World Journal of Advanced Engineering Technology and Sciences
eISSN: 2582-8266
Cross Ref DOI: 10.30574/wjaets
Journal homepage: https://wjaets.com/

(REVIEW ARTICLE)

Check for updates

UNDERSTANDING MATHEMATICAL EQUIVALENCE ACROSS EDUCATIONAL CONTEXTS: A NARRATIVE REVIEW OF EQUAL-SIGN CONCEPTIONS, INCLUSIVE EARLY ALGEBRA AND TEACHERS' PEDAGOGICAL KNOWLEDGE

Konstantinos Angelopoulos [1,*], Garyfalia Charitaki [1], Mesut Ozturk [2] and Fatma Erdogan [3]

[1] Department of Special Education, University of Thessaly, Volos, Greece.
[2] Department of Mathematics and Science, Bayburt University, Bayburt, Türkiye.
[3] Department of Mathematics and Science, Firat University, Elazig, Türkiye.
* Corresponding Author

ORCID Details
Konstantinos Angelopoulos: https://orcid.org/0009-0009-6795-8516
Garyfalia Charitaki: https://orcid.org/0000-0002-1593-2041
Mesut Ozturk: https://orcid.org/0000-0002-2163-3769
Fatma Erdogan: https://orcid.org/0000-0002-4498-8634

World Journal of Advanced Engineering Technology and Sciences, 2026, 21(02), 255–261

Publication history: Received on 09 October 2026; revised on 18 November 2026; accepted on 20 November 2026

Article DOI: https://doi.org/10.30574/wjaets.2026.21.2.0503

Copyright © 2026 Author(s) retain the copyright of this article. This article is published under the terms of the Creative Commons Attribution License 4.0

ABSTRACT

The meaning students assign to the equal sign shapes their progression from arithmetic to early algebra. This narrative review synthesizes literature on operational, relational and substitution-based conceptions of mathematical equivalence, recurring patterns of student errors, curriculum and textbook influences, and teachers' pedagogical content knowledge. Particular attention is given to inclusive mathematics education and the limited evidence concerning students with intellectual disabilities. Cross-national examples, including Greece and Türkiye, demonstrate why instructional opportunities and assessment practices require contextual interpretation. The review proposes an integrative framework linking representational experiences, student conceptions, diagnostic assessment and responsive teaching. It distinguishes findings established in general or mathematics-difficulty samples from hypotheses that still require direct testing in intellectual-disability populations. Priorities include accessible assessment, cross-cultural measurement comparability, and coordinated student–teacher research. The synthesis informs inclusive lesson design while avoiding unsupported claims of universal effectiveness.

Keywords: Early Algebra; Equal Sign; Inclusive Mathematics Education; Intellectual Disability; Mathematical Equivalence; Pedagogical Content Knowledge

1 INTRODUCTION

The equal sign (=) is among the most familiar symbols encountered in school mathematics, yet familiarity does not guarantee conceptual understanding. Many pupils learn to read a statement such as 7 + 5 = 12 as a command to perform an operation and record its answer. Algebra, however, demands that an equation be understood as a statement about two expressions with equal value. This shift from a one-directional computational convention to a relational interpretation is foundational for recognising equivalent expressions, transforming equations and reasoning about unknowns [1, 2]. The transition becomes especially visible in noncanonical statements such as 8 + 4 = □ + 5, where an operational interpretation often impedes a correct solution.

Contemporary accounts extend beyond the simple operational–relational distinction. They consider the interpretation of equality as sameness of value, the substitutability of equivalent expressions, and the coordination of procedural activity with mathematical structure [3, 4]. The relevant question is therefore not merely whether pupils can compute the value of an expression, but whether they recognise why a transformation preserves equality and when two expressions may replace one another.

Opportunities to develop this understanding are unevenly distributed across textbooks, classrooms and national settings. Moreover, evidence about students with mathematics difficulties cannot automatically be generalised to students with intellectual disabilities. A review that connects the conceptual, curricular and pedagogical strands can clarify what is known and what remains to be investigated. This article brings together the literature assembled in a Greece–Türkiye research proposal, but treats the proposed investigation as a future research direction rather than as a source of empirical findings.

The aims are to: (i) analyse major conceptions of mathematical equivalence; (ii) synthesise characteristic errors and their diagnostic meaning; (iii) examine how curriculum, representation and cross-national context may influence learning; (iv) consider the evidence and limits of inclusive instructional approaches; and (v) develop an integrative agenda linking students' understanding and teachers' pedagogical content knowledge (PCK).

2 REVIEW APPROACH AND SCOPE

This is a critical narrative review, not a systematic review or meta-analysis. Its source base is the scholarly literature identified in the supplied research proposal, including theoretical work, empirical investigations, textbook analyses, assessment studies and scholarship on teacher knowledge. References are organised thematically to support an interpretive synthesis; no comprehensive database search, formal quality appraisal or effect-size aggregation is claimed. This distinction matters because the coverage is informative but cannot establish prevalence estimates, exhaustive evidence mapping or pooled intervention effects.

The discussion focuses on late primary and lower secondary education, particularly the transition to algebra, while incorporating earlier-grade findings where they explain later conceptions. Studies from different educational systems are interpreted as context-specific evidence. Intellectual disability, general mathematics difficulty and other special educational needs are treated as distinct categories rather than interchangeable research populations.

3 CONCEPTUAL FOUNDATIONS OF MATHEMATICAL EQUIVALENCE

3.1 Operational, relational and structural meanings

In an operational conception, the equal sign functions as a cue that an answer should follow a calculation. This convention is reinforced when pupils repeatedly encounter expressions of the form a + b = c. By contrast, a relational conception identifies a statement of equivalence between the expressions on either side of the symbol. It remains valid when the order is reversed (12 = 7 + 5), when operations appear on both sides (8 + 4 = 7 + 5), or when a missing value occupies a nonterminal position [1, 5]. Structural understanding additionally attends to the mathematical relationships that make transformations possible without requiring every intermediate arithmetic calculation.

This distinction should not be treated as a binary label for an entire learner. A pupil may correctly identify equality in a familiar arrangement while reverting to an operational strategy with a new expression. The task format, quantities involved, symbols and elicitation method can all influence which understanding is expressed. Studies of learners' explanations therefore offer important interpretive information alongside right–wrong scores [4, 6].

3.2 Sameness, substitutability and the development of algebraic reasoning

A statement that two expressions have the same value is related to, but not identical with, understanding that one may be substituted for the other in a larger expression. Recent instruction-oriented work distinguishes these conceptions and suggests that combining them can better support reasoning about algebraic transformations [3]. The substitution conception requires learners to preserve meaning when an expression is replaced by an equivalent form; this principle underlies equation solving as well as simplification and generalisation [4].

Theoretical work on reification conceptualises mathematical learning as a progression through interiorisation, condensation and the apprehension of processes as objects [7]. Applied to equivalence, this perspective helps explain why learners may fluently execute arithmetic procedures yet struggle to regard an expression as a manipulable mathematical entity. The relationship is theoretically suggestive rather than a deterministic developmental pathway: learners may display several forms of thinking concurrently, and teaching can make structural relationships more or less visible.

3.3 Beyond a single test score

The Mathematical Equivalence Assessment (MEA) brings together diverse types of items, including open equations, judgements about equation structure and interpretations or definitions of the equal sign [8]. Research examining its structure across national samples supports a multidimensional approach rather than assuming that all items measure an undifferentiated skill [9]. It follows that classroom assessment should attend both to performance on equation tasks and to verbal or representational evidence of what learners think equality means. A correct answer obtained through a fragile counting trick may not demonstrate the same conceptual resources as an explanation based on balancing two expressions.

4 RECURRING ERRORS AS EVIDENCE OF UNDERLYING CONCEPTIONS

Noncanonical equations are diagnostically useful precisely because their structure challenges the familiar "calculation = answer" template. In 8 + 4 = □ + 5, the value of the missing term is 7, yet a learner may write 12 because the left-hand sum is interpreted as the answer required immediately after the equal sign. Descriptions of this "answer after the equal sign" response, alongside the summing of all visible numbers and related procedures, recur in studies across educational settings [5, 6, 10]. These patterns show why raw accuracy alone may conceal qualitatively different forms of reasoning.

Another issue is the use of the equal sign to connect a sequence of calculations rather than equivalent expressions, as in 5 + 9 = 14 ÷ 2 = 7 × 3 = 21. The written chain records a procedural history, but the chained equality is mathematically false because its adjacent expressions do not all have the same value [11]. Such responses may reflect classroom conventions that reward getting an answer while placing less attention on the semantics of the notation. Teacher feedback can therefore address the meaning of every equality sign, not only the final numerical result.

Error classification must remain cautious. A response may arise from inattention, task-language demands, arithmetic load or unfamiliar notation as well as from a persistent misconception. Think-aloud protocols, probing questions and varied item formats can help distinguish these possibilities. More nuanced interviews with middle-school learners have likewise been used to examine how interpretations change across arithmetic and algebraic contexts [12]. The pedagogical implication is diagnostic: errors should be treated as hypotheses about reasoning to be tested, not as fixed labels for pupils.

5 TEXTBOOKS, REPRESENTATIONS AND CROSS-NATIONAL CONTEXTS

5.1 The instructional ecology of the equal sign

Children do not encounter mathematical notation in isolation. The frequency and position of particular equation forms, representations of balance, teacher discourse and exercises in school materials create opportunities to notice some mathematical relationships more readily than others. Work on textbook presentation highlights how a high proportion of canonical a + b = c expressions can normalise an operational reading, whereas comparison, reversed equations and multi-term equalities make relational meanings more explicit [13, 14]. The instructional implication is not to eliminate canonical forms, which remain mathematically useful, but to diversify structures and connect them through explicit discussion.

The Greek textbook analysis cited in the source proposal reports that 88.1% of relevant Grade 1 instances used canonical equation formats after an initial introduction involving comparison [13]. This is a finding about the specific analysed textbook material, not a national estimate of students' conceptual knowledge. Other analyses identify context-dependent uses of the same symbol across mathematics and natural science materials, including notation that functions

as a linguistic shorthand rather than an exact mathematical relation [15]. These differences warrant attention when pupils are expected to transfer meaning between subjects.

5.2 International examples and limits of comparison

Literature discussed in the proposal includes work from China, Sweden, Norway, South Africa, the United Arab Emirates and Türkiye. The Chinese example describes early encounters with equality alongside comparison signs, which may provide different initial representational opportunities [16]. Swedish and Norwegian studies investigate learners' interpretations of the symbol in primary or lower-secondary contexts [11, 17]. Studies from South Africa and the United Arab Emirates document interpretive difficulties and underscore the importance of classroom context when comparing response patterns [5, 10, 18]. These examples illustrate heterogeneity; they do not establish an internationally ranked hierarchy of teaching quality.

For Greece and Türkiye, parallel examination of learning materials, tasks and classroom practices could be especially useful. Turkish middle-school investigations address interpretations of equality [12, 19], whereas Greek textbook research highlights the distribution of equation formats and the potential mismatch between intended conceptual meaning and repeated notation [13]. Direct cross-country conclusions about pupils with intellectual disabilities cannot be drawn from these separate sources. Such claims would require harmonised sampling, accessible instruments, careful translation and evidence that scores function comparably across groups.

6 INCLUSIVE EARLY ALGEBRA AND LEARNERS WITH INTELLECTUAL DISABILITIES

Research on mathematical equivalence has been concentrated in general education and in samples identified by mathematics difficulty. Yet inclusion requires asking whether learners with intellectual disabilities can access the relational structure of numerical statements and what forms of support enable them to communicate that understanding. These questions should not be replaced by assumptions that students with intellectual disabilities are limited to procedural arithmetic. Equally, the available source literature does not establish a standard developmental trajectory or a universally effective intervention for this specific population.

An intervention study by Powell and Fuchs examined explicit equal-sign instruction for third-grade pupils with mathematics difficulties and found evidence relevant to the value of focused instructional attention [20]. Importantly, this study does not constitute direct evidence about children with diagnosed intellectual disabilities. Its strategies—explicit mathematical language and carefully selected equation formats—provide potentially transferable design ideas, but transfer must be evaluated, not presumed. The same caution applies to visual balance models, diagrams, concrete manipulatives and realistic contexts discussed in the proposal: they are pedagogically plausible tools for reducing linguistic and working-memory demands, but their effectiveness depends on learner profiles and implementation.

Accessible teaching can use paired physical quantities, balanced scales, matching expressions, colour-supported comparisons and stepwise transitions from objects to symbols. However, visual aids should not become substitutes for mathematical meaning. Teachers need to connect each representation to the invariant claim that both sides have equal value, and to assess whether the learner can move between representations. The range of response modes can include oral explanation, pointing, selecting a matching representation, constructing an equality with objects or using augmentative communication, when appropriate.

Assessment accessibility and construct validity are closely linked. Simplifying instructions, changing response modes or reducing reading load may improve access, but adaptations can also change what an item measures. An inclusive assessment design should therefore document modifications, examine item comprehension through cognitive interviews and test whether measures remain interpretable within and across groups. The current evidence gap is not merely one of small samples; it concerns the absence of sufficiently validated ways to distinguish numerical competence, language demands and relational understanding in students with intellectual disabilities.

7 TEACHERS' PEDAGOGICAL CONTENT KNOWLEDGE AND RESPONSIVE INSTRUCTION

Teachers' pedagogical content knowledge involves understanding not only mathematical relationships but also how they are represented, misunderstood and taught. Ball, Thames and Phelps describe specialised forms of mathematical knowledge for teaching that provide a useful foundation for analysing instructional decisions [21]. In the context of equivalence, teachers need to recognise what is mathematically wrong with a false chain of equalities, anticipate why a pupil writes 12 in the blank of $8 + 4 = \square + 5$, and choose a follow-up that reveals the balancing relation rather than merely the answer.

Studies of teachers' understanding suggest that possessing a relational conception personally does not guarantee that it is made visible in instruction [22]. Teachers can inadvertently reinforce an operational interpretation when examples

are uniformly canonical or explanations portray the sign as "the answer comes next." Conversely, deliberate counterexamples, discussion of reversibility and requests to justify substitutions can foreground mathematical structure. Work on teachers' reasoning about assumptions further suggests the productive role of making implicit premises discussable [23].

For inclusive classrooms, PCK must also include knowledge of accessible representations and differentiated means of participation. A teacher who interprets an incorrect response solely as a calculation deficit may overlook the conceptual or linguistic source of difficulty. Effective diagnostic questioning can separate "What is the sum?" from "How do you know both sides are equal?", while structured comparison of equivalent and nonequivalent expressions helps learners attend to the symbol itself. This is a proposed synthesis of pedagogical principles, not the outcome of an experimentally validated teacher-development programme.

8 AN INTEGRATIVE FRAMEWORK FOR RESEARCH AND PRACTICE

The literature suggests a four-part framework linking (a) representational opportunities in curriculum and teaching, (b) students' operational, relational and substitution-based conceptions, (c) task- and response-level diagnostic evidence, and (d) teacher interpretation and instructional response. Feedback is central: the teacher's choice of tasks shapes the opportunities learners encounter, while learners' responses provide evidence for adjusting those tasks. Accessibility affects each component, including the choice of materials, wording, response mode and assessment interpretation.

This framework is an interpretive proposal derived from the reviewed material; it is not presented as an empirically tested causal model. Its value lies in making the connections among research strands explicit and in identifying observable questions that future studies can address. For instance, researchers can test whether teachers' ability to diagnose equivalence errors predicts the diversity of tasks they provide, and whether such task diversity relates to students' conceptual explanations after controlling for age, arithmetic fluency and context.

Table 1: Components of the proposed integrative framework

Component	Illustrative evidence	Research or teaching implication
Representation and context	Canonical/noncanonical equations, balance models, curriculum language	Audit the range of representations and make equivalence explicit
Student conception	Operational answers, relational explanations, valid substitutions	Assess multiple item types and explanatory responses
Accessible assessment	Task language, response mode and cognitive interview evidence	Document adaptations and examine comparability
Teacher PCK and response	Error diagnosis, questioning and differentiated instruction	Study the link between teacher knowledge and learner reasoning

Note: The framework is a conceptual synthesis from the literature reviewed here; it has not been independently validated.

9 RESEARCH AGENDA AND METHODOLOGICAL CONSIDERATIONS

A coherent next step is to examine mathematical equivalence and teacher knowledge together in coordinated studies of general and special education. The supplied Greece–Türkiye proposal outlines a sequential explanatory mixed-methods approach, initially assessing students and teachers quantitatively and subsequently exploring selected responses through interviews. This is a research plan, not an implemented study. Proposed student measures include the MEA, while teacher questionnaires and semi-structured interviews would investigate PCK, instructional strategies and differentiation [8, 21].

Cross-cultural comparison must establish more than linguistic translation. Multi-group confirmatory factor analysis can be used to examine configural, metric and scalar properties, although appropriate models and sample sizes depend on the data and item structure. Prior work has investigated the measurement invariance of mathematical equivalence assessment across six countries [9], providing a methodological precedent rather than a guarantee of equivalence between Greece and Türkiye or among disability groups. Comparisons of latent means should be conditioned on defensible evidence of measurement comparability.

Future sampling should explicitly specify how intellectual disability is identified, avoid grouping it indiscriminately with other special educational needs, and document relevant accommodations. Inclusive interviewing could use adapted

think-aloud tasks and multimodal prompts, supported by ethical approvals, parental or guardian consent where required, and developmentally appropriate learner assent. The proposed integration of error-pattern analysis with qualitative thematic analysis could illuminate when a seemingly identical answer reflects different conceptual processes [24]. The design should also address nested school–classroom structures if sample characteristics permit.

Four priorities emerge: establish accessible and conceptually interpretable assessment procedures; identify mechanisms, rather than simply correlations, connecting representations to relational understanding; investigate teachers' diagnostic PCK in inclusive settings; and distinguish cultural or curricular differences from artifacts of measurement. Intervention trials should measure transfer to unfamiliar equation forms and substitutions, not merely improvement on practiced questions. Such work could inform curricula and teacher development without assuming that one instructional sequence fits every learner.

10 DISCUSSION AND LIMITATIONS

Across the cited literature, a common thread is that the equal sign is a relational statement even when classroom routines present it primarily as a computational prompt. Task variation and explanation are therefore central to understanding learners' reasoning. But the synthesis also identifies important distinctions: sameness is not synonymous with substitution, a correct answer is not necessarily conceptual mastery, and an accessible test is not automatically measurement-equivalent across populations. These distinctions matter particularly when general-education findings are used to motivate hypotheses for learners with intellectual disabilities.

The evidence assembled here is limited in several ways. The source set comes from an existing proposal rather than an exhaustive search, so publication coverage and selection bias cannot be ruled out. Included studies span different countries, grades, designs and definitions; direct quantitative comparison is therefore not justified. Some claims about inclusive strategies are theoretical or extrapolated from mathematics-difficulty samples, not demonstrated specifically for students with intellectual disabilities. Finally, proposed teacher–student links and Greece–Türkiye contrasts remain research questions until supported by empirical data.

11 CONCLUSION

Understanding mathematical equality requires more than calculating an answer: learners must recognise equivalent expressions, justify transformations and, ultimately, use substitution in algebraic reasoning. Textbook formats, representations and teacher practices can either reveal or obscure this structure. A productive inclusive agenda combines diversified equation tasks, accessible multimodal assessment and teachers' informed interpretation of student errors. The literature supports the importance of these connections but leaves direct evidence concerning students with intellectual disabilities and cross-national comparability notably incomplete. The proposed integrative framework offers a basis for focused, ethically designed research and for reflective development of inclusive early-algebra instruction.

STATEMENTS ON COMPLIANCE WITH ETHICAL STANDARDS

Acknowledgments

This research is conducted in the operating framework of the University of Thessaly Innovation, Technology Transfer Unit and Entrepreneurship Center "One Planet Thessaly", under the "University of Thessaly Grants for Scientific Publication Support" action and is funded by the Special Account of Research Grants of the University of Thessaly.

Funding Source

This research was funded by the Special Account of Research Grants of the University of Thessaly, under the "University of Thessaly Grants for Scientific Publication Support" action.

Disclosure of conflict of interest

The authors declare no conflict of interest.

REFERENCES

[1] Knuth EJ, Alibali MW, Hattikudur S, McNeil NM and Stephens AC. (2008). The importance of equal sign understanding in the middle grades. *Mathematics Teaching in the Middle School,* 13(9), 514–519.

[2] Chimoni M, Pitta-Pantazi D and Christou C. (2018). Examining early algebraic thinking: insights from empirical data. *Educational Studies in Mathematics*, 98, 57–76.

[3] Donovan AM, Stephens A, Alapala B, Monday A, Szkudlarek E, Alibali MW and Matthews PG. (2022). Is a substitute the same? Learning from lessons centering different relational conceptions of the equal sign. *ZDM – Mathematics Education,* 54, 1199–1213.

[4] Ntow FD, Adom G, Essel S and Kenney S. (2024). Students' understanding of the equal sign when solving arithmetic tasks. *African Journal of Educational Studies in Mathematics and Sciences, 20(2),* 293–306.

[5] Essien A and Setati M. (2006). Revisiting the equal sign: Some Grade 8 and 9 learners' interpretations. African Journal of Research in Mathematics, *Science and Technology Education,* 10(1), 47–58.

[6] Alexandrou-Leonidou V and Philippou G. (2007). Elementary school students' understanding and use of the equal sign. In: *Pitta-Pantazi D and Philippou G (Eds), Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education*, 825–834.

[7] Sfard A and Linchevski L. (1994). The gains and the pitfalls of reification: The case of algebra. *Educational Studies in Mathematics*, 26, 191–228.

[8] Matthews P, Rittle-Johnson B, McEldoon K and Taylor R. (2012). Measure for measure: What combining diverse measures reveals about children's understanding of the equal sign as an indicator of mathematical equality. *Journal for Research in Mathematics Education*, 43(3), 316–350.

[9] Simsek E, Jones I, Hunter J and Xenidou-Dervou I. (2021). Mathematical equivalence assessment: Measurement invariance across six countries. *Studies in Educational Evaluation*, 70, 101046.

[10] Wardat Y, Jarrah AM and Stoica G. (2021). Understanding the meaning of the equal sign: A case study of middle school students in the United Arab Emirates. *European Journal of Educational Research*, 10(3), 1505–1514.

[11] Madej L. (2022). Primary school students' knowledge of the equal sign—the Swedish case. *International Journal of Science and Mathematics Education*, 20 (2), 321–343.

[12] Türk A and Öztürk M. (2026). How to conceptualize the equal sign by middle school students? *Mathematics Education Research Journal*. https://doi.org/10.1007/s13394-026-00573-4.

[13] Voutsina C. (2014). An analysis of the presentation of the equals sign in Grade 1 Greek textbooks. In: Jones K (Ed.), *Proceedings of the International Conference on Mathematics Textbook Research and Development (ICMT-2014)*, 483–488. University of Southampton, Southampton, UK.

[14] Pang J and Lee Y. (2025). The equal sign and equivalence: Interplay between textbook research and curriculum development. *ZDM – Mathematics Education*. https://doi.org/10.1007/s11858-025-01707-x.

[15] Moutsios-Rentzos A, Pinnika V, Kritikos G and Kalavasis F. (2020). Appearances of the equals sign in primary school mathematics and natural sciences: An interdisciplinary, systemic approach. *Quaderni di Ricerca in Didattica*, 7, 285–294.

[16] Sun J and Gu Y. (2023). A further investigation to introducing the equal sign in China. In: Reid-O'Connor B, Prieto-Rodriguez E, Holmes K and Hughes A (Eds.), *Weaving Mathematics Education From All Perspectives: Proceedings of the 45th Annual Conference of the Mathematics Education Research Group of Australasia*, 483–491. MERGA, Newcastle, Australia. ISBN: 978-1-920846-34-3.

[17] Opsal H. (2019). How students in 5th and 8th grade in Norway understand the equal sign. *Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education.*

[18] Machaba F and Makgakga S. (2016). Grade 9 learners' understanding of the concept of the equal sign: A case study of a secondary school in Soshanguve. In: *Towards Effective Teaching and Meaningful Learning in Mathematics, Science and Technology Education*. Proceedings of the 9th Annual ISTE Conference.

[19] Özdemir E. (2022). Middle school students' understanding of equal sign. *Shanlax International Journal of Education*, 10(4), 23–38.

[20] Powell SR and Fuchs LS. (2010). Contribution of equal-sign instruction beyond word-problem tutoring for third-grade students with mathematics difficulty. *Journal of Educational Psychology*, 102(2), 381–394.

[21] Ball DL, Thames MH and Phelps G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389–407.

[22] Vermeulen C and Meyer B. (2017). The equal sign: Teachers' knowledge and students' misconceptions. *African Journal of Research in Mathematics, Science and Technology Education*, 21(2), 136–147.

[23] Stylianides GJ and Stylianides AJ. (2024). Promoting elements of mathematical knowledge for teaching related to the notion of assumptions. *Mathematical Thinking and Learning*, 26(4), 382–410.

[24] Braun V and Clarke V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology*, 3(2), 77–101.