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(Research Article)

NOISE-INDUCED DYNAMICS IN A SURFACE ACOUSTIC WAVE DELAYED-FEEDBACK OSCILLATOR

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ABSTRACT

This study investigates the influence of additive white noise on a surface acoustic wave (SAW) delayed-feedback oscillator system. In the deterministic regime, the oscillator stabilizes at discrete frequencies centered around the intrinsic resonant frequency of the SAW delay line. Upon the introduction of additive white noise, however, the system exhibits noise-induced transitions among these coexisting multistable oscillatory states. We analyze the resulting spectral frequency distributions and characterize the scaling behavior of the average residence time as a function of key system parameters, including delay duration, SAW coupling strength, and orbit frequency. The empirical results reveal that oscillatory orbits closer to the central resonance frequency exhibit significantly longer residence times, demonstrating enhanced robustness against stochastic perturbations. These findings provide deeper insights into noise-mediated multistability in delayed-feedback systems and offer practical design principles for optimizing the phase stability of SAW-based oscillators under environmental noise.

Keywords: Surface Acoustic Wave (SAW) Device; White Noise; Stochastic Resonance; Multistability; Mode Hopping

1 INTRODUCTION

Delayed-feedback systems constitute an important class of nonlinear dynamical systems in which the present state depends not only on the instantaneous system variables but also on their past values. Time delays arise naturally in a wide variety of physical, biological, electronic, optical, and engineering systems because of finite propagation times, signal-processing intervals, transport processes, and feedback mechanisms. Unlike conventional dynamical systems described by ordinary differential equations, delayed systems possess an effectively infinite-dimensional phase space and can therefore exhibit a rich variety of dynamical behaviors, including periodic oscillations, multistability, synchronization, bifurcations, and chaotic dynamics [1, 2]. The presence of a sufficiently long feedback delay is particularly significant because it can generate multiple coexisting oscillatory states, even in systems with relatively simple underlying dynamics.

Surface acoustic wave (SAW) devices provide a particularly important physical platform for investigating delayed-feedback dynamics. A SAW is an elastic wave that propagates along the surface of a solid, with its energy primarily confined near the surface. Such waves can be efficiently generated and detected using interdigital transducers fabricated on piezoelectric substrates. Because the propagation time between the transmitting and receiving transducers can be precisely controlled through the geometry of the device, SAW delay lines provide a natural and experimentally accessible source of delayed feedback. SAW technology has consequently found widespread applications in radio-frequency communication systems, signal processing, frequency control, and sensing [3, 4].

When a SAW delay line is incorporated into the feedback loop of an electronic amplifier, a delay-line oscillator can be formed. The oscillation frequency is determined by the combined amplitude and phase conditions imposed by the feedback circuit and the frequency response of the acoustic delay line. Earlier studies have demonstrated that the delay associated with acoustic-wave propagation plays a fundamental role in determining the operating frequencies and stability properties of SAW oscillators [5]. In particular, mathematical models based on delay differential equations have shown that stable oscillations can emerge through Hopf bifurcations and that multiple oscillatory modes may occur within the frequency range supported by the SAW device. The operating frequencies are generally located near the central frequency of the corresponding open-loop response but can be modified by the effective propagation delay and other circuit parameters.

The frequency stability of SAW delay-line oscillators is of considerable practical importance. In sensing applications, small variations in environmental conditions or surface properties may modify the acoustic propagation velocity and, consequently, the delay experienced by the signal. These changes can be converted into measurable variations in the oscillator frequency. The sensitivity and reliability of such devices therefore depend strongly on the stability of the underlying oscillatory state. SAW delay-line oscillators have been investigated as frequency-stabilizing elements and sensing platforms, and previous studies have reported high levels of short-term frequency stability under appropriately controlled operating conditions.

Despite these advantages, practical oscillator systems are inevitably exposed to fluctuations originating from thermal processes, electronic components, environmental disturbances, and measurement systems. Noise can influence both the amplitude and phase of an oscillator and may produce frequency fluctuations or transitions between different dynamical states. In systems possessing a single strongly attracting stable state, weak noise generally produces fluctuations around the deterministic solution [6, 7]. However, the effect of noise becomes substantially more interesting in multistable systems, where several stable attractors coexist. Under these conditions, stochastic perturbations can drive the system away from one attractor and induce transitions toward another. The resulting dynamics are characterized by random switching events and by finite residence times within individual stable states [8].

The interaction between time delay, multistability, and stochastic forcing has attracted considerable attention in nonlinear dynamics. A time delay can substantially increase the number of available dynamical states and thereby create a complex landscape of coexisting periodic solutions. Noise then enables the system to explore this landscape through transitions that would not occur in the deterministic limit. Previous investigations of delay-coupled oscillators have demonstrated that noise can induce switching among stable periodic orbits with different frequencies and oscillation patterns [9, 10]. These studies have shown that the statistical distribution of the observed frequencies and the corresponding residence times can provide important information about the stability of individual oscillatory states and the underlying delayed-feedback mechanism.

Noise-induced transitions in systems with delay are fundamentally related to the competition between stochastic forcing and deterministic stability. In a multistable oscillator, each stable periodic orbit is associated with a basin of attraction. Random perturbations continuously displace the system from its deterministic trajectory, and sufficiently large fluctuations may cause the system to cross an effective boundary between neighboring basins of attraction. The probability of such transitions depends on the noise intensity as well as on the local stability of the competing states. Consequently, the time spent by the system in a particular oscillatory state, commonly referred to as the residence time, provides a useful statistical measure of its robustness against noise-induced perturbations.

For SAW delay-line oscillators, these issues are particularly important because the acoustic delay line simultaneously determines the frequency-selective characteristics of the system and introduces a finite propagation delay into the feedback loop. As a result, the oscillator may support several stable frequencies located within or near the transmission band of the SAW device. Deterministic models of SAW delay-line oscillators have previously associated the occurrence of different oscillatory modes with delay-induced bifurcation phenomena and mode selection [11, 12]. Mode jumping and synchronization effects have also been identified as important dynamical issues in SAW-based oscillator and sensor systems. However, a systematic understanding of how additive stochastic noise drives transitions among coexisting frequency states in a SAW delayed-feedback oscillator remains essential for evaluating the practical stability of such systems [13, 14].

The concept of residence time is particularly relevant for delayed-feedback oscillators because the delay itself contributes directly to the formation of multiple oscillatory states. The number and spacing of the possible frequencies are influenced by the delay duration and by the frequency-dependent characteristics of the feedback loop. Therefore, changing the delay may not merely shift the oscillation frequency; it may alter the structure of the multistable state space. Similarly, variations in the coupling strength or feedback gain can modify the stability of the periodic solutions and influence the transition probabilities between them. Theoretical work on stochastic switching in delayed oscillators has shown that the number of coexisting stable periodic states can depend strongly on the delay and coupling parameters, whereas the statistical occupation and residence times of these states are governed by a more subtle interplay between deterministic dynamics and stochastic fluctuations [15, 16].

The investigation of noise effects in delayed systems has a longer history in the broader context of stochastic nonlinear dynamics. Noise can modify the behavior of systems near bifurcation points, induce transitions between stable dynamical states, and significantly influence the statistical properties of delayed oscillations. Studies of stochastic delay differential equations have demonstrated that the ratio between the feedback delay and the characteristic response time can substantially affect the dynamical consequences of noise. Such observations highlight the fact that delayed-feedback systems cannot always be understood simply as conventional oscillators with an additional perturbation. Instead, delay and noise interact in a manner that can fundamentally reshape the observed dynamics.

An important question in this context concerns whether all coexisting oscillatory states possess the same degree of robustness against noise. The frequency response of a SAW delay line is generally centered around an intrinsic central frequency, where the device exhibits its most favorable transmission characteristics. Oscillatory states located near this central region may therefore experience stronger effective feedback or more favorable phase and amplitude conditions than states closer to the edges of the accessible frequency range. It is consequently reasonable to expect that the stability of the different frequency states is not uniform. A detailed statistical analysis of noise-induced switching can reveal these differences and provide a quantitative measure of the relative robustness of the coexisting oscillatory orbits.

The present study investigates the influence of additive white noise on a SAW delayed-feedback oscillator operating in a multistable regime. In the deterministic limit, the system stabilizes on well-defined oscillatory states characterized by discrete frequencies located near the intrinsic central frequency of the SAW delay line[7, 8]. When additive white noise is introduced, stochastic perturbations induce transitions between these coexisting states, resulting in a fluctuating frequency trajectory and a statistical distribution of occupied oscillatory modes. By examining this distribution, the underlying preference of the noisy system for particular frequency states can be identified.

A central objective of this work is to quantify the average residence time associated with the different oscillatory states. The residence-time analysis provides a direct measure of the persistence of each orbit under stochastic forcing and enables a comparison of their relative stability. Particular attention is given to the dependence of the residence time on three fundamental parameters: the delay duration, the coupling strength of the SAW delay line, and the frequency of the corresponding oscillatory orbit. These parameters represent distinct physical aspects of the delayed-feedback system [17, 18]. The delay duration determines the temporal structure of the feedback and influences the number of accessible oscillatory modes, whereas the coupling strength governs the effective influence of the delayed signal on the oscillator dynamics. The orbit frequency, in turn, determines the position of a given state relative to the intrinsic frequency characteristics of the SAW delay line.

The analysis presented in this work demonstrates that noise-induced switching is not equally probable among all available oscillatory states. In particular, oscillatory orbits located closer to the central frequency of the SAW delay line exhibit significantly longer average residence times than states further away from this region. This result indicates that frequency states near the central frequency possess enhanced resistance to stochastic perturbations and therefore represent dynamically more robust operating conditions. The relationship between orbit frequency and residence time provides a useful framework for understanding how the frequency-selective properties of the delay line influence noise-mediated multistability.

2 SURFACE ACOUSTIC WAVE (SAW) DELAY LINE OSCILLATOR

The variations in the propagation delay of a surface acoustic wave (SAW) device caused by typical physical measurands are generally extremely small, often ranging from parts per million (ppm) to parts per trillion (ppt). Direct measurement of such minute changes in delay is therefore challenging. A practical and effective approach is to incorporate the SAW delay line into the feedback path of an electrical oscillator. In this configuration, a variation in the SAW delay is converted into a corresponding change in the oscillator frequency, which can be measured more conveniently and accurately. Consequently, SAW feedback-controlled electrical oscillators, commonly known as SAW oscillators, have become widely used in the development of high-sensitivity sensor systems.

Fig. 1 illustrates the schematic layout of a SAW delay-line oscillator circuit. The system consists of a nonlinear amplifier whose feedback is controlled by a SAW delay line. The output of the amplifier is applied to the input of the SAW delay line. After experiencing attenuation and a finite propagation delay, the signal emerging from the SAW delay line is fed back to the input of the amplifier, thereby forming a closed-loop circuit. Initiated by the unavoidable noise fluctuations present in the circuit, the system can develop self-sustained oscillations when the appropriate amplitude and phase conditions are satisfied. These conditions are commonly referred to as the Barkhausen criteria for feedback oscillators.

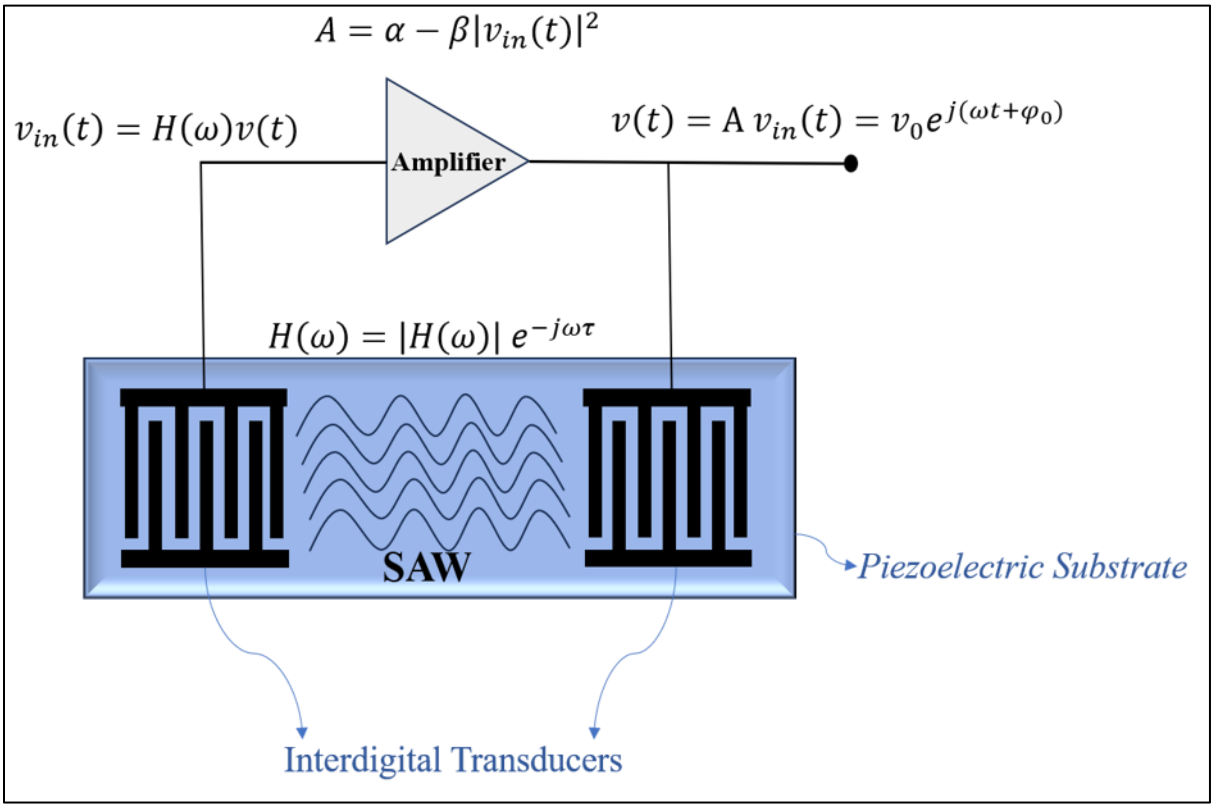

Figure 1: Schematic of SAW delay line feedback oscillator supported by an amplifier with cubic nonlinearity gain.

For sustained oscillations, the magnitude of the open-loop gain must compensate for the losses in the feedback path, while the total phase shift around the closed loop must be an integer multiple of 2π. If $A(\omega)$ denotes the amplifier gain and $L(\omega)$ represents the insertion loss of the SAW delay line at the angular frequency ω, the amplitude condition requires that the amplifier gain be sufficient to overcome the losses introduced by the SAW device. The phase condition requires that the signal, after completing one complete loop through the circuit, returns to the amplifier input in phase with the original signal. Thus, the total phase shift around the loop must satisfy

$$\phi(\omega)=2\pi n \quad (1)$$

where n is an integer.

The amplifier gain can generally be represented in complex form, with its phase contribution assumed to be approximately independent of frequency over the operating range. In contrast, the SAW delay line introduces a frequency-dependent phase shift that is determined by the propagation delay. If τ denotes the delay time of the SAW device, the corresponding phase shift is proportional to $-\omega\tau$. Therefore, the total phase accumulated around the feedback loop determines the permissible frequencies at which self-sustained oscillations can occur. Once the amplitude condition is fulfilled, the oscillation frequency is primarily governed by the phase condition. As a result, even a very small change in the SAW propagation delay produces a corresponding shift in the oscillator frequency, forming the fundamental principle behind SAW oscillator-based sensing.

A SAW delay-line oscillator can also be regarded as an autonomous, self-sustained limit-cycle oscillator. It contains all the essential components required for autonomous oscillatory behavior: an energy source supplied through the DC voltage of the electronic circuit, dissipation resulting from losses in the SAW delay line and other circuit components, and nonlinear dynamics introduced by the nonlinear voltage transfer characteristic of the amplifier [9]. Unlike a driven oscillator, the system does not require an external periodic signal to initiate or maintain its oscillations. Instead, the unavoidable intrinsic noise and fluctuations present in the electronic components provide the initial perturbations from which oscillations develop. The amplitude of the oscillation subsequently increases until nonlinear effects in the amplifier limit further growth and establish a stable steady-state amplitude.

The oscillator output voltage can be conveniently expressed using complex notation as

$$v_{out}(t) = v_0 e^{j(\omega t + \varphi_0)}$$

where *A* represents the stable, time-independent amplitude of oscillation, ω is the angular frequency, and ϕ_0 is a constant initial phase. The initial phase reflects the phase accumulated during the transient process through which the oscillations evolve from noise-induced fluctuations to a stable steady-state condition. Once the transient dynamics have decayed, the system settles onto a stable limit cycle characterized by a well-defined amplitude and frequency. This autonomous oscillatory behavior, combined with the high sensitivity of the oscillation frequency to changes in the SAW propagation delay, makes the SAW delay-line oscillator an effective platform for precision sensing and for investigating the effects of noise, delay, and nonlinear feedback on multistable dynamical systems.

3 SURFACE ACOUSTIC WAVE (SAW) DELAY LINE OSCILLATOR WITH ADDITIVE WHITE GAUSSIAN NOISE (AWGN)

Additive White Gaussian Noise (AWGN) can significantly influence the dynamical behavior of phase-coupled surface acoustic wave (SAW) oscillators by affecting the stability, phase evolution, and synchronization properties of the system. Noise in an oscillator has traditionally been associated primarily with random phase fluctuations and the consequent broadening of the oscillator spectrum. However, stochastic perturbations may have a more complex influence on nonlinear oscillator dynamics. In addition to producing phase diffusion, noise can modify the statistical properties of phase evolution and, under certain conditions, lead to changes in the mean oscillation frequency. These effects are particularly important in SAW-based delayed-feedback systems, where small variations in phase and frequency can substantially influence synchronization, mode selection, and spectral stability [12, 13].

In many oscillator systems, the amplitude dynamics are strongly stabilized by nonlinear saturation mechanisms, whereas the phase remains comparatively sensitive to external and internal perturbations. Consequently, stochastic forcing often affects the phase dynamics more strongly than the amplitude dynamics. This separation of time scales makes phase-reduced models particularly useful for investigating the influence of noise on coupled oscillator systems. Such reduced descriptions are closely related to phase-oscillator and Kuramoto-type models, which provide a convenient theoretical framework for analyzing synchronization in the presence of coupling and stochastic fluctuations.

The presence of white Gaussian noise can produce phase diffusion, leading to an increase in the spectral linewidth and a reduction in frequency stability. In coupled SAW oscillator systems, noise can also influence the phase-locking condition and modify the effective frequencies of the interacting oscillators. As a consequence, sufficiently strong stochastic fluctuations may weaken synchronization, induce intermittent loss of phase locking, or promote transitions between different oscillatory modes. In oscillator arrays and communication systems, these effects can degrade spectral purity and reduce synchronization accuracy. Therefore, a comprehensive description of SAW oscillator dynamics should account for both the random diffusion of the phase and possible noise-induced modifications of the average phase evolution [14].

The influence of noise becomes even more significant in delayed-feedback SAW oscillators. The propagation time through the SAW delay line introduces a finite feedback delay, which can produce a large number of coexisting stable dynamical states. This delay-induced multistability implies that the system may possess several stable phase-locked or periodic solutions simultaneously. In the deterministic limit, the oscillator remains confined within the basin of attraction associated with its initial state. When stochastic fluctuations are introduced, however, the noise can drive the system across the boundaries separating neighboring basins of attraction, resulting in transitions between different stable oscillatory states.

Thus, AWGN acts as a stochastic excitation mechanism that enables the oscillator to explore the multistable dynamical landscape generated by the delayed feedback. These noise-induced transitions are particularly relevant for understanding the residence time of different oscillatory states and their relative stability. States with stronger deterministic stability generally exhibit longer residence times, whereas less robust states are more readily destabilized by random fluctuations. The resulting stochastic switching process can lead to observable changes in the oscillator frequency and phase dynamics.

Equation of motion for phase dynamics of SAW Oscillator with AWGN is written as –

$$\dot{\varphi}(t) = \omega_0 + k\cos[\varphi(t-\tau) - \varphi(t)] + \xi(t) \tag{2}$$

where:

ω_0 = *natural frequency of SAW oscillator*
τ = *feedback delay*
$k = \omega H(\omega)$, coupling strength
$\xi(t)$ = Additive Gaussian white noise, with $\langle \xi(t)\xi(t_0)\rangle = 2D\delta(t-t_0)$

Without Noise, the oscillator may reside in one of the frequencies $\dot{\varphi} = \Omega$, given by –

$$\Omega = \omega_0 + k\cos(\Omega\tau) \tag{3}$$

Where:

$$\Omega = [\varphi(t) - \varphi(t-\tau)]/\tau \tag{4}$$

The term Ω corresponds to the average of the instantaneous frequency $\dot{\varphi}(t)$ over the past delay interval.

4 GRAPHICAL REPRESENTATION OF COEXISTING FREQUENCIES WITH SAW DELAYED FEEDBACK

The transcendental relationship governing the phase-locked dynamics of the system is illustrated by mapping the self-consistent solutions of the eq. (3). In this graphical representation fig. 2, the synchronized orbital frequencies Ω emerge from the geometric intersections between the linear progression of the state variable and the oscillatory modulation imposed by the time-delayed feedback loop. Each discrete intersection point localized on the graph denotes a stable, phase-locked frequency capable of sustaining steady-state periodic oscillations within the delayed-feedback architecture.

Under conditions characterized by a long feedback delay or strong coupling strength ($k\tau$ >>1), the slope of the linear boundary becomes sufficiently shallow relative to the rapid oscillations of the cosine feedback term. Consequently, for the dominant modes operating in close proximity to the natural frequency ω_0, the phase-locking conditions simplify significantly, allowing the localized solutions to be closely approximated by the localized resonance condition $\Omega\tau \approx 2m\pi$ (where m is an integer). Due to the bounded nature of the harmonic feedback component, all physically realizable frequency solutions are strictly confined within the finite spectral band defined by $\Omega\text{-}k \leq \Omega \leq \Omega\text{+}k$. Within this bounded region, the high-dimensional phase space of the system accommodates a multitude of multi-stable states, where the total number of coexisting stable orbits can be asymptotically estimated by the ratio $k\tau/\pi$. The color modulation of the intersection points highlights the varying stability criteria or phase relations of these coexisting modes across the multi-stable landscape.

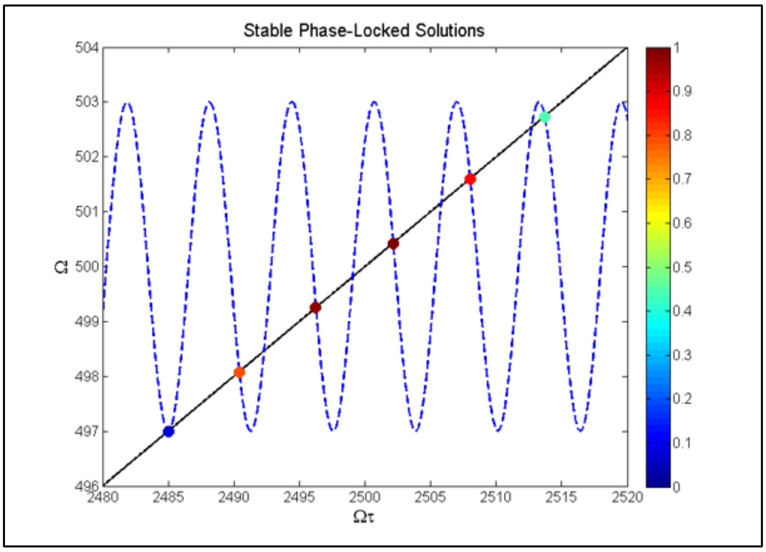

Figure 2: Stable phase locked solutions with parameters: $\Omega = 500MHz$, $k = 3$ and $\tau = 5\mu s$

5 PHASE EVOLUTION $[\varphi(t) - \omega_0 t]$ AND FREQUENCY VARIATION Ω WITH TIME

The temporal dynamics of the delayed-feedback system, subject to stochastic perturbations or thermal noise, are characterized by capturing the simultaneous evolution of the instantaneous frequency variation given by eq. (4) and the corresponding phase deviation $\varphi(t) - \omega_0 t$. As illustrated in fig. 3, the system does not remain permanently anchored to a single orbital state; instead, it exhibits distinct, abrupt jumps between well-defined, deterministic frequencies. This stochastic switching behavior directly indicates mode hopping among the coexisting multistable states previously identified in the transcendental phase-locking landscape. The system spends macroscopic intervals fluctuating around a primary frequency locked near $\Omega \approx 500.4\ MHz$ before undergoing rapid, noise-induced transitions to an adjacent stable lower-frequency mode localized around $\Omega \approx 499.3\ MHz$.

This frequency-switching mechanism is perfectly mirrored in the complementary phase evolution plot. Because the frequency represents the time-derivative of the phase, each deterministic frequency state establishes a constant linear slope in the quantity $\varphi(t) - \omega_0 t$. Consequently, the mode hopping events manifest as sharp, discontinuous slope changes within the piecewise-linear phase trajectory. The prolonged stays at the higher frequency mode yield a positive linear progression, whereas transitions to the lower frequency mode cause a sharp inversion to a negative slope. These synchronized transitions demonstrate that the phase-locked orbits serve as stable attractors, while external fluctuations drive the intermittent, statistical migration of the system across the underlying multi-basin potential energy landscape.

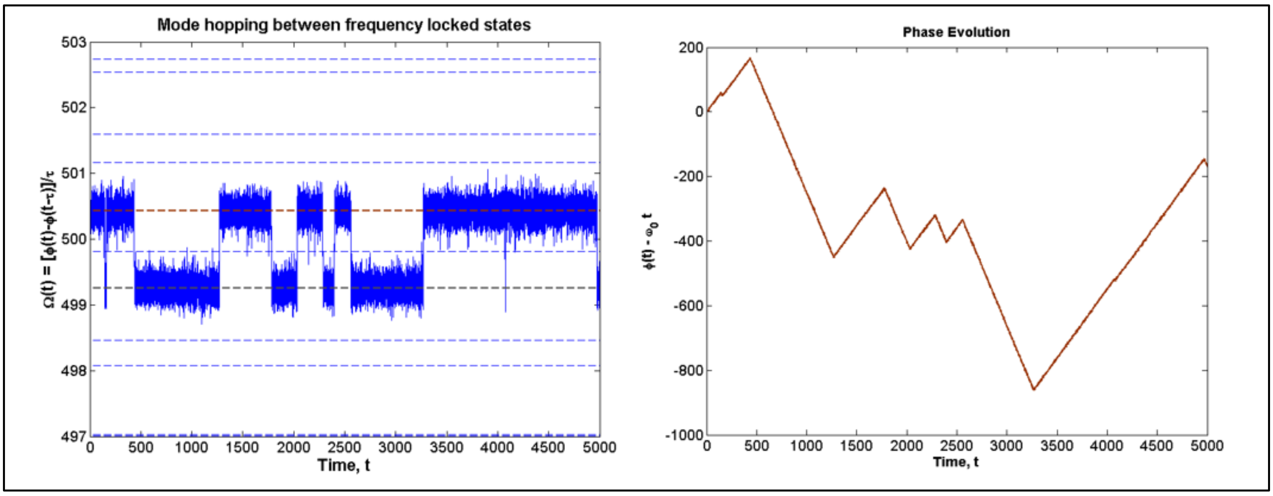

Figure 3: Mode hopping and phase evolution between Stable phase locked states with parameters: $\Omega = 500MHz,\ k = 3\ and\ \tau = 5\mu s$

6 THE FREQUENCY DISTRIBUTION FOR SAW FEEDBACK OSCILLATOR WITH AWGN

The statistical behavior of the surface acoustic wave (SAW) delayed-feedback oscillator under the influence of Additive White Gaussian Noise (AWGN) is elucidated through the stationary probability density distribution of the tracking frequencies $p(\omega)$ [15]. Mathematically, this steady-state distribution is governed by the relationThe stationary distribution of frequencies is given by -

$$p(\omega) = e^{-\frac{\tau}{4D}(\omega-\omega_0)^2} e^{\frac{k\, sin\omega\tau}{2D}} \qquad (5)$$

Black dotted curve represents Gaussian envelope with mean ω_0 and variance -

$$\sigma^2 = \frac{2D}{\tau} \qquad (6)$$

Within this enveloping profile, the fine structure of the distribution reveals fairly distinguishable multiple maxima that align precisely with the deterministic phase-locked frequencies derived from the system's transcendental mechanics. The presence of these sharp, localized peaks provides clear statistical evidence of multistability, confirming that the noise-driven system treats these discrete coordinates as stable thermodynamic attractors.

Crucially, the probability amplitudes of these coexisting states are strongly modulated by their spectral distance from the center. The frequencies closest to the natural frequency ω_0 specifically the dominant peaks emerging near near $\Omega \approx 500.4\ MHz$ and near $\Omega \approx 499.3\ MHz$ exhibit the highest probability densities. This indicates that the oscillator resides within these central basins of attraction for the vast majority of its operational duration, making them the most frequently visited states during stochastic mode-hopping events. In stark contrast, the states positioned further down the wings of the Gaussian envelope exhibit near-zero probability amplitudes. This implies that the system spends negligible time in these peripheral frequencies, as the quadratic restoring force of the effective potential barrier suppresses long-term activation into these outer spectral modes.

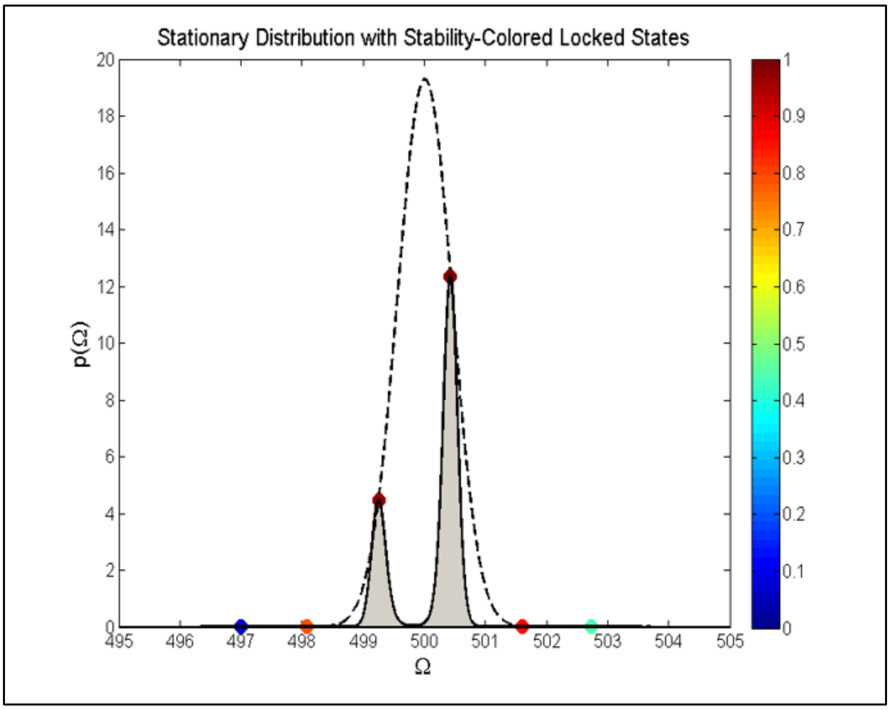

Figure 4: Distribution with stable phase locked states with parameters: $\boldsymbol{\Omega = 500MHz,\ k = 3\ and\ \tau = 5\mu s}$

7 MEAN ESCAPE TIME

The lifetime and relative stability of the coexisting phase-locked orbits are quantified by analyzing the mean escape time $T_0(\Omega)$, which characterizes the average duration the oscillator resides within a specific basin of attraction before a noise-induced transition occurs. Analytically, this stochastic residence metric is governed by the expression [16]:

$$T_0 = \frac{\pi e^{\frac{k}{D}+\frac{\pi^2}{4\tau D}}}{2k\cosh\left[\frac{\pi(\Omega-\omega_0)}{2D}\right]} \tag{7}$$

Fig. 5 demonstrates a highly localized, bell-shaped distribution of escape times centered symmetrically around the oscillator's natural frequency ω_0. This profile uncovers a strong dependency on the operational frequency coordinates and the underlying potential barrier heights.

A primary feature of the system's dynamics is the sensitive dependence on feedback and noise. The mathematical structure dictates that the mean escape time T_0 increases exponentially with higher feedback strengths k, which effectively deepens the localized potential wells, while it decreases rapidly with increasing noise intensity D, as stronger stochastic perturbations accelerate barrier-crossing events.

Regarding the role of delay, for a fixed tracking frequency Ω, the feedback delay τ exerts only a limited effect on the mean escape time, and for sufficiently long delays ($k\tau$ >>1), this parametric dependence asymptotically vanishes as the secondary exponential factor approaches unity.

Consequently, the graphical landscape highlights that only robust frequencies located in close proximity to the natural frequency ω_0 exhibit significant residence times, making them highly resilient against external AWGN. As depicted, the orbits at $\Omega \approx 500.4\ MHz$ and near $\Omega \approx 499.3\ MHz$ (marked by deep red indicators corresponding to a high stability index) possess elevated T_0 values, meaning they act as dominant metastable states. Conversely, peripheral orbits lying along the wings of the curve display vanishingly small mean escape times approaching zero. This confirms that their corresponding potential wells are exceptionally shallow, leading to immediate evacuation. The spectral width of this robust frequency zone is explicitly scaled by the noise strength D and remains fundamentally independent of both the delay τ and coupling coefficient k.

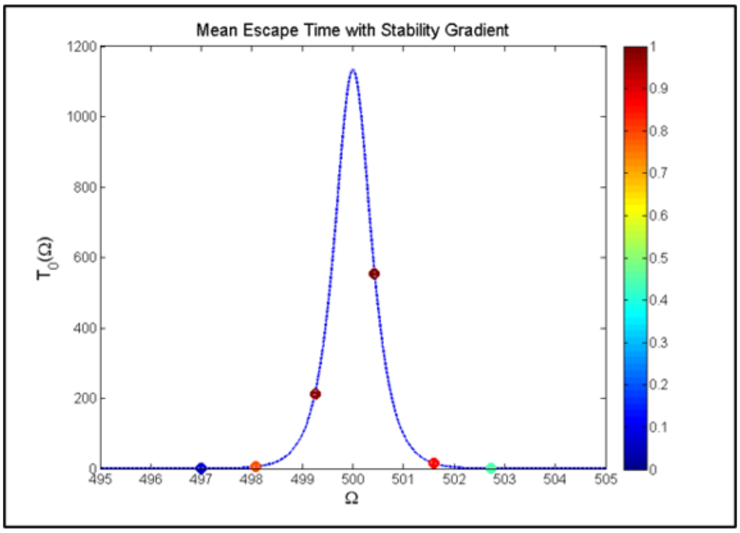

Figure 5: Mean escape time with stability gradient for parameters: $\Omega = 500MHz$, $k = 3$ and $\tau = 5\mu s$

8 RESULTS AND DISCUSSION: PARAMETRIC DEPENDENCIES AND TOPOLOGICAL TUNING OF MULTI-STABLE STATES

The structural evolution of the phase-locked orbital network under varying operational constraints is systematically unravelled by mapping the system's transcendental equilibrium profile across discrete coupling gains (k) and feedback path delays (τ). The collective visual evidence highlights that these two parameters serve as distinct topological controls that modulate the absolute quantity, spectral density, and local stability profiles of the coexisting steady-state frequencies.

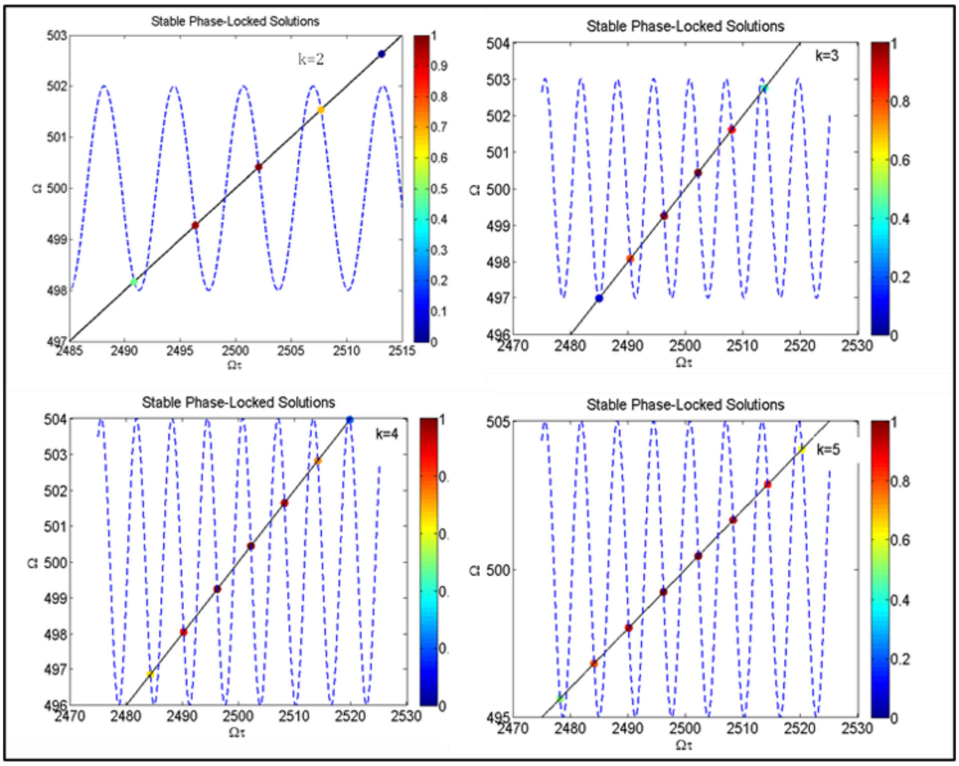

Figure 6: Stable phase locked frequencies for k = 2, 3, 4 & 5 with $\Omega = 500MHz$ and $\tau = 5\mu s$

Evaluating the system at a fixed long delay while stepping the feedback coupling coefficient through *k = 2, 3, 4,* and *5* reveals a distinct spectral widening mechanism. Mathematically, the amplitude of the cosmic feedback curve is directly proportional to k. Consequently, as k scales upward, the crests and troughs of the blue dashed modulation waveform expand vertically along the frequency (Ω) axis. This transformation modifies the geometric intersections with the linear state progression line by stretching the physical limits of the active frequency bands according to the boundary condition $\omega_0 - k \leq \Omega \leq \omega_0 + k$. This expansion allows peripheral modes to emerge at coordinates further removed from the natural frequency ($\omega_0 = 500\ MHz$). Looking at the right-hand color gradient, increasing k effectively deepens the potential energy wells of the central modes ($\Omega \approx 500.5$), which deepens their stability metrics toward a value of 1.0 (dark red markers), while simultaneously shifting the newly emerged outermost modes into highly volatile, low-stability regimes near 0.0 (dark blue markers).

In contrast to the vertical stretching driven by the coupling coefficient, adjusting the physical propagation delay time ($\tau = 2, 3, 4, 5\mu s$) under fixed coupling conditions applies a powerful horizontal compression across the phase space. Because the horizontal coordinate maps to the cumulative phase argument $\Omega\tau$, lengthening the time delay shortens the effective period of the cosine modulation relative to the frequency axis Ω. This geometric compaction forces the linear sweep line to puncture through a rapidly increasing number of harmonic waves, driving up the total count of coexisting phase-locked orbits. At long delays ($\tau = 5$), the slope of the intersection line becomes shallow, causing the localized mode solutions to cluster near the ideal resonance peaks defined by $\Omega\tau \approx 2m\pi$. The system transitions from a sparse, highly stable low-order regime ($\tau = 2$, showing fewer coexisting frequencies) into a high-density, multi-stable state environment ($\tau = 5$). This progression confirms that the product $k\tau$ serves as the foundational parameter governing high-dimensional chaos and state capacity in SAW delayed-feedback architectures.

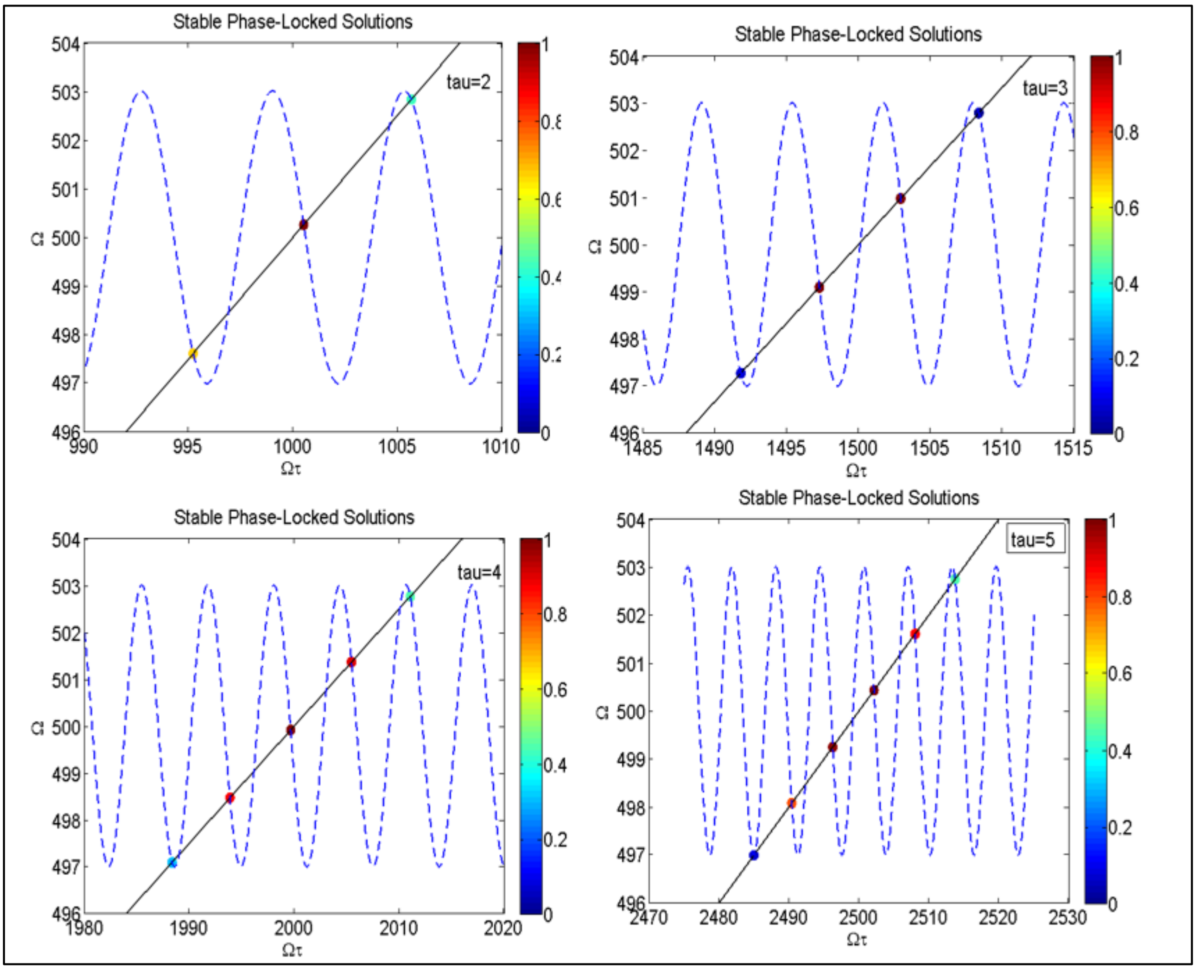

Figure 7: Stable phase locked frequencies for τ = 2, 3, 4 & 5μs with $\Omega = 500MHz\ and\ k = 3$

9 CONCLUSION

This study investigated the influence of additive white noise on the dynamics of a surface acoustic wave (SAW) delayed-feedback oscillator operating in a multistable regime. In the absence of noise, the system exhibits stable oscillatory states at discrete frequencies near the intrinsic central frequency of the SAW delay line. The introduction of additive white noise induces stochastic transitions among these coexisting states, resulting in random switching between different oscillatory frequencies. The statistical analysis of the frequency distribution and average residence time provides valuable insight into the relative stability and robustness of the individual oscillatory states. The results demonstrate that the residence time is significantly influenced by the delay duration, coupling strength, and frequency of the oscillatory orbit. In particular, states located closer to the central frequency of the SAW delay line exhibit longer average residence times, indicating greater resistance to noise-induced perturbations and enhanced dynamical stability. These findings highlight the important role of the frequency-selective properties of the delayed feedback system in determining its response to stochastic disturbances. The present work therefore contributes to a better understanding of noise-mediated multistability in SAW delayed-feedback oscillators and may provide useful guidance for designing oscillator and sensing systems with improved stability under noisy operating conditions.

Future research can extend this work by investigating the effects of different noise intensities and types, including colored and multiplicative noise, as well as fluctuations in the delay parameter. Further theoretical studies may establish quantitative scaling laws for residence times and transition rates using stochastic delay differential equation models. Experimental validation using practical SAW oscillator systems would also be valuable for examining the influence of real-world factors such as temperature variations, electronic noise, and device imperfections. In addition, future investigations could explore feedback-control strategies, multiple-delay configurations, and adaptive coupling mechanisms to suppress undesirable noise-induced switching and selectively stabilize preferred oscillatory states. Such developments may contribute to the realization of more robust, reliable, and high-performance SAW-based oscillators and sensing devices operating under stochastic conditions.

STATEMENTS ON COMPLIANCE WITH ETHICAL STANDARDS

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Disclosure of conflict of interest

The author(s) declare no conflict of interest to be disclosed.

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