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(Research Article)

CHOOSING THE NUMBER OF SAMPLES TO USE IN THE HISTOGRAM TEST OF ADCS

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ABSTRACT

This paper studies how to choose the input and sampling frequencies, together with the number of samples to acquire, so that coherent sampling is achieved. This choice matters when analog-to-digital converters (ADCs) are tested with the histogram method, the discrete Fourier transform (DFT) method or the sine-fitting method.

It is shown that a trade-off exists between how closely the actual frequencies can be kept to the desired ones and the duration of the test, which is governed by the number of acquired samples.

Several algorithms are presented and compared in terms of their complexity, advantages and drawbacks.

Keywords: Analog-to-digital converter; ADC testing; Coherent sampling; Histogram test; Frequency errors; Number of samples; Cumulative histogram.

1 INTRODUCTION

The most common ADC tests are based on sampling a sinusoidal input signal at a constant rate (the sampling frequency, f_s) and using the ADC output codes to estimate several characteristics of the converter. The histogram method [1][2][3], for instance, uses the output codes to build a histogram of the occurring codes and, from it, estimates the transition voltages, which in turn allow the code bin widths, the gain and offset errors and the integral and differential non-linearities (INL and DNL) to be computed [1]. Other test methods do not rely on a histogram of the samples, such as the static test method [4] or others [5]. The sine-fitting method performs a least-squares fit of a sinusoid to the ADC output codes; subtracting the fitted sinusoid from the codes yields an error signal from which the additive noise, the phase noise and the aperture uncertainty (jitter) can be estimated. The DFT test [1] applies a discrete Fourier transform to the output codes and, from the resulting spectrum, estimates the ADC additive noise (SNR), the harmonic distortion (THD, SFDR) and the spurious distortion (TSD, SFDR).

All these methods assume that the input signal is a perfect sinusoid sampled at specific time instants, so that the amplitude distribution of the sampled voltages is known. In principle, this would require the acquisition to span exactly one period of the stimulus signal. However, the large number of samples (often in the millions) needed for good precision would force the stimulus frequency to be much lower than the sampling frequency, which is undesirable because the performance of an ADC depends on the input frequency [6]. The same difficulty arises in other systems that measure a signal and subsequently sample and quantize it [7][8].

To overcome this limitation, time-equivalent sampling is normally used. The samples are then acquired over more than one period of the sinusoid, and care must be taken to ensure that the sample phases (relative to the sinusoid) are uniformly distributed over the interval 2π. This is achieved when the number of periods (J) over which the acquisition takes place is an integer that is mutually prime with the number of acquired samples (M). The sinusoid and sampling frequencies must therefore satisfy [9]

$$\rho = \frac{f}{f_s} = \frac{J}{M} \qquad (1)$$

Figure 1 shows the case in which 40 samples are acquired over 2 periods of the stimulus signal. Because the integers 2 and 40 are not mutually prime (they share a common factor greater than 1, namely 2), only 20 distinct phases are obtained, as can be seen in Figure 2.

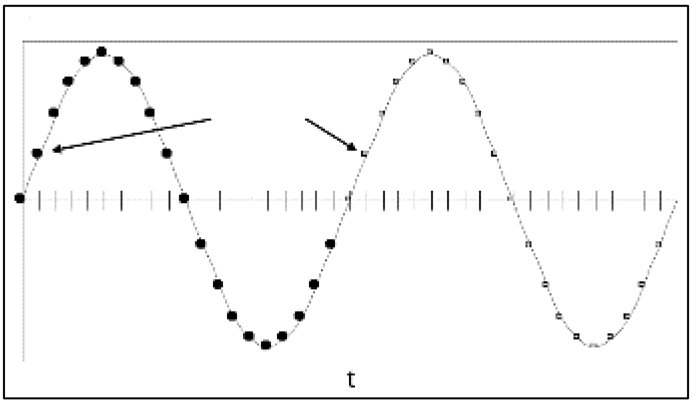

Figure 1: Representation of 2 periods of a sinusoid during which 40 samples were acquired.

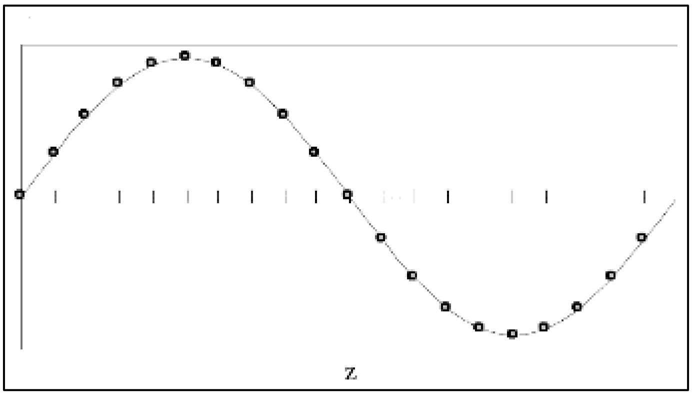

Figure 2: Representation of the sample phases (z), normalized from 0 to M, for the case of 40 samples acquired during 2 sinusoid periods.

Acquiring 39 samples over the same 2 periods (Figure 3) instead yields 39 distinct sample phases, because the integers 2 and 39 are mutually prime, as shown in Figure 4.

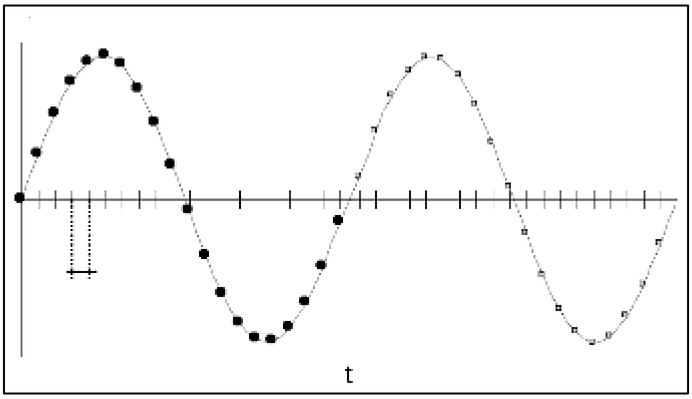

Figure 3: Representation of 2 periods of a sinusoid during which 39 samples were acquired.

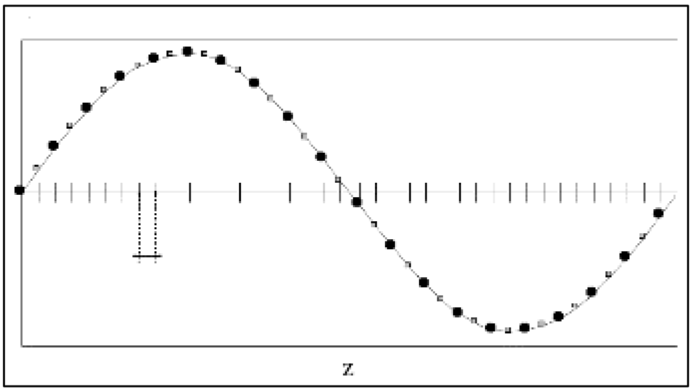

Figure 4: Representation of the sample phases (z), normalized from 0 to M, for the case of 39 samples acquired during 2 sinusoid periods.

2 FREQUENCY ERRORS

The unavoidable frequency errors of the sinewave and sampling generators prevent the sample phases from being perfectly uniformly distributed. To ensure that an error Δρ in the frequency ratio ρ produces a deviation of the phase distribution smaller than 50% of the ideal phase spacing, the number of samples must satisfy

$$\frac{\Delta\rho}{\rho} \leq \frac{1}{2 \cdot J \cdot M}. \qquad (2)$$

This limit also guarantees that the variance of the count of the cumulative histogram is smaller than 1/4 [10][11][12].

Inserting (1) into (2) gives

$$\Delta\rho \leq \frac{1}{2M^2} \qquad (3)$$

where

$$\Delta\rho = |\rho_{\text{ideal}} - \rho|. \qquad (4)$$

Using (1) again gives

$$\Delta\rho = \left|\frac{f_{\text{ideal}}}{f_{s,\text{ideal}}} - \frac{f}{f_s}\right| \qquad (5)$$

If the actual frequencies are affected by maximum relative errors of $\pm\varepsilon_f$ and $\pm\varepsilon_{fs}$, equation (5) can be written as

$$\begin{aligned}\Delta\rho &= \left|\frac{f_{\text{ideal}}}{f_{s,\text{ideal}}} - \frac{f_{\text{ideal}}\cdot(1\pm\varepsilon_f)}{f_{s,\text{ideal}}\cdot(1\pm\varepsilon_{f_s})}\right| = \\ &= \frac{f_{\text{ideal}}}{f_{s,\text{ideal}}}\left|1 - \frac{(1\pm\varepsilon_f)}{(1\pm\varepsilon_{f_s})}\right| = \\ &= \frac{f_{\text{ideal}}}{f_{s,\text{ideal}}}\left|\frac{\pm\varepsilon_{f_s}-(\pm\varepsilon_f)}{1\pm\varepsilon_{f_s}}\right|\end{aligned} \quad (6)$$

Expression (6) implies that

$$\frac{f_{\text{ideal}}}{f_{s,\text{ideal}}}\left|\frac{\varepsilon_{f_s}-\varepsilon_f}{1+\varepsilon_{f_s}}\right| \le \Delta\rho \le \frac{f_{\text{ideal}}}{f_{s,\text{ideal}}}\left|\frac{\varepsilon_{f_s}+\varepsilon_f}{1-\varepsilon_{f_s}}\right|. \quad (7)$$

If the right-most term is smaller than $1/(2M^2)$, then $\Delta\rho$ is smaller than that value as well, so the number of samples must satisfy

$$\frac{f_{\text{ideal}}}{f_{s,\text{ideal}}}\left|\frac{\varepsilon_{f_s}+\varepsilon_f}{1-\varepsilon_{f_s}}\right| \le \frac{1}{2M^2} \quad (8)$$

This finally yields an expression for the maximum number of samples that can be acquired consecutively:

$$M \le \sqrt{\frac{1}{2}\frac{f_{s,\text{ideal}}}{f_{\text{ideal}}}\frac{|1-\varepsilon_{f_s}|}{\varepsilon_{f_s}+\varepsilon_f}} = M_{\max}. \quad (9)$$

3 ALGORITHMS

Choosing an input frequency and a number of samples that satisfy both (1) and (9) is not straightforward. Several algorithms are examined in the following.

3.1 Standard

The IEEE waveform digitizer standard [1] proposes an algorithm based on the fact that an integer is mutually prime with any of its multiples decreased by one [9]. The algorithm proceeds as follows:

Find an integer, n, such that the desired frequency (f_d) is approximately f_s / n.

Let $J = \text{int}(M / n)$ = the number of full cycles that can be recorded at this frequency.

Let $f = J \times f_s / (nJ-1)$.

This guarantees $nJ-1$ distinct sample phases.

This algorithm is ambiguous in the choice of the integer n (first step), which is the number of samples acquired during one period of the sinusoid, because it does not specify whether the ratio f_s/f_d should be rounded up or down,

$$n = \left\lceil\frac{f_s}{f_d}\right\rceil \quad \text{or} \quad n = \left\lfloor\frac{f_s}{f_d}\right\rfloor \quad \text{respectively} \quad (10)$$

Figure 5 shows the actual frequency ratio f/f_s as a function of the desired frequency ratio f_d/f_s for large values of M (>100), where $nJ - 1 \approx nJ$ and $\rho \approx 1/n$. The thin and thick lines correspond, respectively, to rounding the ratio f_s/f_d up and down. When the desired frequency is half the sampling frequency ($\rho_d = 1/2$), the ratio f_s/f_d is exactly 2 and no rounding is needed ($n = f_s/f_d = 2$); this is the centre of the figure and means that 2 samples are taken per sinusoid period. If the desired frequency lies between half the sampling frequency and the sampling frequency, $1/2 < \rho_d < 1$ (right-hand side of the figure), the ratio f_s/f_d falls between 1 and 2. Rounding it up would give 2 samples per period ($n = 2$) and hence always $\rho = 1/2$, producing a large gap between the desired and the actual sinusoid frequencies. This is an artefact of restricting the number of samples per period to an integer (first step of the algorithm); recall that the requirement stated earlier was that the acquisition span an integer number of sinusoid periods, which is not the same thing.

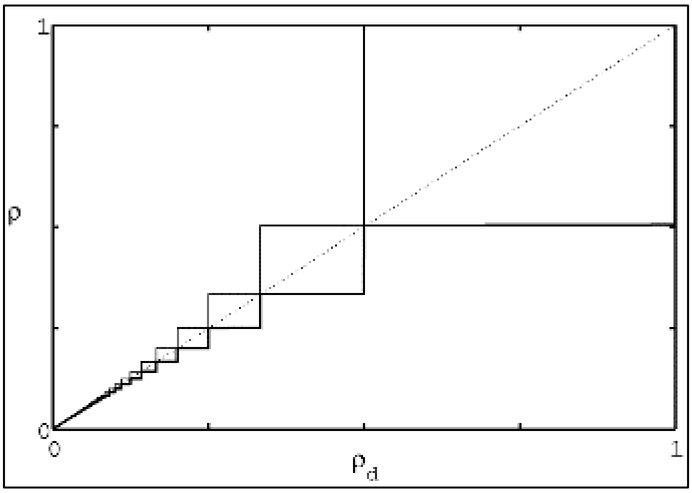

Figure 5: Representation of the actual frequency ratio as a function of the desired frequency ratio. The thin line represents the situation where f_s/f_d is rounded up and the thick line when it is rounded down.

The algorithm proposed in [1] also does not indicate which value should be used for M. Following the previous section, where the number of samples was required to be smaller than $M_{\max}$ given by (9), one would naturally use that value in place of M when determining J. There is, however, a difficulty: although the algorithm guarantees that the number of samples to acquire ($nJ - 1$) is smaller than the value used for M, it may lead to a frequency higher than the desired one, which lowers the bound $M_{\max}$ in (9) and can make $nJ - 1$ exceed $M_{\max}$. This is visible in Figure 6 (thin line), where the relative difference between the actual number of samples and the maximum limit given by (9) is computed as

$$\varepsilon_M = \frac{M - M_{\max}}{M_{\max}} \qquad (11)$$

This occurs when the ratio f_s/f_d is rounded down to obtain n. The correct choice is therefore to always round f_s/f_d up. As the thick line in Figure 6 shows, condition (9) is then always satisfied. This choice also implies a frequency lower than the desired one, as illustrated by the thick line in Figure 5.

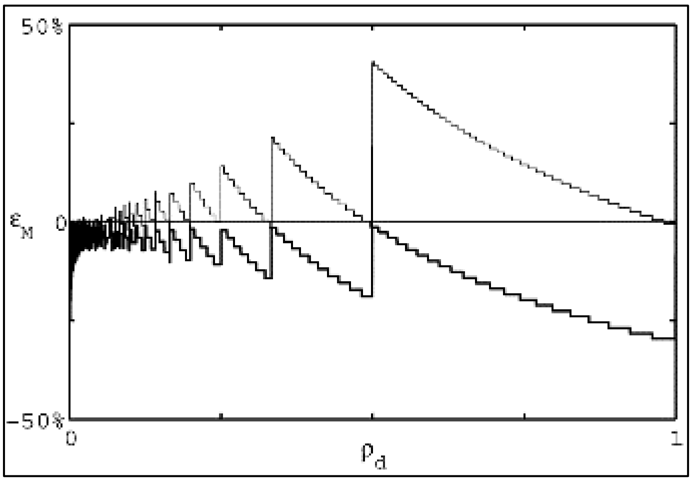

Figure 6: Representation of the relative difference of the number of samples defined by (11) as a function of the desired frequency ratio. The thin line represents the situation where f_s/f_d is rounded up and the thick line when it is rounded down. Frequency errors of 25 ppm were used.

The resulting actual frequency ratio and number of samples to acquire are

$$\rho = \frac{J}{M} \quad \text{and} \quad M = nJ - 1 \quad \text{with} \quad J = \left\lfloor \frac{M_{\max}}{n} \right\rfloor \quad \text{and} \quad n = \left\lceil \frac{f_s}{f_d} \right\rceil \qquad (12)$$

3.2 Analytical

Note that the frequency ratio obtained from (12) depends on the maximum number of samples, $M_{\max}$, which itself depends on the frequency ratio through (9). The two expressions can be combined as follows. Substituting $f_{s,\text{ideal}}/f_{\text{ideal}}$ by M/J in (9) gives

$$M \le \sqrt{\frac{1}{2}\frac{M}{J}\frac{|1-\varepsilon_{f_s}|}{\varepsilon_{f_s}+\varepsilon_f}} \quad (13)$$

Using $M = nJ - 1$, expression (13) can be rewritten as

$$M \le \sqrt{\frac{1}{2}\frac{M}{\frac{M+1}{n}}\frac{|1-\varepsilon_{f_s}|}{\varepsilon_{f_s}+\varepsilon_f}} \qquad (14)$$

After some simplification,

$$M(M+1) \le n \cdot \frac{1}{2}\frac{|1-\varepsilon_{f_s}|}{\varepsilon_{f_s}+\varepsilon_f} \quad (15)$$

Solving this second-order equation gives

$$M \le \frac{-1+\sqrt{1+2n\cdot\frac{|1-\varepsilon_{f_s}|}{\varepsilon_{f_s}+\varepsilon_f}}}{2} \qquad (16)$$

Reinserting $M = nJ - 1$ gives

$$nJ - 1 \le \frac{1}{2}\left(-1+\sqrt{1+2n\cdot\frac{|1-\varepsilon_{f_s}|}{\varepsilon_{f_s}+\varepsilon_f}}\right) \qquad (17)$$

Finally, solving for J and rounding the right-hand side of (17) down gives

$$J = \left\lfloor \frac{1}{2n}\cdot\left(1+\sqrt{1+2n\cdot\frac{|1-\varepsilon_{f_s}|}{\varepsilon_{f_s}+\varepsilon_f}}\right)\right\rfloor \quad (18)$$

In terms of the number of samples, this is a more efficient way of computing J than the one given by (12), as shown by the thick line in Figure 7.

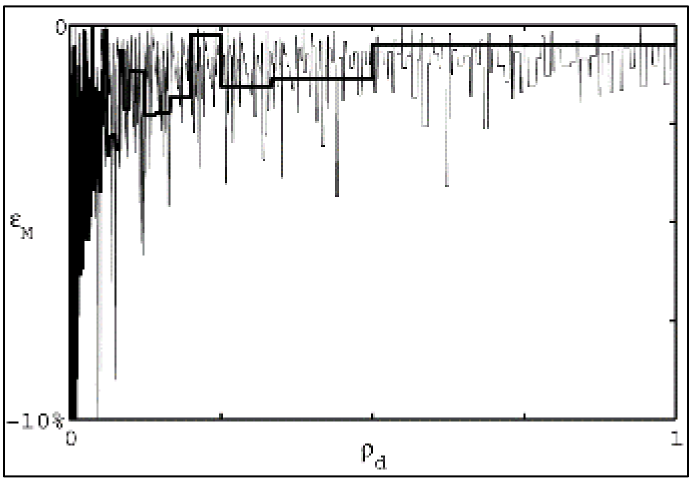

Figure 7: Representation of the relative difference of the number of samples defined by (11), using expression (18), as a function of the desired frequency ratio. The thin line represents the optimal solution described in 3.3 and the thick line the use of equation (18). Frequency errors of 25 ppm were used.

Even so, the frequency may still differ substantially from the desired one in some cases, as shown by the thick line in Figure 8.

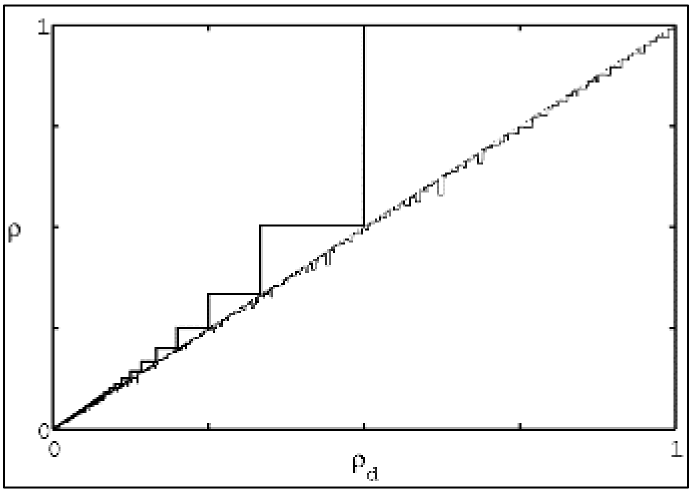

Figure 8: Representation of the actual frequency ratio as a function of the desired one. The thick line represents the situation when expression (18) is used and the thin line when the optimal solution is used (finding *J* that is mutually prime with *M* given by (9)). Frequency errors of 25 ppm were used.

3.3 Optimal

An optimal solution is to take the number of samples given by (9),

$$M = \lfloor M_{\max} \rfloor \qquad (19)$$

and to look for an integer J smaller than

$$J_{\max} = \lfloor \rho_d \cdot M \rfloor \quad (20)$$

such that J and M share no common divisors. This requires a computer, which is rarely a limitation since one is used anyway to process the results or even to run the ADC test. The efficiency of this approach can be seen in Figure 7 (thin line) and, especially, in Figure 8 (thin line), where the actual frequency ratio is practically equal to the desired one.

4 CONCLUSIONS

The requirement of keeping the phase samples as uniformly distributed as possible, even in the presence of frequency errors, constrains the number of consecutive samples that can be acquired. The algorithm suggested in [1] was clarified, and a better solution was proposed that determines the number of samples more efficiently (equation (18)).

The efficiency of the optimal solution, based on an exhaustive search for two mutually prime integers, was also demonstrated. This alternative offers clear advantages in terms of closeness to the desired input frequency (thin line of Figure 7) and of the maximum number of samples that can be used (thin line of Figure 8), at the cost of some added complexity and the need for a computer.

STATEMENTS ON COMPLIANCE WITH ETHICAL STANDARDS

Disclosure of conflict of interest

No conflict of interest to be disclosed.

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