that the physical distance between any two points grows even if their comoving coordinates remain fixed. This concept is fundamental to understanding why distant galaxies appear to recede from us and why the "farthest point" concept becomes dynamic rather than static.

2.3. Density Parameter and Critical Density

The concept of critical density, which determines the geometry of the universe, is derived from the Friedmann equation:

$$\rho_c = 3H^2/8\pi G$$

The density parameter is defined as $\Omega = \rho/\rho_c$. According to Planck Collaboration (2020) observations, $\Omega = 1.000 \pm 0.004$, indicating that the universe is nearly perfectly flat. However, even this small uncertainty can have critical implications for the topology of the universe.

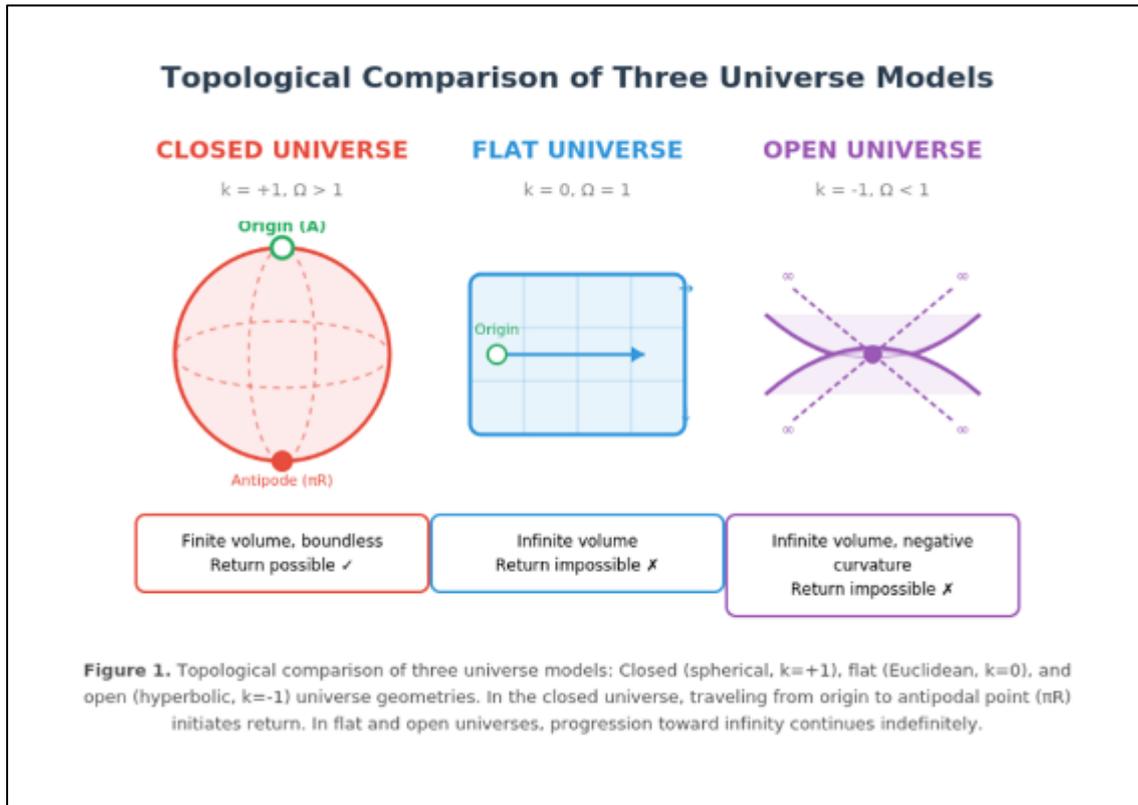
The remarkable precision of this measurement—accurate to within 0.4%—represents one of the greatest achievements of observational cosmology. Yet this precision also highlights a profound mystery: why should the universe be so precisely balanced at the critical density? This question, known as the flatness problem, motivated the development of inflationary cosmology and continues to drive theoretical investigations into the fundamental nature of spacetime.

2.4. Cosmological Horizons

In an expanding universe, several distinct horizons limit our ability to observe and interact with distant regions. The particle horizon defines the maximum distance from which light could have reached us since the Big Bang—approximately 46.5 billion light-years in comoving coordinates. The event horizon, approximately 16 billion light-years

away, represents the boundary beyond which light emitted today will never reach us due to accelerating expansion (Davis & Lineweaver, 2004).

These horizons are not physical barriers but rather consequences of the interplay between the finite speed of light and the expansion of space. They fundamentally constrain any discussion of "farthest" and "nearest" points in the real universe, as regions beyond the event horizon are causally disconnected from our future light cone.



Schematic representation of closed (spherical, $k=+1$), flat (Euclidean, $k=0$), and open (hyperbolic, $k=-1$) universe geometries with their characteristic features

Figure 1 Topological comparison of three universe models

3. Model Analyses

3.1. Static Closed Universe Model ($\Omega > 1$)

The closed universe model played a central role in Einstein's (1917) cosmological thinking. In this model, the universe is conceptualized as the surface of a three-dimensional sphere (3-sphere or S^3). Just as the two-dimensional surface of a sphere is finite but boundless, a closed universe has finite volume but no edge or boundary.

The fundamental mathematical properties of this model are as follows:

- Topology: 3-sphere (S^3), positive curvature ($k = +1$)
- Maximum distance: πR (π times the universe radius)
- Circumference: $2\pi R$ (finite and measurable)
- Antipodal point: Located at distance πR from the origin, reachable from any direction at equal distance

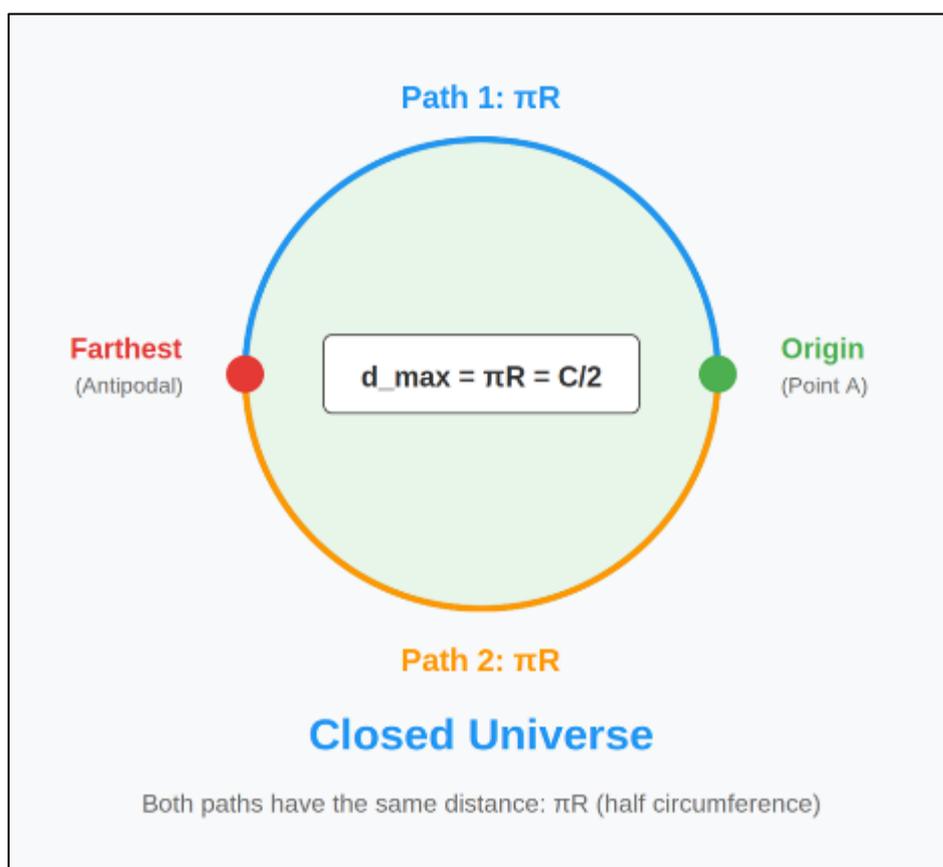
The elegance of this model lies in its self-consistency: a traveler moving in any direction through such a universe would eventually return to their starting point, having circumnavigated the entire cosmos. This property, while counterintuitive, is a direct consequence of positive spatial curvature and has profound implications for our understanding of cosmic geometry.

3.1.1. The Antipodal Point Paradox

In the closed universe model, the concept of "farthest point" manifests as the antipodal point. This point represents the maximum distance achievable when traveling in any direction from the starting point and corresponds to exactly half of the universe's circumference (πR). The proposition "The point infinitely approached is the farthest point, and at the same time the nearest point" is mathematically valid in this model (Luminet et al., 2003).

The mathematical explanation of this paradox is as follows: Moving from starting point A on a circle, upon reaching the exact opposite point (antipode), this point is simultaneously the farthest point (at half the circumference distance) and the point where the return journey begins. In other words, once this threshold is crossed, one begins approaching the starting point. This is a fundamental property of non-Euclidean geometries (Riemannian geometry).

Consider an analogy with Earth's surface: if you travel due north from any point on the equator, you will reach the North Pole—the farthest point in that direction. Continue in the same direction (now south), and you begin approaching your starting point. The North Pole represents a two-dimensional analog of the antipodal point in a three-dimensional closed universe.



The observer travels from the starting point to the antipodal point and returns to the origin, demonstrating the circular nature of geodesics in positively curved space

Figure 2 Journey simulation in the closed universe model

3.1.2. Evaluation of the Propositions

The examined propositions can be evaluated for the closed universe model as follows:

Proposition 1 ("Farthest = Nearest"): MATHEMATICALLY VALID. When explained through the limit concept, at the antipodal point, when an infinitesimally small distance remains to complete the full circuit, the traveled path is maximum, but in terms of position, one is at the nearest point to the origin. This is a natural consequence of Riemannian geometry and represents one of its most elegant properties.

Proposition 2 ("46 billion years = half circle"): PARTIALLY CORRECT. If the universe circumference is approximately 250 billion light-years (Vardanyan et al., 2011), 46 billion light-years corresponds to only 18.4% of the total. For half the circumference, 125 billion light-years would be required. This proposition significantly underestimates the scale of the universe.

3.2. Flat or Semi-Open Universe Model ($\Omega \leq 1$)

The flat universe model ($k = 0$) is the model most strongly supported by observational data. Bennett et al.'s (2013) WMAP observations and Planck Collaboration (2020) data reveal that the universe's curvature is very close to zero ($\Omega = 1.000 \pm 0.004$). In this model, the universe has the topology of three-dimensional Euclidean space.

The fundamental mathematical properties of this model are as follows:

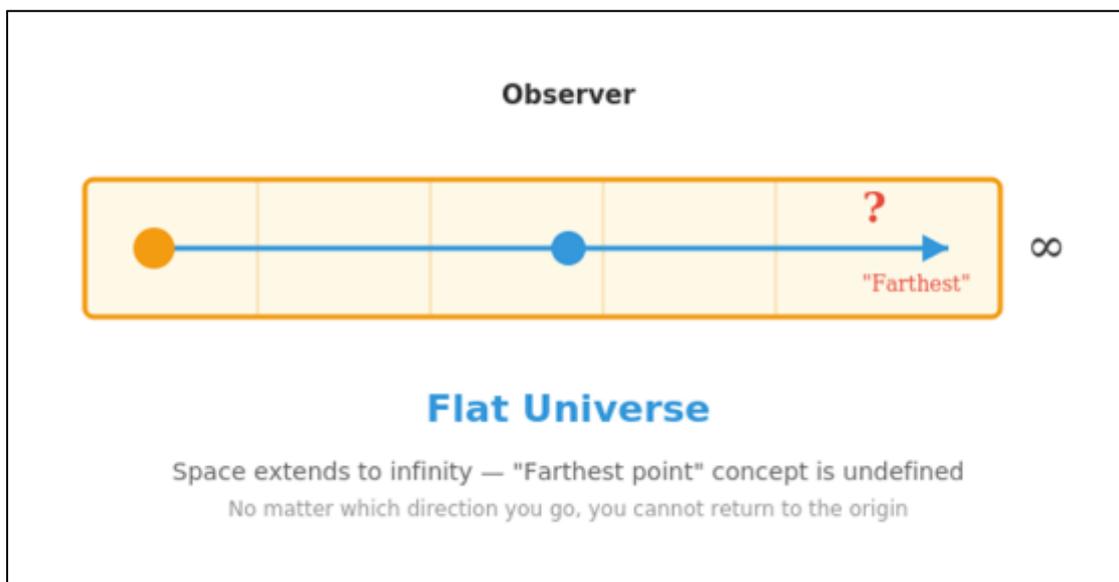
- Topology: Euclidean (\mathbb{R}^3), zero curvature ($k = 0$)
- Maximum distance: Undefined (infinity)
- Circumference: None (infinite space)
- Antipodal point: Conceptually non-existent

The flat universe model aligns most closely with our everyday intuition about space. Parallel lines never meet, the angles of a triangle sum to exactly 180 degrees, and space extends infinitely in all directions. However, this intuitive simplicity comes with a cost: the concept of a "farthest point" becomes meaningless, as there is always a more distant location.

3.2.1. Infinity and the Concept of Farthest Point

In the flat universe model, the concept of "farthest point" becomes synonymous with mathematical infinity (∞). In an infinite Euclidean space, one can travel an infinite distance in any direction and never return to the starting point. This situation, while consistent with our everyday intuitions, renders the concept of "farthest point" meaningless; because there is always a more distant point.

This infinite regress has profound philosophical implications. In a truly flat, infinite universe, every point is equally "central"—there is no privileged location, no edge, and no boundary. The universe simply extends forever in all directions, with an infinite number of galaxies, stars, and perhaps even copies of ourselves at unimaginably vast distances.



The observer progresses limitlessly in any direction through infinite Euclidean space, never returning to the origin

Figure 3 Infinite linear motion in the flat universe model

3.2.2. Evaluation of the Propositions

The examined propositions can be evaluated for the flat universe model as follows:

Proposition 1 ("Farthest = Nearest"): INVALID. Since the "farthest point" in a flat universe is infinity, this concept is undefined. Approaching infinity does not require approaching the starting point; rather, it signifies continuous recession. The paradox is valid only in closed topologies.

Proposition 2 ("46 billion years = half circle"): INVALID. Since the concept of a circle does not exist in a flat universe, this statement is meaningless. 46 billion light-years corresponds to the radius of the observable universe, but has no relation to the total size of the universe, which may well be infinite.

3.3. Dynamic Expanding Universe Model (Λ CDM)

The Lambda-CDM (Λ CDM) model is accepted as the standard model of modern cosmology. This model explains the accelerating expansion of the universe revealed by supernova observations of Riess et al. (1998) and Perlmutter et al. (1999). The model includes dark energy (Λ) and cold dark matter (CDM) components.

The fundamental properties of this model are as follows:

- Topology: Time-varying metric, scale factor $a(t)$
- Hubble constant: $H_0 \approx 67.4$ km/s/Mpc (Planck Collaboration, 2020)
- Observable universe radius: ~ 46.5 billion light-years
- Event horizon: ~ 16 billion light-years (beyond is unreachable)

The Λ CDM model represents our best current understanding of the universe's large-scale structure and evolution. It successfully explains a wide range of observations, from the cosmic microwave background to the distribution of galaxies and the apparent acceleration of cosmic expansion. However, it also introduces profound complications for the concept of cosmic distances and reachability.

3.3.1. Hubble Flow and Superluminal Recession

In the expanding universe, distant galaxies recede according to Hubble's law: $v = H_0 \times d$. This relationship implies that sufficiently distant objects can recede faster than the speed of light. Davis and Lineweaver (2004) explained that this "superluminal recession" does not violate special relativity; because what is involved is the expansion of space itself, not the motion of objects through space.

At the Hubble radius ($c/H_0 \approx 14$ billion light-years), the recession velocity of galaxies equals the speed of light. Galaxies beyond this distance are receding relatively faster than light. Galaxies at the boundary of the observable universe (46.5 billion light-years) are receding at approximately $3c$ velocity.

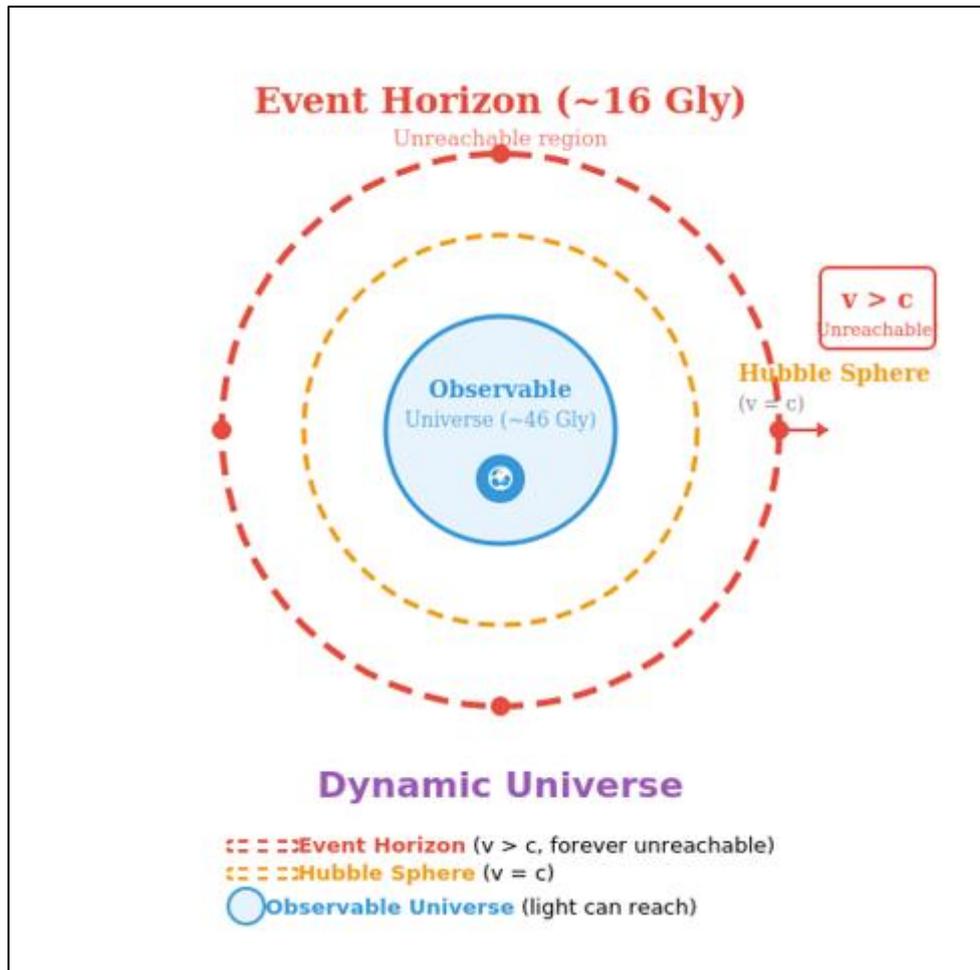
This distinction between motion through space and expansion of space is crucial. A spaceship traveling at near light speed could, in principle, reach a galaxy currently receding at less than the speed of light. However, for galaxies beyond the event horizon, no amount of speed would suffice—the expansion of intervening space would always outpace any possible journey.

3.3.2. Cosmological Event Horizon

Harrison (1991) defined the concept of "event horizon" in the expanding universe. The event horizon is located at approximately 16 billion light-years, and beyond this boundary, nothing can ever be reached in any way. A light signal sent today will never reach regions beyond the event horizon due to the accelerating expansion of the universe.

This concept fundamentally changes the "farthest point" paradox: In the expanding universe, the "farthest point" is not fixed; it is continuously receding. Moreover, regions beyond the event horizon become physically unreachable. This situation makes the concept of "going and returning" impossible.

The existence of the event horizon means that the observable universe is not merely the portion we can currently see—it represents the absolute limit of our causal contact with the cosmos. Galaxies currently visible will eventually fade from view as they cross this horizon, and no signal we send will ever reach them. The universe is effectively fragmenting into causally disconnected regions.



Visual representation of Hubble flow, event horizon, and galactic recession demonstrating the impossibility of return journeys in an accelerating cosmos

Figure 4 Dynamic expanding universe model simulation

3.3.3. Evaluation of the Propositions

The examined propositions can be evaluated for the dynamic expanding universe model as follows:

Proposition 1 ("Farthest = Nearest"): **PHYSICALLY IMPOSSIBLE**. In the expanding universe, the target point is continuously receding. Reaching beyond the event horizon is physically impossible. Even if the universe is closed, the expansion rate may prevent the return journey. The concept of "farthest point" becomes a dynamically changing quantity.

Proposition 2 ("46 billion years = half circle"): **MISLEADING**. 46 billion light-years is the radius of the particle horizon (observable universe). Galaxies at this distance are currently receding at $3c$ velocity. The concept of "half of the universe" is meaningless for an expanding universe; because the total size of the universe is constantly changing and likely infinite.

4. Comparative Analysis

4.1. Summary Table of Three Models

Parameter	CLOSED UNIVERSE ($\Omega > 1, k = +1$)	FLAT UNIVERSE ($\Omega = 1, k = 0$)	DYNAMIC UNIVERSE (Λ CDM Model)	Description
Topology	3-Sphere (S^3)	Euclidean (R^3)	Expanding Metric $a(t)$	Geometric structure of space
Curvature (k)	+1 (Positive)	0 (Zero)	≈ 0 (Nearly Flat)	Friedmann k parameter
Farthest Point	πR (Antipode) Defined and Reachable	∞ (Infinity) Undefined	Dynamic & Partially Unreachable Event Horizon: ~ 16 Gly	Maximum geodesic distance
Universe Circumference	$2\pi R \approx 250+$ Gly (Finite)	∞ (Infinite)	Observable: 46.5 Gly (Continuously Expanding)	Total size of universe
Return to Origin	✓ POSSIBLE	✗ IMPOSSIBLE	✗ IMPOSSIBLE (Expansion prevents)	Ability to complete full circuit
PROPOSITION 1: "Farthest = Nearest"	✓ VALID Mathematically correct	✗ INVALID Concept undefined	✗ IMPOSSIBLE Physically unrealizable	Antipodal point paradox evaluation
PROPOSITION 2: "46 Gly = Half circle"	⚠ PARTIAL Only 18.4% (Half = 125 Gly)	✗ INVALID No circle concept	✗ MISLEADING 46 Gly = Particle horizon radius	46 billion light-year distance evaluation
Observational Support	Weak $\Omega = 1.000 \pm 0.004$	Strong Planck 2020 data	Strongest Standard Model	Planck Collaboration (2020)

Note: Gly = Billion Light Years (Giga Light Year). Ω = Density parameter. k = Curvature parameter. Λ CDM = Lambda-Cold Dark Matter model.
Sources: Einstein (1915, 1917), Friedmann (1922), Planck Collaboration (2020), Davis & Lineweaver (2004), Luminet et al. (2003), Vardanyan et al. (2011).

Comparison of key parameters including topology, curvature, farthest point definition, return possibility, and proposition validity across closed, flat, and dynamic models

Figure 5 Comparative summary table of three universe models

As shown in Table 1, the concept of "farthest point" carries a different meaning in each model. In the closed universe model, this concept is defined and reachable (πR). In the flat universe model, it is undefined as infinity. In the dynamic universe model, it appears as a continuously changing and partially unreachable boundary.

4.2. Model-Based Validity Status of Propositions

Proposition 1 ("The point infinitely approached is the farthest point, and at the same time the nearest point") validity status: Mathematically VALID in closed universe, conceptually INVALID in flat universe, physically IMPOSSIBLE in dynamic universe.

Proposition 2 ("When traveling 46 billion years away, half of the circle is completed") validity status: Numerically INCORRECT in closed universe (only 18.4%), conceptually INVALID in flat universe, physically MISLEADING in dynamic universe.

4.3. Implications for Cosmic Exploration

The analysis presented here has profound implications for any discussion of interstellar or intergalactic travel. In the closed universe model, circumnavigation of the cosmos would be theoretically possible given sufficient time and technology. In the flat model, exploration could continue indefinitely without ever retracing one's path. In the dynamic model—our actual universe—the accelerating expansion places fundamental limits on how far humanity, or any civilization, could ever travel.

Even at the speed of light, we could only ever reach galaxies currently within our future light cone. The vast majority of the observable universe is already beyond our reach, and this fraction will only increase as expansion accelerates. The "farthest point" is not merely distant—it is receding faster than we could ever pursue it.

5. Discussion

This study demonstrates how a philosophical proposition can be evaluated differently according to cosmological models. The paradox "The farthest point is at the same time the nearest point" is an elegant consequence of Riemannian geometry and maintains its validity in closed topologies. However, observational data indicates that our universe is either flat or very slightly closed and is definitely dynamically expanding.

Alternative topologies such as the dodecahedral universe hypothesis proposed by Luminet et al. (2003) present various variations of the closed universe model. However, Planck Collaboration (2020) data has not been able to provide strong evidence supporting these hypotheses. Vardanyan et al.'s (2011) Bayesian analysis shows that the universe's curvature radius must be at least 5-10 times larger than the observable universe.

As Davis and Lineweaver (2004) explained, the concept of "superluminal recession" in the expanding universe does not contradict relativity theory. This concept relates to the expansion of space itself and should not be confused with the motion of objects through space. This distinction is critically important when evaluating the concept of "journey to the farthest point."

The event horizon concept defined by Harrison (1991) determines the "reachability" boundary in the expanding universe. This boundary explains why the closed universe paradox is invalid in the dynamic universe: When the target point is continuously receding and some regions become permanently unreachable, the concept of "going and returning" loses its meaning.

It is worth noting that even within the Λ CDM framework, significant uncertainties remain. The nature of dark energy—whether it is truly a cosmological constant or some evolving field—affects long-term predictions about cosmic evolution. Some alternative models suggest the expansion might eventually slow or even reverse, which would dramatically alter our conclusions about reachability and return journeys.

Furthermore, the possibility of non-trivial topologies in a flat or nearly-flat universe cannot be entirely excluded. A flat universe could still be finite if it has the topology of a torus or some more complex quotient manifold. Such possibilities, while not currently supported by observational evidence, remind us that the relationship between local geometry and global topology is subtle and that surprises may yet await future observations.

6. Conclusion

This study has comprehensively analyzed how the concepts of "farthest" and "nearest" points change depending on the geometric structure of the universe. Research results can be summarized as follows:

First, the proposition "The point infinitely approached is the farthest point, and at the same time the nearest point" is mathematically valid only in the static closed universe model. In this model, the antipodal point functions both as the maximum distance and as the return threshold.

Second, in the flat universe model, this proposition is conceptually invalid; because in infinite Euclidean space, the definition of "farthest point" cannot be made. Observational data indicates that our universe is very close to this model.

Third, in the dynamic expanding universe model (our actual universe), this proposition becomes physically impossible. The cosmological event horizon creates permanently unreachable regions, and the accelerating expansion of the universe prevents any "return" journey.

Fourth, the value of "46 billion light-years" represents not half of the universe, but the radius of the observable universe. Even if the universe is closed, the estimated circumference (~250+ billion light-years) is more than five times this distance.

Fifth, the distinction between different cosmological horizons—particle horizon, Hubble sphere, and event horizon—is essential for understanding the limits of observation and interaction in an expanding universe. Each defines a different aspect of cosmic reachability.

In conclusion, the examined philosophical proposition reflects a deep insight into the idealized closed universe model. However, observational cosmology reveals that our universe differs significantly from this idealized model. The universe is nearly perfectly flat and is expanding with acceleration, and under these conditions, the concept of "farthest point" transforms from a static geometric threshold into a dynamically changing and partially unreachable boundary.

The beauty of the original paradox lies not in its universal applicability but in what it reveals about the relationship between geometry and topology. While it may not describe our actual universe, it illuminates the profound ways in which the shape of space determines the nature of distance, journey, and return. As we continue to refine our understanding of cosmic geometry through ever more precise observations, we may yet discover that the universe holds topological surprises that bring new relevance to these ancient geometric intuitions.

Compliance with ethical standards

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Disclosure of conflict of interest

The author declares no conflict of interest.

Statement of ethical approval

Because this study did not include human participants, clinical interventions, personal data, or identifiable information, ethical approval and informed consent were not required.

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